A TREATISE ON SURVEYING

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SURVEYING

COMPILED BY

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PREFACE.

In the United Kingdom, and with Ordnance Maps to various scales, and more or less corrected to date, always at hand, the surveyor seldom, if ever requires to put into practice a knowledge of high class, or Geodetic Surveying. The result is, that diplomas are granted to students who possess but a very limited knowledge of this class of work.

In the principal Colonies, however, these conditions do not obtain, and their governments, not being satisfied with the limited acquirements of many English surveyors, insist on a local training, or apprenticeship and the possession of one of their own diplomas. This restriction has, in many cases, proved somewhat arbitrary.

About three years ago, a number of gentlemen interested in this question, and in the improvement of the standard of qualifications for English diplomas as Surveyors, met at the Surveyors' Institution by kind permission of the Secretary, and formed a committee to consider what steps might best be taken to secure a better position for the English student who might wish to seek employment in one of our Colonies.

The Council of the Surveyors' Institution was approached, and their Secretary, Mr. Julian Rogers, informed the committee that their members were prepared to look favourably on any efforts made in the direction indicated.

It was arranged that the committee should prepare, and submit a Text-Book, which the Council agreed to adopt, if satisfied with the same.

The present Treatise on Surveying is the result, and in order to meet the requirements of a body of students who may seldom, if ever, practise the art of Geodetic Surveying, but to whom a knowledge of the practices of ordinary surveying is a necessity, the work has been divided into two Parts, so that problems requiring Geodetic treatment, and Astronomical determinations, as well as the manner of Conducting Marine, Route, and other special surveys, are discussed in the Second Part.

The Authors of this work make no general claim to originality, since most of the information given is to be found in some one or other of the many existing treatises on surveying; but, so far as they know, no one book contains in a practical form the information now offered to the student.

In addition to the various methods of surveying limited areas, as usually practised, every effort has been made to point out clearly, the degree of accuracy which can readily be obtained with the appliances available, and which is essential with regard to the purpose for which the survey is undertaken.

Further, the effects of different sources of error (avoidable, or otherwise) on the ultimate accuracy of the work, have been discussed, so that the surveyor may decide how advisedly to apportion the time and care that he should bestow on the various processes of his work. Some space has also been devoted to the considerations which affect the distribution of residual errors, in a simple yet systematic manner.

The Authors beg to offer the above remarks as their justification for submitting this work to the verdict of public opinion, and whilst making no claim to having compiled a complete treatise on so large a subject as surveying, still it is hoped that all the information necessary to enable a student to acquire the knowledge required of a qualified surveyor is to be found in it.

The following is a list of the Contributors; and it is deeply to be regretted that the sections written by Mr. LEANE (who died at sea when on his return from Africa) and by Major-General WOODTHORPF. C.B.

(appointed Surveyor-General of India just before his death) have had to be produced without the advantage to the Editor of being able to submit proofs for their revision :—

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The thanks of the Authors are due to Mr. M. T. ORMSBY, Demonstrator in Surveying at University College, Gower Street, London, for assisting in the editing of the work, and in the preparation of drawings, tables, &c. It is believed that all matter taken from other sources has been acknowledged in the text.

PREFACE TO THE SECOND EDITION.

IN the present Edition it will be found that the clerical errors which existed in the First have been corrected, and a few Appendices have been added dealing with some omissions which had inadvertently occurred and escaped the notice of the compilers.

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CHAPTER I.

CHAIN SURVEYING.

Introductory Remarks. LAND SURVEYING with the chain, (with the addition of a cross staff, offset staff, a few poles, and the occasional use of a rough kind of clinometer) was at one time almost the only method of making plans employed, and it still is, and must ever be, an essential part of a surveyor's education.

A method by which the necessary amount of accuracy may be obtained, is given in the following pages, together with an approved form of field-book.

It is, of course, impossible to become a reliable surveyor without a certain amount of instruction and practice in the field, but if the student will follow, step by step, the directions here given, in conjunction with a careful study of the plans and field books, and himself lay down the lines, and plot the work in the examples illustrated, he should be prepared to make a small survey on his own account.

The Chain. Chains for surveying purposes are of various kinds and of different lengths, but whatever unit of measure is adopted, whether it be the foot, the link, or part of a metre (usually 2 decimetres), the chain is subdivided into 100 equal parts, each tenth part being marked by a piece of brass of a particular shape, to indicate its distance from either end.

If a chain is spoken of without reference to the unit of measure, Gunter's chain of 66 feet divided into 100 links is meant, this being the most convenient for land surveying.

Its advantages are, that it is an exact decimal part of an acre, and of a mile, there being 10 square chains in an acre, and 10 lineal chains in 1 furlong, of which there are 8 to a mile.

The Steel Tape. Steel tapes have to a large extent superseded the old type of chain, especially for work requiring great accuracy, and for many reasons, are to be preferred. They are much lighter to handle, are not liable to vary in length by use (except by wear in the handles), and can be quickly and easily rolled up into the case provided for that purpose.

The principal objections urged against them are, that in case of breakage, they cannot be so readily repaired in the field as the ordinary chain, that somewhat greater care has to be taken to avoid kinking, and that when used upon stiff, wet, clayey soils, the brasses soon get obscured by soil adhering to them. This last objection is, however, common to all chains, and as the steel tapes are extremely light the difficulty may be got over by carrying them bodily off the ground in such cases.

Arrows, or Pins. Arrows, or pins, are used to mark the end of each chain, and are made of stout iron or steel wire, about 15 inches in length, one end being formed into a ring for facility of handling, and to which a piece of red tape is fixed. Ten such arrows are generally used, but this is not always the case, as some surveyors prefer to work with only nine, while others find virtue in using eleven, but in every case the object is, to obtain a correct record of the number of chains run, and especially of every tenth chain.

It is convenient to allude to the person drawing the chain as the 'leader' and to the person following and directing it as the 'follower.'

Those who use nine arrows allow the 'leader' to extend the tenth chain without any arrow, and make him hold the handle of the chain in position until the 'follower,' arriving at the leading end, places his offset staff at the end of the chain, instead of an arrow, and giving up the nine arrows in his hand allows the 'leader' to proceed as before.

Those who work with ten arrows instruct the 'leader' to mark as usual with the tenth pin, and to wait until the 'follower' substitutes his rod for it. While the 'follower' is coming forward it is well for the 'leader' to mark the end of the tenth chain by means of a peg or stick for future reference.

It is a great convenience to have this tenth arrow of special make, suitable for dropping, instead of a plumb-bob, when measuring on steep places or over banks and fences, also to insert on occasions when it is necessary to mark a portion of a chain instead of the whole.

It is obvious that this arrow must never be retained by the 'follower.'

Those who use eleven arrows, allow the 'leader' to lay out and mark the eleventh chain. The 'follower' then gives up the ten arrows in his hand, but the eleventh remains and counts as the first chain in the next set of ten.

It may, however, be considered as fairly established, that, taken all round, ten is the most convenient number of arrows to use, particularly if the tenth be of special pattern as before mentioned.

Offset Rod. The offset rod, is a light rod shod with iron (fig. 1), usually ten links in length, and marked at each link. It is used for taking short offsets and for marking positions, and should be formed at the top into a hook of such construction that the handle of the chain may, with equal facility, be pushed or pulled through a fence with it.


FIG. 1.

Ranging Rods. Ranging rods are light poles shod with iron, painted and marked in feet or links, and used for ranging straight lines, or as flag poles to mark important stations upon the survey lines. When using a Gunter's chain, 10 links is a useful length.

Flags. Flags should be either red or white, or a combination of the two colours, and should be formed with a sleeve on one side into which to thrust a rod or pole, and furnished with a tape at the lower end of the sleeve with which to make it secure. These flags are less cumbersome and quite as conspicuous if made in the shape of a triangle, the sleeve side and the bottom forming a right angle.

The Optical Square. The optical square consists of a small brass box in which are fixed two mirrors, set at a permanent angle of 45° to each other, and so arranged that an object at 90° from the line will be reflected, and will coincide with the direct view of the object in the line of sight.

This instrument requires occasional testing and should be capable of adjustment by means of a screw which will slightly turn one of the mirrors. *Vide* Chapter on Instruments.

The Clinometer. The clinometer is an instrument by which the slope of the ground may be ascertained.

A very primitive form consisted of simply a piece of board or cardboard, upon which a quadrant, divided into degrees was marked.

The sight was taken along one side of the quadrant and the angle of inclination indicated by a small plumb-bob suspended from the centre.

A more convenient instrument (Watkins's pattern), is made in watch form. The degrees are marked upon a segment of a circle, balanced vertically by a weight, so that the zero has a tendency to keep in a horizontal position, and coincides with the zero of the line of sight when that is horizontal.

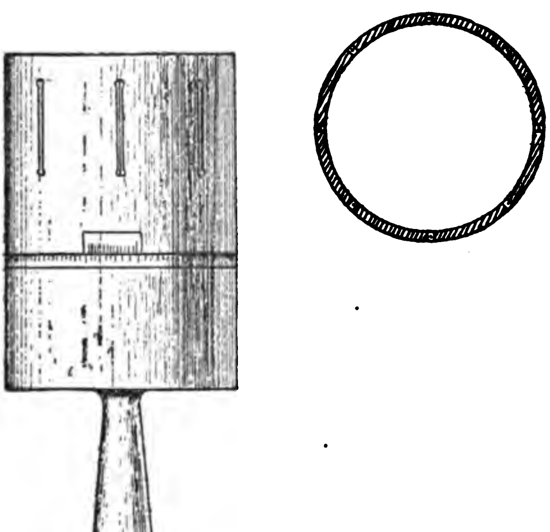


FIG. 2.

The point observed is seen through small apertures in the sides, between which a magnifying reflector is so adjusted that the angle of elevation or depression can be read off with considerable accuracy.

The Cross Staff. The old-fashioned cross staff head of wood, has been superseded by one of brass (*vide* fig. 2), with a pair of sights at right angles to each other, and another pair making angles of 45° with the first. Some again have the lower portion divided in a similar manner to a theodolite, and the upper portion made to revolve, thus admitting of angles of any number of degrees being laid off.

These heads are adapted to fit on special staves shod with iron, and about 5 feet in length. They are exceedingly useful in small intricate surveys.

Stations. Stations, are marks for reference, left upon the ground when setting out, or chaining, the main survey lines.

When the survey is a small one, these may be of a very temporary character, but if the lines being set out, or chained, form part of a large survey, care should be taken to select points for stations which are not likely to be disturbed by the plough, or by cattle.

A favourite method of making a mark, is to cut out a sod of turf in the form of an acute isosceles triangle, pointing in the direction of the line (for this purpose a light spade, or a chopper and spade combined, is carried). A peg should then be driven at the apex of the triangle. Such marks neatly made on meadow land, or on the turf at the sides of ditches or roads, may be easily found and identified months afterwards, but upon ploughed land, pegs, sticks, and mounds, are all that can be used.

Stations should mark the angles of all triangles chained, and also where subsidiary lines are likely to be required, or where main lines intersect. Here more than one mark should be cut, so that the apex of each triangle should point to the centre peg, and they should thoroughly indicate the directions of the off-going lines by the manner in which the triangles are disposed.

Chainmen. Much depends upon the choice of a suitable man to act as 'chainman,' or 'leader,' and to assist in ranging the lines, &c.

He should be active, intelligent, possessed of good eyesight, and capable of ranging a line with rods or sticks.

Such a man is not always easy to find in rural districts, but the selection of a suitable one will amply repay the surveyor for the trouble.

Woodmen, who are accustomed to range lines for fencing and tree-planting, frequently make the best of chainmen, and being experts in the use of the bill-hook do not mind carrying one in their pocket.

When it becomes necessary to cut through a fence or wood, such a man will do it with greater neatness, and more quickly than an ordinary labourer, and without causing any unnecessary damage. Moreover, he will in a minute or two repair the fence which one may have been obliged to cut, in a workmanlike manner, thus, probably obviating any unpleasant remarks from the owner.

A sharp boy who will assist in carrying rods, and running errands, is also a desideratum.

If the survey be one of considerable extent, it is well to train a second man to act as 'follower,' and to assist with offsets, so that the surveyor is free to attend to his offsets and booking. In this way the chain can be kept almost continually upon the move.

It is not intended, however, to deal here with the filling in of the detail of large surveys, but only with such work as a beginner may be supposed to undertake, such as the survey of estates and farms of moderate size.

Method of Chaining.

To chain a given line with accuracy and despatch, is not quite so simple a matter as the uninitiated might suppose, and proficiency can only be attained by practice. Accuracy first, despatch second, must be the rule.

Various detailed methods of chaining are advocated by surveyors, but a very good way, possibly as good as any, is for the 'follower' (in this instance the surveyor himself) to approach the arrow holding his handle of the chain in his right hand, his field-book in his left, his pencil or pen between the fingers of his left hand (or, if he fears losing it, tied by a string to a button of his coat), his offset rod under his left arm, and any arrows he may have already collected hanging upon the little finger of his left hand.

His right hand is thus left free to direct the chain, to use his rod, or to make notes in his book, as the case may require.

On arriving at the arrow he plants his right foot with the toes nearly touching it, his left foot somewhat to the rear, and his right hand holding the chain handle exactly in the line, slightly in advance of the arrow, and about three feet above it.

This attitude is well adapted to admit of attention, and to resist without exertion any sudden, or accidental pull, from the 'leader.'

The 'leader' is so trained that he at once faces the 'follower' and by the aid of some back object or mark, sets himself approximately in line, the chain being held sufficiently tight to appear straight.

The 'follower' by a movement of the thumb, and by a slight lateral motion imparted to the chain indicates his wish for the 'leader' to move a little to the right or the left, as the case may be, and when satisfied that the direction is perfect, he stoops down, thrusts his thumb through the loop, and brings the handle of the chain into contact with the arrow.

The 'leader,' seeing the 'follower' bending, also stoops down, and while the 'follower' holds his handle to the arrow, the 'leader' still keeping the chain moderately tight, thrusts in another arrow, and is ready to proceed again.

The 'follower' as he rises, brings up the arrow upon his thumb, and transfers it to the little finger of his left hand, as he walks along.

A little practice will enable the 'follower' to so hold his hand when directting the 'leader,' that as he stoops, he will bring it exactly to the arrow, without further movement either backwards or forwards, and thus ensure the best possible results in the quickest time.

Care must be taken that the 'leader' when stooping and thrusting in his arrow, does not deviate from the line given him, and the 'follower' must see to this before withdrawing his arrow.

The 'leader' should carry his arrows in his left hand, and draw one with his right hand as he goes along ready for use on arriving at the chain end, and must be careful that at the moment of thrusting it into the ground, the arrow is perfectly upright, and touching the handle of the chain.

The 'follower' should avoid all drag upon the chain when it is being pulled along, and be careful to check the 'leader' when he arrives at the proper distance, without jerk, which not only injures the chain but ruffles the 'leader's' temper.

In order to be capable of undertaking large surveys when occasion requires, every surveyor should accustom himself to chain long lines towards a distant object, without much setting out with rods, and with care, this can be accomplished as follows:—

Before commencing to chain a line the follower should plant his offset rod at its commencement, and retiring a few yards behind it, examine the ground between it and the distant fore-mark, for any well-defined objects that may exist in the exact line.

In cases where no such objects are exactly upon the line, there may be some very near to it, which will be useful to give a general indication of direction, and a little training to the eye will enable him to select and always identify more minute objects. Advantage should be taken of a gap, peculiarly shaped bush or bough in a fence, a conspicuous stone or clod, a thistle, a dock, or a tuft of grass, which may be in the exact line.

The first object, if small, should not be at too great a distance, and before it is passed, a careful review of the line should be made, and as new ground is opened up to the eye, fresh objects should be selected.

On moderately level ground, and with a well-defined fore-mark, such as a corner of a house, a church spire, or a single tree, lines of considerable length, up to half a mile, may be chained with perfect straightness, and without any previous setting out with rods.

The 'leader' must be well trained to select objects behind the starting point, marked by a flag, if necessary, to keep a constant check upon the direction, and should at once remark any accidental deflection from the true line.

Where no objects of sufficient prominence intervene between the starting point and the fore-mark, and especially, if there should be any offsets to take, the lad, or spare man, should be sent forward with a rod and a few prepared sticks. He will use the rod for the 'follower' to give him the correct line, and will then substitute a stick for it and go forward to another position. Chaining may proceed immediately the first stick is in place. It is not an easy matter to drive a line perfectly straight with only the one distant mark to go upon, and the best surveyor may be excused for getting his line a little twisted.

The natural tendency is to continually drive the line on one particular side, every man seeming to have a bias for one side or the other, so that the law of chances does not apply, by which, a nearly correct result is obtained from a multitude of small errors which balance one another.

Limit of Error Permissible.

If it were possible to chain with the utmost accuracy over all sorts of ground, under all conditions, and with reasonable expedition, then instrumental work would, to a large extent, be unnecessary.

With reasonable care, however, upon fairly level ground, and with a carefully tested chain, it should be possible to rely upon distances to within $\frac{1}{1000}$ part of the whole, or one link in 10 chains, but when rough or somewhat hilly ground has to be measured it is not easy to chain within $\frac{1}{500}$ or two links in 10 chains, while upon very hilly and rocky ground, or on bad boggy land, chaining by ordinary methods becomes unreliable. If great accuracy is necessary, some special means for securing it must be resorted to. In laying down lines which do not fit exactly, though within the margin of permissible error, care should be taken to spread the difference equally throughout the whole line. Thus, if a line measures 10,000 units on the ground, and there is room on the plan for 10,010, then it is evident that each 1000 mark should be plotted off as 1001.

Inaccuracies in Chaining. Inaccuracies in chaining may result from many causes, some of which are here indicated.

1. Through the chain not being hauled fairly taut at the moment that the 'leader' puts in the arrow.

2. Through the 'leader' not holding his arrow exactly at the chain end, or not having it exactly perpendicular when thrusting it into the ground.

3. Through the 'follower' not bringing his end of the chain fairly to the arrow, or allowing the 'leader' to drag his arrow out of the perpendicular.

4. Through kinks or knots in the chain, especially upon first opening it out.

5. Through inattention to the variations in length to which an ordinary chain is liable, either through wear in the many joints, or through the stretching or opening of the links, and want of care in testing for length.

6. Through inaccurate allowance upon sloping ground, or want of care in chaining over obstacles such as rocks, boulders, or fences.

7. Through not keeping in proper alignment.

8. Through variations in temperature from which the chain expands or contracts.

Though this last should not be lost sight of, still it is trifling when compared with the preceding causes of error, as will be seen by the following remarks.

A variation of 80° Fahr., which will more than cover the ordinary range of winter and summer temperature in the British Isles, makes a difference of about $\frac{1}{1800}$ part, or ·44 of an inch in a 66-foot chain.

Standards of length, are accurate at about 60° Fahr., the mean temperature between our extremes of heat and cold, so that errors from this cause can scarcely ever exceed $\frac{1}{4}$ inch per chain of 66 feet.

Testing Chains and Tapes for Length. Chains and linen tapes used for surveying purposes, are liable to become inaccurate from various causes. The former, sometimes get stretched by rough handling in pulling them through hedges and over rough ground, or they are shortened by links getting trodden on, or twisted and thus bent. Tapes may be stretched by continued use in windy weather or shrunk by being soaked with rain.

For this reason they should be frequently compared with some standard measure, or the work done becomes unreliable.

In most large towns, standards such as that at Trafalgar Square, London, (which have been carefully measured with rods of standard length at a known temperature), are available for this purpose, but in the field, recourse must be had to some provisional method. It is then best to make use of a good steel tape, or failing that, of a levelling staff. The distance being accurately measured off, two pegs are driven into the ground, and nails are inserted in their heads marking the exact length required, the chains can be compared with these at regular intervals during the survey, and either lengthened or shortened as required, by means of the adjustable links at the handles.

Tapes, not being provided with any means of adjustment, any inaccuracy found to exist in their length must be noted and '*added to*' or '*deducted from*' the measurements taken.

Errors in Booking.

Errors in booking the chainage are of frequent occurrence with beginners, and the following are some of the most likely to occur.

1. Through reading from the wrong end of the chain, i.e. mistaking a 40 brass for a 60, a 30 for a 70, and a 10 for a 90 (these pairs being fashioned alike in most chains), or through the brasses becoming clogged, and so liable to be mistaken one for the other.

2. Through mis-counting the arrows, or through dropping or leaving one behind. To avoid such errors it will be found of great advantage to instruct the 'leader' to check the number of chains being booked by the number of arrows he still retains in his hand.

3. Through want of care in changing arrows at each ten chains.

4. Through omitting to adjust the chainage after crossing a pond or other obstacle, where it had been necessary to mark a portion of a chain upon the first side, in order that a full length of the chain may be available for the width.

Chaining over Obstacles.

To chain over an obstacle, such as a fence, boulder, or bank, the following is a convenient, expeditious, and accurate method. Supposing the obstacle to occur at about the 70th link, the 'leader' pulls up the chain and marks, say the 60th link, with his special arrow which he leaves, and himself goes round, or over the obstacle.

The 'follower' upon arriving at the 60th link substitutes his offset rod for the special arrow, and passes the chain handle forward along with the arrow. The 'leader' then stretches the remainder of the chain, and both the 'leader' and 'follower' hold the chain taut and horizontal at such a height as to just clear the obstacle, and when in true alignment the 'leader' drops his special arrow, and immediately replaces it by another, unless this happens to be the end of the tenth chain.

The crossing of the fence is read and booked, when the chaining proceeds as usual. In the case of fences without banks, the chain handle may be thrust through by means of the offset rod.

Chaining on sloping ground.

In the case of a steep, short slope, falling in the direction of the chainage, it is usual to cause the chain to be pulled forward to its full length, and then for the leader to return to some convenient brass upon it, hold this point level with the follower's hand, and, when in true alignment, drop the special arrow, the follower then comes forward, substitutes his rod for the special arrow, and the operation is repeated until the whole chain is out.

When the slope is rising in a forward direction, the chain is pulled out to its full length, and the 'follower' plants his offset rod vertically in the place of the last arrow down, or, using a plumb-line for this purpose, directs the 'leader' to return to a convenient brass mark, and to hold the chain at the ground, he then holds his end of the chain level with the leader's hand, the chain is pulled taut, and when truly aligned, the special arrow is temporarily inserted at the brass mark, the 'follower' then advances with his rod, and the operation is repeated until the whole chain is marked out.

It is obvious that if the rod be not vertical, or the portion of the chain in use be not held horizontal, the correct chainage cannot be obtained in this manner, but it is usually sufficiently near for practical purposes upon short, sharp inclines, and where great accuracy is not necessary.

When, however, there is a considerable length of uniformly sloping ground to be chained, it is better to ascertain the angle of slope by means of a clinometer carried by the 'follower,' and to allow each chain to be pulled forward the full length plus the necessary amount of allowance.

If the allowance is thus made in the field, it obviates the necessity of having to make deductions from the book chainage when plotting, but a note should nevertheless be made in the field-book of the 'allowance per chain' which has been made, and this is convenient in case of other lines subsequently closing upon the one being chained.

A convenient method of ascertaining the angle of slope is to place a ranging rod at the next change of incline, having a flag or piece of paper tied at the height of the observer's eye, who then, standing erect at his end of the line, sights the mark on the forward rod with his clinometer, records the angle and notes the allowance per chain to be made, or, he lays his rod upon the ground on the incline, and places his clinometer upon it and so ascertains the angle of inclination.

After a few trials the surveyor will be able to judge the angle near enough and the necessary allowance for all moderate inclines up to, say 4° or 5°. At 4°, which is a considerable incline, it is only about ¼ link per chain.

From a table of natural secants, to a radius of 1, the exact length that a chain should be upon the incline, to represent one upon the horizontal, can be found, and the surveyor should allow for the excess.

The most accurate method is to place another arrow in advance of the one already in the ground, the exact allowance for the next chain being made with a pocket-rule, or by means of the offset rod (which may have one of its links specially marked in decimal parts for this purpose), and to lay the chain from this new arrow—being careful however, to lift both before going forward.

The surveyor should make a note in his field-book, of the natural secants likely to be required (if his memory be not retentive enough), remembering to remove the decimal place two points to the right, so that instead of reading chains and decimals of a chain, it will read links and decimals of a link.

Table of Natural Secants.

For one degree of elevation the natural secant is 100·015, making a difference of only about ⅛ of an inch in a chain, a quantity not worth considering, but the correction rapidly increases with the incline, as the following table of natural secants will show. The radius is assumed to be 100 links, and the equivalent of one horizontal chain for each degree of elevation is represented by the amount set opposite the number of degrees.

	links.		links.
1°	= 100·015	8°	= 100·983
2°	= 100·061	9°	= 101·247
3°	= 100·137	10°	= 101·543
4°	= 100·244	15°	= 103·528
5°	= 100·382	20°	= 106·418
6°	= 100·551	25°	= 110·338
7°	= 100·751	30°	= 115·470

Offsets. Offsets are measurements taken at right angles to the chain (unless otherwise specially indicated by arrow heads in the field-book), to fences, corners of buildings, or other objects, which it is necessary to show upon the survey.

Where more distances than one, have to be taken upon the same offset, they may either be taken and booked separately, or in running measure. When using the offset rod, the former is the more convenient plan, but when a tape or the chain itself can be stretched, the latter is preferable. Whichever method is adopted, there must be no confusion in the field-book, and no possibility of mistaking the one form of entry for the other, when plotting, and it will be found of great service, when replotting from old books (when possibly all details of this kind have long since been forgotten), if a uniform system has been adhered to.

When the measurements are taken with an offset rod, it will be found to be difficult to keep up a continuous running measure without incurring a great risk of error, and thus it becomes more convenient to take and enter each dimension separately, the sign + (plus) being understood between each.

Thus supposing two fences (such as those of a lane) to be both upon the same side of the survey line, and the distance upon the offset to the first fence to be 25 links, this should then be entered in the field-book, between the line and the sketch of the first fence, then, if the distance to the second fence be 30 links more, this 30 would be entered by the side of the 25, but between the two fences, and the total distance from the survey line to the farthest fence would be 25 + 30 or 55 links.

On the other hand, if a tape or chain be stretched, it would be inconvenient to separate the dimensions in this manner, and, consequently, a running or continuous measurement would be taken, and in order to avoid the chance of being misunderstood, the field-book should be turned upon one side, and the entries made in the same manner as a chain-line would be booked. In this way any number of dimensions may be entered upon the same offset without confusion.

For example of these two methods of booking, see line 16 in the field-book, (p. 20) where it will be seen that the offsets at 200 and 216 are in accordance with the second method.

No necessary measurements must be left to memory, and sufficient dimensions should be booked to enable an assistant to plot the work, with equal accuracy to the surveyor himself.

Obstructed Intervals in Chaining.

There are many ways of ascertaining the distance of obstructed intervals in chaining—some of which are given below:—

1. Where the obstruction is a hayrick, building, or walled enclosure, which it is not possible to chain over, it is generally sufficient to erect two perpendiculars of equal length upon the chain-line—one

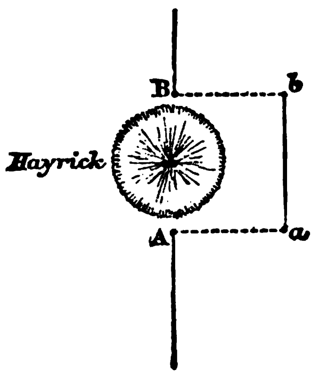


FIG. 3.

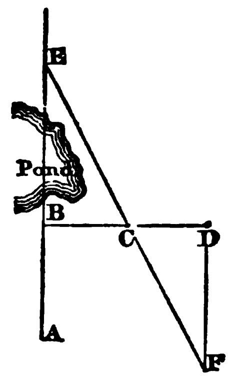


FIG. 4.

on each side of the obstruction. A line joining the two ends of these perpendiculars will be of the same length as the portion of the chain-line between them.

Thus upon the sketch (fig. 3) A B = *a b*.

2. If a pond obstruct the chain-line A B E (fig. 4). On arriving at B set a

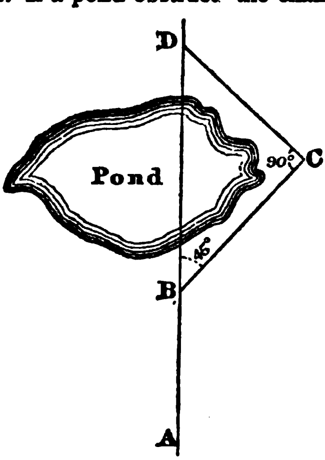


FIG. 5.

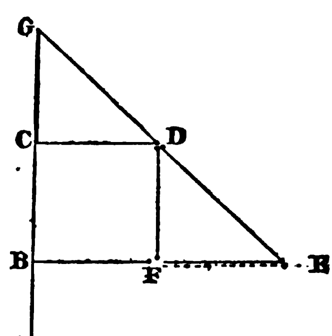


FIG. 6.

mark at C, chain B C and produce it to D, make D C = B C. Then select another point E on the chain-line, make C F = C E. Then F D = B E, the required distance.

3. It may not be possible to secure clear space in the rear, as in the last case, or to have room for right angles, as in the first case.

Set off from B (fig. 5) by means of the cross-staff an angle of 45° to C. On arriving at C set off a right angle B C D, meeting the chain line, continued, at D. Then B C and C D will be equal, and $BC^2 + CD^2 = BD^2$ ∴ $BD = \sqrt{BC^2 + CD^2}$. Or, multiply B C by 1·4142 (nat. secant of 45°) and result gives length of B D.

4. On line A B C G (fig. 6) the distance C G is required. From points B and C set off the equal lines B F and C D at right angles to A B. Produce G D to E in line with F B, then E F D and E B G are similar triangles ∴ E F : F D : : E B : B G or D C : C G.

5. On the line A B F (fig. 7) required the distance B F. From B set off B C D at right angles to A B, making B C equal to C D.
From D set off a right angle C D E and make E in line with C and F. Then E D is the distance required.

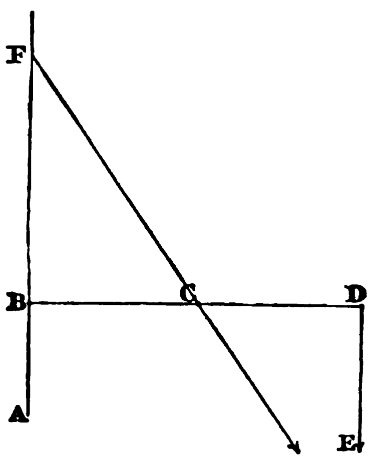


FIG. 7.

There are many other ways of obtaining distances without the use of angular instruments, other than the cross-staff. It may be useful to know how to obtain the distances to the intersection of two lines, which may fall in a wide river, or deep pond, or other inaccessible place.

On the lines A B O and C D O (fig. 8) required the lengths B O and D O.

Select any convenient points upon the two lines, as B and D. Produce A D and B D to E and F, making D E = A D and D F = B D. Produce E F until it cuts O D C at C. Then C D = D O and C F = B O. Should it be inconvenient to set the lines back so far, set each back, say ½ or ⅓ the respective distances, and multiply the results by 2 or by 3 as the case may be.

Ranging Lines. Before commencing to range out the main lines, it is necessary to have acquired some knowledge of the general contour of the ground to be surveyed, and in which direction the longest line can be run so as to be, if possible, entirely within it.

If there are no serious obstacles, such as a village, thick woods, or a lake upon it, the longest line obtainable, is generally the best as a base line.

Having obtained a general idea of its direction, proceed to that point upon it which commands the best view, and thence determine its exact direction, so as to avoid, as far as possible, all obstacles to both setting out, and chainage. If the position is an elevated one, some object, at, or beyond the boundary of the property, may be visible, and if the line can be directed towards it, the setting out will be much facilitated, and it will always exist as a ready means of identifying the line.

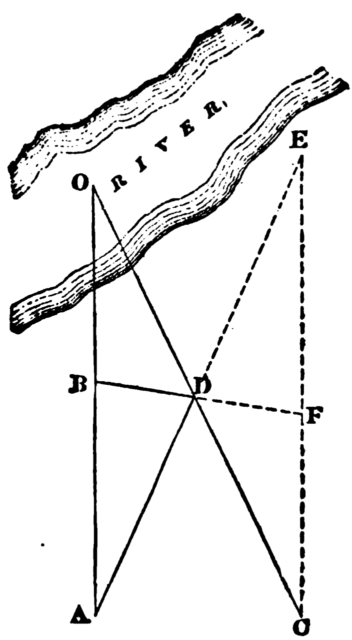


FIG. 8.

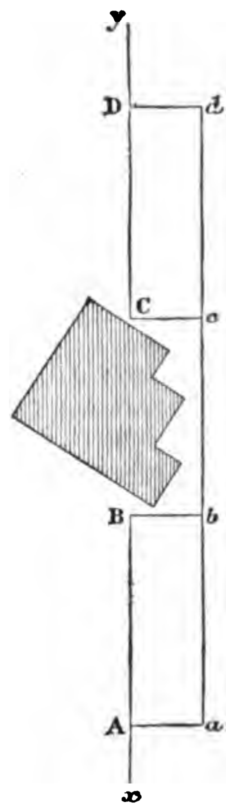


FIG. 9.

Such a mark, too, will exist as a proof that the line has been accurately set out, after perhaps, having been lost sight of in passing through low ground.

The operation of ranging a straight line has been often described, but the whole secret of perfection lies in being able to plant a rod firmly and truly perpendicular in the ground, in such a position, that, upon retiring a few yards behind it, and looking in the direction from which the line has been driven, all other rods and marks upon that line are hidden by the last rod.

The rods, when set, can be taken up (being replaced by cleft sticks with pieces of paper inserted in them), and used over and over again, but in no case should the last two or three rods be disturbed until others have been placed in position.

In addition to cutting occasional marks, alluded to under the head of 'stations,' the tops of all high fences whether crossed at right angles, or upon the skew, should be cut off so as to appear as a narrow slit in the direction of the line.

Ranging Past Obstructions.

In the case of a line being obstructed by buildings or stacks, it is often desirable to range it over them, because, had they been visible from the point from which the line was selected, it would in all probability have been so arranged as to avoid such obstructions.

Where, however, it is not easy to range the line over the obstruction, and it is necessary to continue it, this object may be accomplished by means of a parallel line in the following manner.

Let $x\ y$ (fig. 9) be the line, obstructed between the points B and C.

Leave rods at A and B and erect the perpendiculars Aa and Bb, make them of equal length, and sufficiently long to avoid the obstruction. Produce $a\ b$ to c and d and let fall the perpendiculars Cc and Dd, making them also equal to one another, and to Aa, Bb. C and D are then upon the line $x\ y$ and can be produced to the required distance. The lengths of A B C D should never be less than three times that of the perpendiculars Aa and Cc.

It is self-evident that great care must be exercised in the measurements, or the result will be anything but satisfactory. The direction of the line, too, should be tested (as soon as it is possible to see over the obstructions), by means of the distant objects before alluded to, and by the back-marks.

Sometimes it will be found worth while to erect long poles to get over an obstacle, but such details must be left to the judgment of the surveyor.

It frequently happens that long lines can be ranged from hill to hill for miles, in which case stout poles with large flags are necessary, and a binocular field-glass will be most useful. The intermediate portions of such a line can be filled in afterwards.

When crossing the summit of a hill, or the bottom of a valley, very great care must be exercised in setting the rods perfectly in line, and vertical, or, as the intermediate distances are necessarily short, the least deviation from the straight, will soon become serious, necessitating the re-setting out of that portion of the line.

It is of advantage to leave as many marks as possible upon the line, and here and there, in particularly good places, special marks, such as a flag or extra large piece of paper, which will be visible again after some distance has been passed, and act as a check upon the direction of the line.

Traversing with the Chain.

Traversing with the chain only, is not to be commended, but it sometimes becomes necessary for the purpose of continuing a road, stream, or boundary. By exercising great care to secure the best possible ties at the junctions of the lines, and measuring the same carefully, very good work may be done, but where long tie-lines cannot be obtained, the difficulty of obtaining accurate results is much increased,

and such a method only becomes permissible when there is no intention of filling in the intervening work upon the plan.

Arranging Lines, and Plan of Yeldon Farm.

In arranging the main lines for a chain survey, it must be borne in mind that they should be laid out in such a manner as to form triangles, otherwise they cannot be plotted upon the paper with accuracy.

For example, looking at the plan of Yeldon Farm (Plate I.) it will be observed that the base line No. 1 is, so to speak, the backbone of the survey, and is about as long a line as it is possible to obtain within the estate, without encountering any serious obstacle.

Lines 1, 2, and 3 form the largest triangle, while lines 4 to 14 are subsidiary lines, no one of which could possibly fit into its place if inaccurate in length, whilst all of them are necessary in order to pick up the work. Line 10 is the principal tie-line for the main triangle and should be laid down next after 1, 2, and 3, so as to prove them.

All the remaining lines are, more or less, proofs of the work, and are, it will be observed, all run as near as possible to the fences and other landmarks, which have to be picked up upon each, so as to avoid long offsets. Lines 15 and 16, together with a part of line 1, form another triangle, and this is really proved correct without the tie-line 17, because line 16 is a prolongation of line 10, hence its position and direction are already fixed, and if line 15 fits, the other lines must be in agreement. Line 17 is, however, absolutely necessary, in order to delineate the position of a crooked fence, and is an additional proof, as it could not fit in if the other lines, or any of them, were incorrect.

Lines 18 and 20, with part of line 1 form another triangle, to which line 19 is the tie, while 20*a* is an additional proof of the accuracy of the rest.

Line 20 is also produced from line 18, which fixes its direction, while line 21 checks its length.

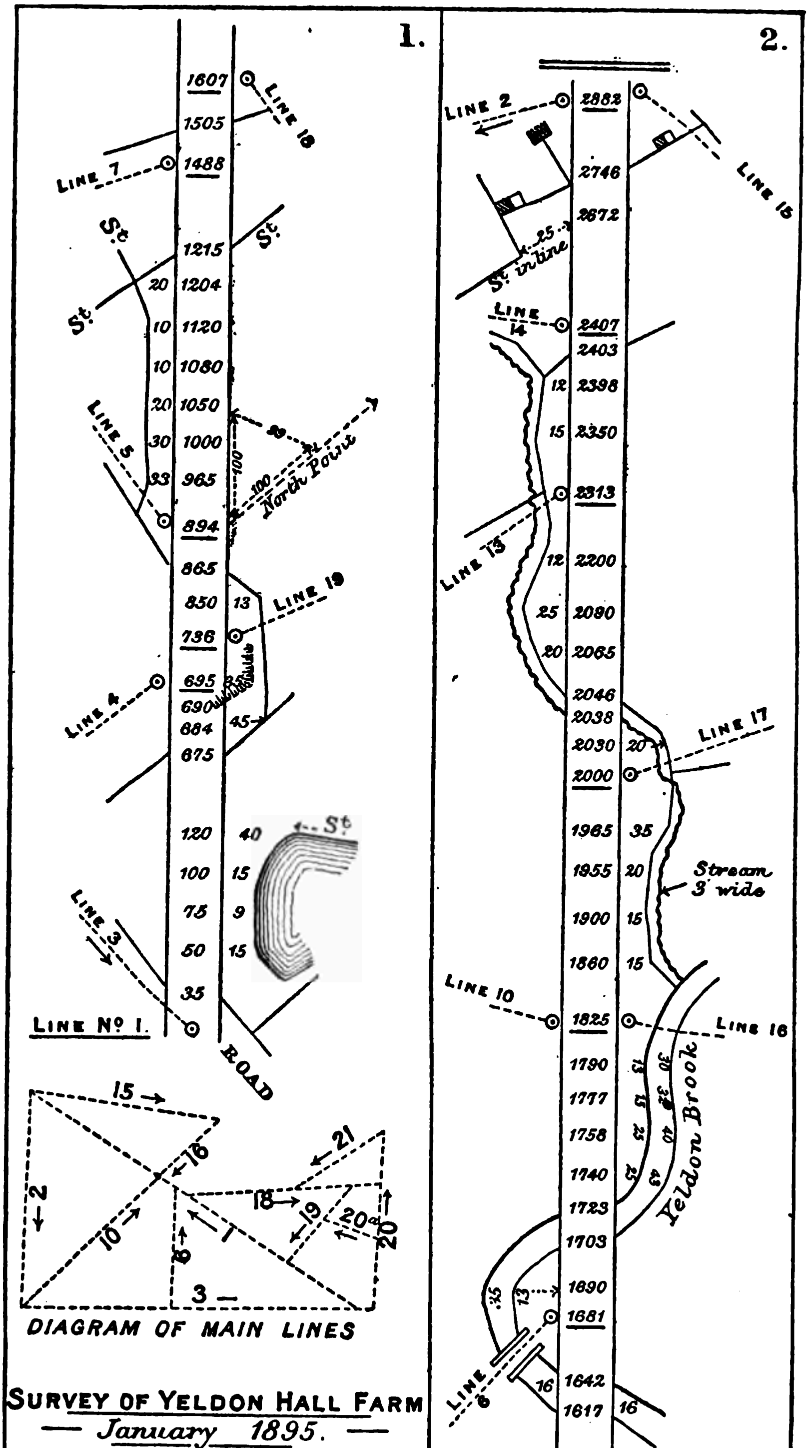
The objects to be aimed at in determining the arrangement of the lines of a survey, are always the same, viz.:—

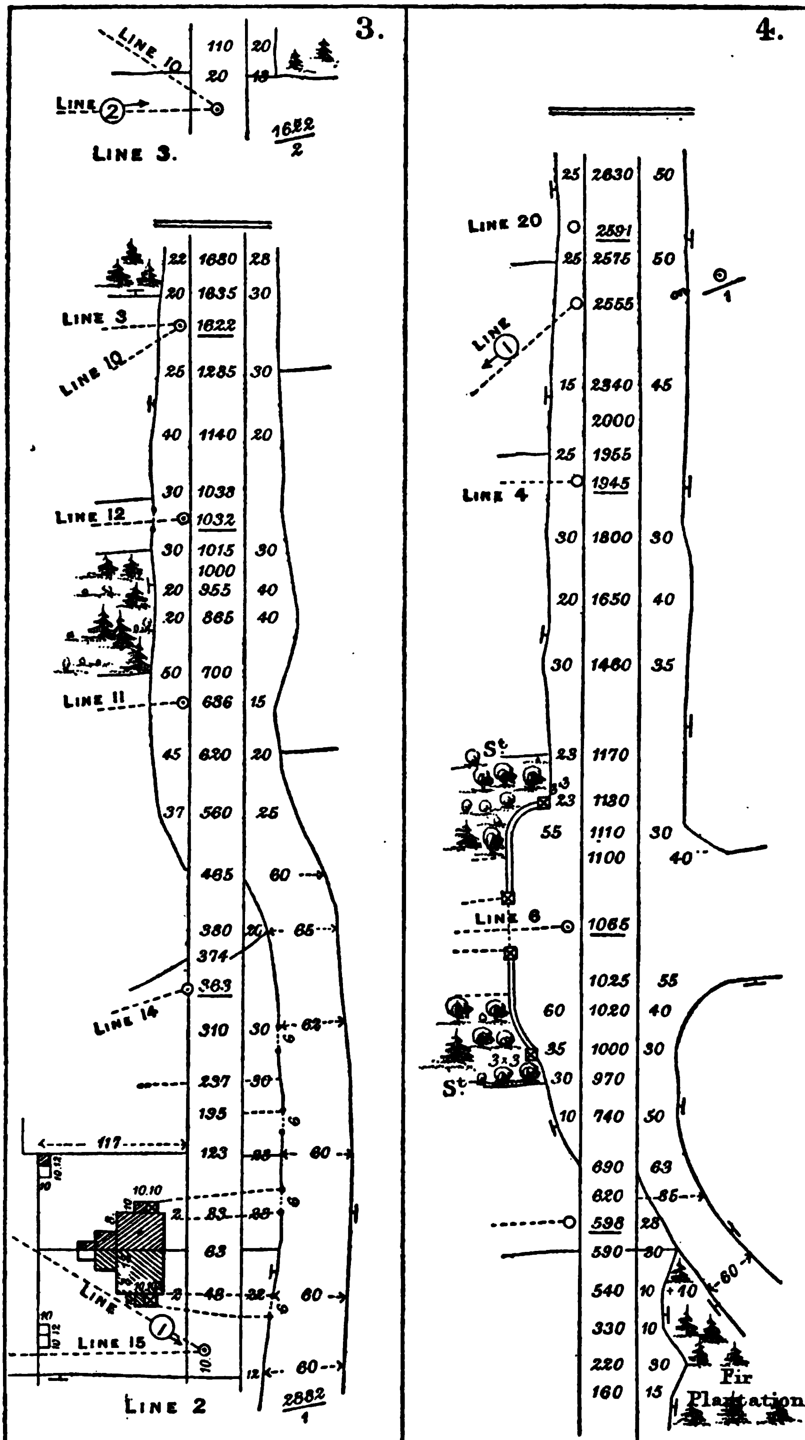
1. A good main line to form a 'backbone' upon which to build the framework.

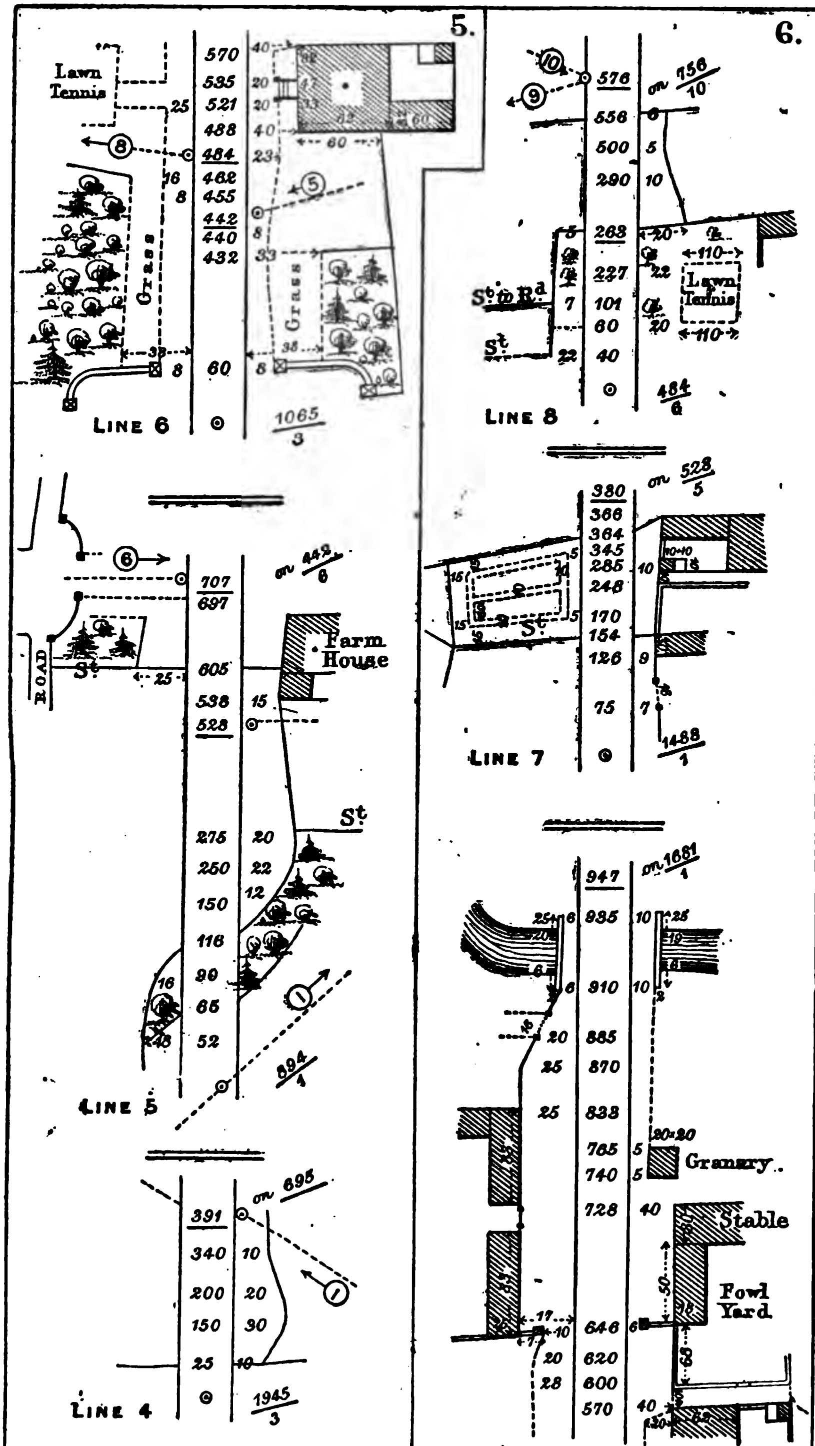
2. Main triangles to embrace as many as possible of the fences and other objects to be delineated in the plan, and thus to avoid, as far as is practicable, building out again and again.

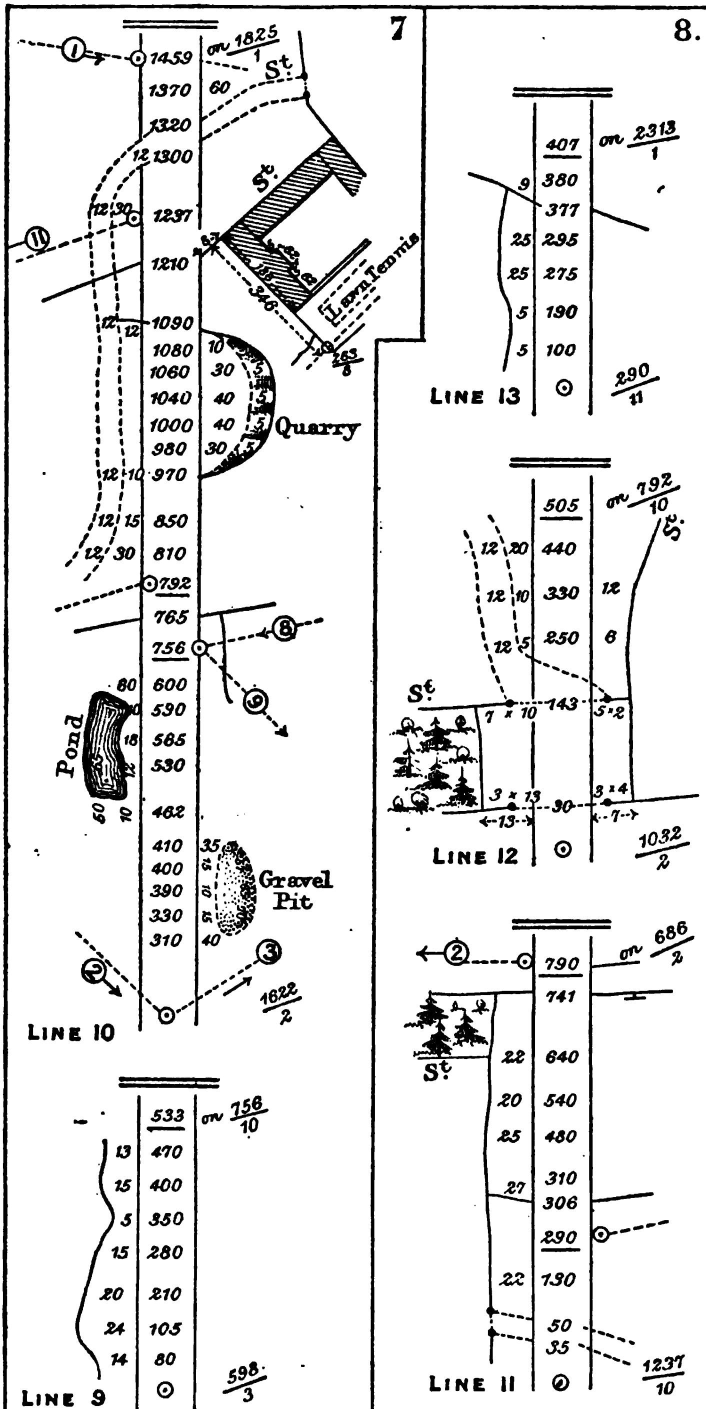
3. Lines interior to the main triangles, which, while serving to pick up all the work, are as few in number as is consistent with obtaining short offsets to every point to be fixed. It may be set down as a rule, which should rarely be broken, that no offset should exceed one chain in length, unless checked by a tie-line, in which case it is no longer an offset pure and simple, but a subsidiary survey line.

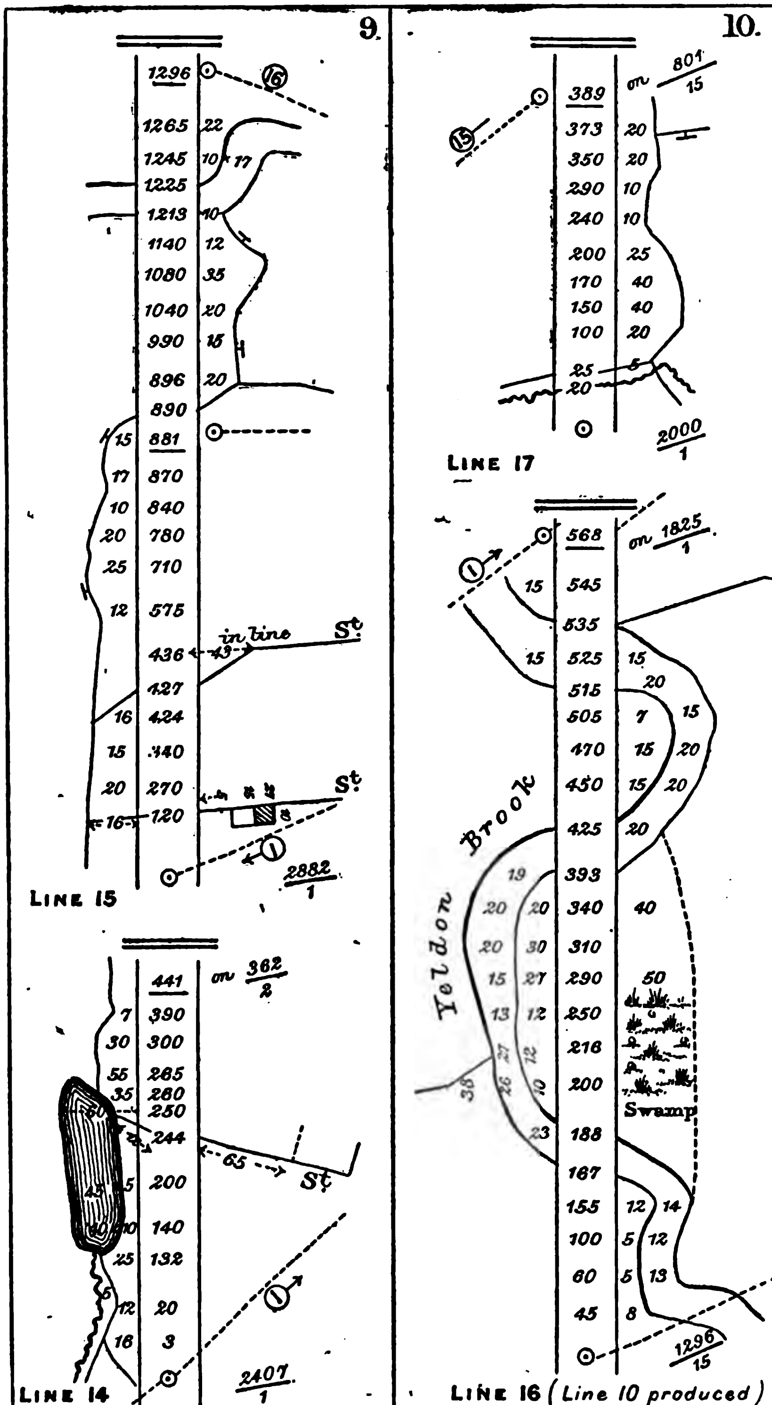
Should small portions of the ground still lie outside the lines forming triangles, such as occur on line 18, a perpendicular as there shown, may be built out (see field-book) by means of the optical square or cross-staff, and if of any considerable length, should be tied back, even if the tie-line be not required as in this case, to pick up work.

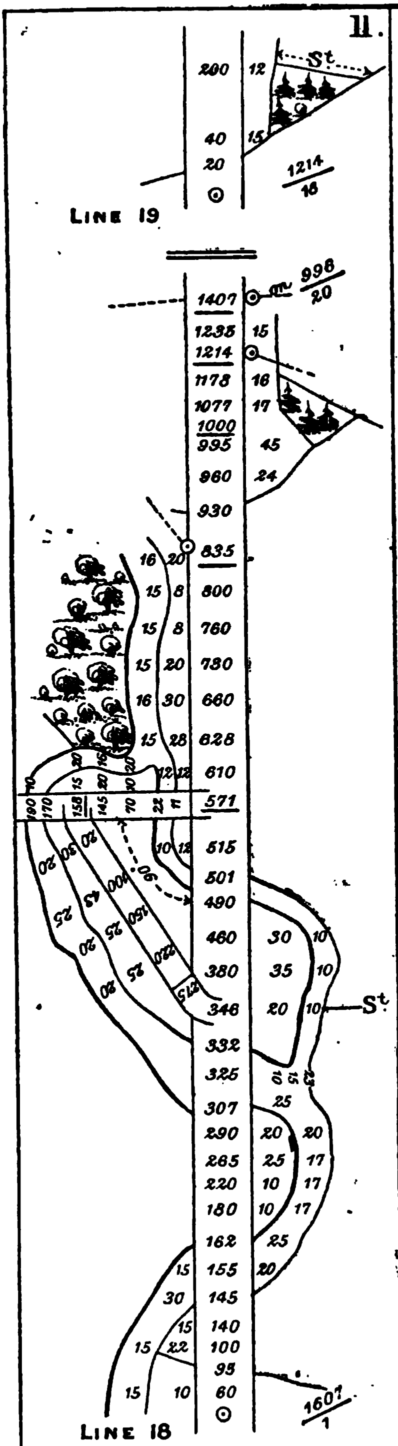

1.
2.
1607
1505
1488
1215
1204
1120
1080
1050
1000
965
894
865
850
736
695
690
684
675
120
100
75
50
35
Line No. I.
Line 7
Line 18
Line 5
Line 19
Line 4
Line 3
St.
North Point
Road
DIAGRAM OF MAIN LINES
SURVEY OF YELDON HALL FARM
— January 1895. —
Line 2
Line 15
Line 14
Line 13
Line 17
Line 10
Line 16
Line 8
2882
2746
2672
St. in line
2407
2403
2398
2350
2313
2200
2090
2065
2046
2038
2030
2000
1965
1955
1900
1860
1825
1790
1777
1758
1740
1723
1703
1690
1681
1642
1617
Stream 3' wide
Yeldon Brook

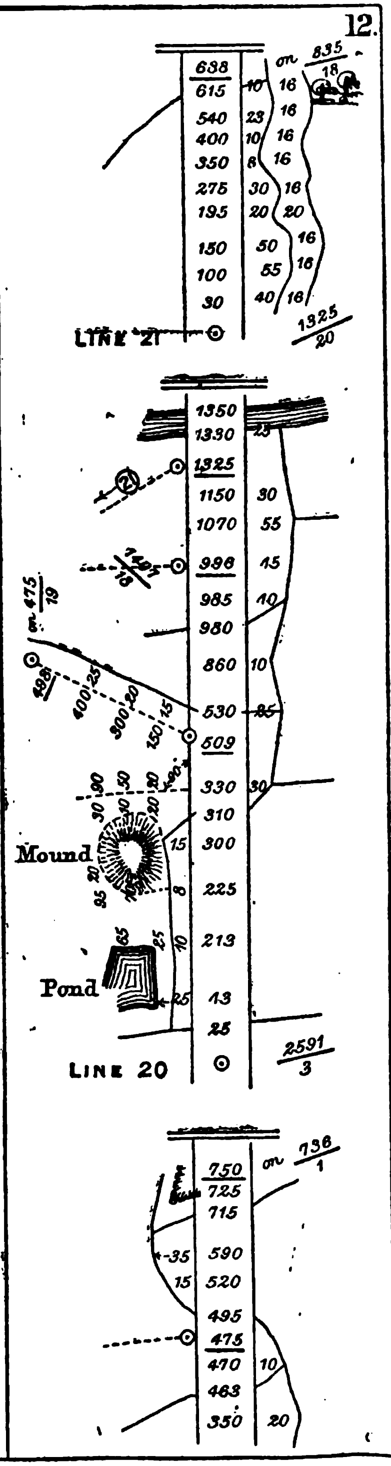

3.
LINE 10
LINE 2
LINE 3.
1622
2
1680
1635
LINE 3
1622
LINE 10
1285
1140
1038
LINE 12
1032
1015
1000
955
865
700
LINE 11
686
620
560
465
380
374
363
LINE 14
310
237
195
117
123
10.10
83
63
LINE 1
48
LINE 15
LINE 2
2882
1
4.
2630
LINE 20
2591
2515
2555
LINE 1
2340
2000
1955
LINE 4
1945
1800
1650
1460
St.
1170
1130
1110
1100
LINE 6
1065
1025
1020
1000
3x3
St.
970
740
690
620
598
590
540
330
220
160
Fir
Plantation


5.
6.
Lawn Tennis
Grass
LINE 6
1065
3
LINE 8
on 756
10
St to Rd.
Lawn Tennis
484
6
on 528
5
LINE 7
1488
1
on 448
6
Farm House
ROAD
St.
LINE 5
894
4
on 1681
1
Granary
Stable
Fowl Yard
on 695
LINE 4
1945
3


7
8.
on 1825/1
St.
1459
1370
1320
1300
1237
1210
1090
1080
1060
1040
1000
980
970
850
810
792
765
756
600
590
565
530
462
410
400
390
330
310
Lawn Tennis
Quarry
Pond
Gravel Pit
1622/2
LINE 10
533
470
400
350
280
210
105
80
on 756/10
598/3
LINE 9
407
380
377
295
275
190
100
on 2313/1
290/11
LINE 13
505
440
330
250
143
on 792/10
1032/2
LINE 12
790
741
640
540
480
310
306
290
130
50
35
on 686/2
1237/10
LINE 11


9.
10.
LINE 15
LINE 14
LINE 17
LINE 16 (Line 10 produced)
in line
St.
St.
St.
Yeldon Brook
Swamp


11.
LINE 19
LINE 18
St


12.
LINE 21
LINE 20
Mound
Pond

Chain Survey of a Single Field.

In order to illustrate the various processes required to make and plot a small survey, let the student take for an example, the single field, in shape approximately a parallelogram (fig. 10).

For the purposes of a survey of this nature, a chain, 10 pins or arrows, a few poles, an offset rod, and a cross staff or optical square, will be required.

One man to draw the chain will be necessary, but as in this case the surveyor will himself follow and direct the chain, no other assistance will be required, though a lad to carry poles, &c., may be useful.

Time will probably be well spent by the student if he first walks round the boundary and himself selects the points of junction A, B, C and D of the lines intended to be run, and sets up flags or marks at them.

When experience has been gained, he will probably dispense to a large extent, with poles and flags, and, upon entering a field will at once be able to determine the best way to set about surveying it.

The points A, B, C and D, should, if the ground will permit, be selected within view of each other, and in such positions that lines connecting them may come within easy offsetting distance of the boundaries of the field.

Having placed marks at these points, the student must stand behind A and direct the chainman to go forward towards the point C, with two rods or papered sticks, and to set one at E, exactly in line between A and C, and approximately opposite the point B and the other at F nearly opposite D, in convenient positions to run the lines E B and F D, which are necessary as ties to prove the accuracy of the survey.

It must be a hard and fast rule, that no triangle shall be accepted as accurate until the correctness of its dimensions has been proved by a suitable tie-line, otherwise no reliance can be placed upon the work.

Setting out a Right Angle, with the Optical Square.

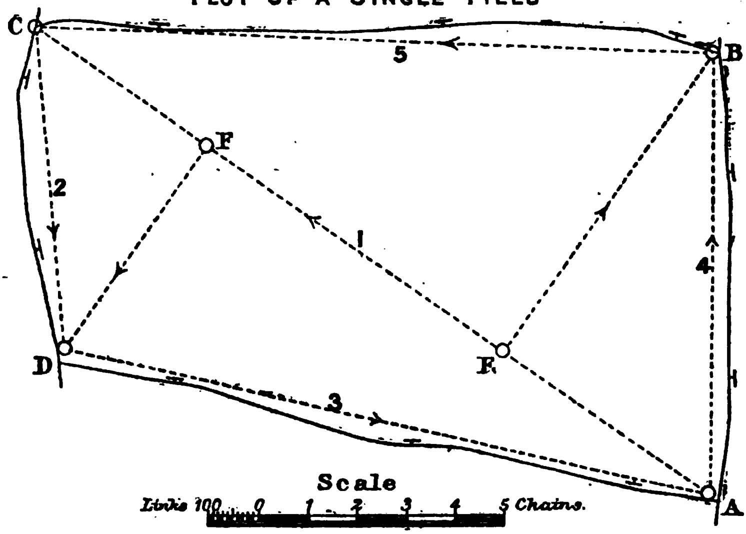
Commence chaining from point A towards C, and call this line No. 1.

If it is intended to obtain the area of the field, without in the first instance making a plan of it, *but not otherwise*, it will be necessary that the tie-lines E B and F D be perpendicular to the base line A C, so upon arriving at E and F these must be corrected by means of the cross-staff, or optical square.

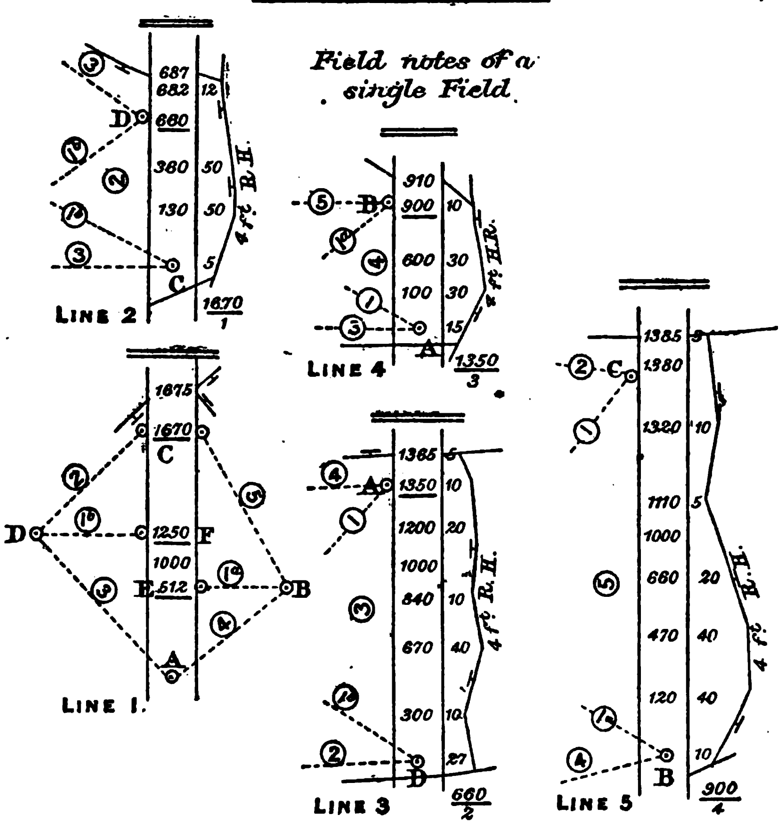
A little practice will enable the student to fix these points within a few links by the use of the offset rod and eye, alone, by standing facing the point B with his feet apart, toes about touching the chain-line, and holding his rod, as nearly as he can judge, at right angles to the chain. Then, glancing in the direction in which the rod is pointing, and he may be sure he is not very far out if it is fairly on point B, and he can now proceed to fix his position exactly, as follows.

As the chain lies in position along the line A C, let the student, with his face towards the point C, and with one foot on either side of the chain, hold the optical square in his left hand, exactly over the chain, and look through it towards C. If E is at the correct place, the reflected image of the mark at B will coincide with the mark seen at C.

PLOT OF A SINGLE FIELD



Field notes of a single Field.



Should this not be so, the student must move backward or forward, as the case may be, keeping his eye directly above the chain as before, until the two marks at B and C do coincide, then the point on the chain immediately beneath the instrument will be the point E required.

After a little proficiency has been acquired, the whole operation will not occupy more than a minute or two.

To check the angle, and the adjustment of the instrument, take the optical square in the *right* hand, and face towards the mark at A. The reflected image of the mark at point B should then coincide with the mark seen at A.

If this be not so, either the optical square is out of adjustment, or the line A, E, F, C is not perfectly straight. In the former case the point E should be fixed exactly halfway between the two points found, and in the latter case the line A, E, F, C must be corrected and the point E tested again.

The distance of point E upon the chain-line, must now be entered in the 'field-book,' and the chaining continued until opposite the point D, when the point F must be determined, and distance entered in the same manner, and the chainage continued to the point C, at the fence, when the *total distance* is entered and the fence noted.

On arriving at C proceed with the chaining of line C D and call it No. 2.

While chaining this line, on arriving opposite the first bend in the fence the distance upon the chain line must be entered in the field book, and a measurement taken to the fence at right angles to the chain by means of the offset staff, and noted.

If care be taken to draw the fence approximately as it appears, but as an exaggeration or caricature of the reality, it will be a great aid to the memory when plotting, and will tend to prevent any serious errors from creeping into the plan.

Fences and Boundaries.

The boundary fences should in all cases be described, and the side to which the fence belongs noted, and in some cases the positions of any large trees that may happen to stand in it.

It is the custom, now adopted by the Ordnance Survey Department, to survey to the centre of the fences, but it must be borne in mind that the centre of a fence is not always the true boundary of a property. In the case of a fence growing upon a bank, and having a ditch, or a wall with buttresses or indents, the true boundary is usually at a distance of 4 feet or so from the centre of the hedge or wall. This distance, however, is a varying quantity, and the practice of the district should be enquired into, the usual rule being, that the material of which the bank is formed has been originally taken from the land of the owner of the fence. Thus, as the annexed sketch shows (fig. 11) the land belonging to A extends from arrow to arrow.

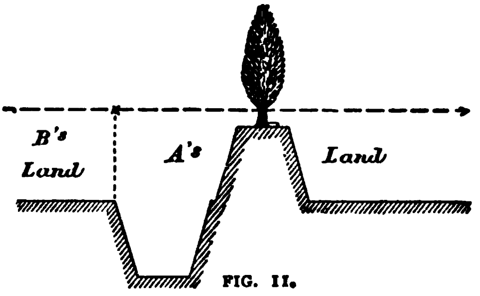


FIG. 11.

A convenient method of noting this in the 'field-book' (as also upon the plan), is by a mark thus $\frac{\mathsf{T}}{\text{4 feet R. H.}}$, the mark T stands upon the side to which the fence belongs, and the remark '4 feet R. H.' means that the true boundary of the property is 4 feet from the root of the hedge, which, of course, must be allowed for when computing the area of the field.

Methods of Keeping Field Notes.

Field notes are most conveniently made with an indelible pencil, or dry ink. Liquid ink, although sometimes insisted upon, is troublesome, especially in unfavourable weather, being liable to blot and run, from rain. Figures, when once entered should not be erased, but if a correction is necessary should be boldly scored out, and the new figures inserted, and if this should cause any confusion, the *whole* or the *part* of the *page* should be copied clearly, and a marginal note added stating where the clear copy may be found.

The Field-Book, with Example from Yeldon Farm Survey.

All 'field-books' should be paged for convenience of reference, and are best of the form usually sold for the purpose with either one or two central lines ruled, as the surveyor may prefer.

On reference to F. B. of Yeldon Farm (page 16), it will be observed that the length measurements of survey lines are entered by commencing at the bottom of the page and working upwards, instead of commencing at the top, and a moment's consideration will show that this is the more convenient method of proceeding. When holding the book in the hand and following the chain, dimensions taken from the chain-line fall naturally into place, i.e. those taken upon the *right side* of the chain are entered upon the *right* in the book, and those taken upon the *left side* are entered upon the *left* in the book, which would not be the case if the chain measurements were commenced at the top of the page.

In order to save as much writing as possible in the field, it is advisable to adopt a short and clear method of making the necessary notes of the commencements and terminations of the survey lines, and other remarks.

The following illustrate what is referred to.

Instead of writing '*Line* 12 *commencing from* 940 *on line* 1,' it is only necessary to write '*Line* 12 $\frac{940}{1}$.' Again, at closing, instead of writing '*Closing on* 1450 *line* 2,' write '*on* $\frac{1450}{2}$.' Supposing line 12 to close not upon the registered station of 1450 on line 2, but, say, 15 links beyond or short of that station, write, 'on $\frac{\substack{1450\\+15}}{\underline{1465}\\2}$,' or 'on $\frac{\substack{1450\\-15}}{\underline{1435}\\2}$,' as the case may be.

This will give a full record of how the closing point has been arrived at, and as mistakes will sometimes occur, no matter how careful one may be, this method affords an easy means of checking that particular point.

When making large surveys, it is always advisable to lay down the lines on paper as soon as possible, in order that, in case they do not fit, any error may be quickly discovered while the marks are still upon the ground, and the whole work fresh in the mind of the surveyor. The adoption of this rule will help to make the work of surveying a pleasure.

A distance underlined thus '<u>920</u>' in the field-book denotes that a 'station' or mark for reference' has been left upon the ground, while a circle and dotted line thus, '○⋯⋯' indicates that another survey line is arranged to be run in the direction of the dotted line.

If any particular form of mark has been cut upon the ground it is just as well to sketch it in the book as a ready means of identification.

'St.' on a fence means that the fence is straight to the next point surveyed upon it, on whatever line it may be. 'Stm.' means 'stream'—the direction of the current is usually denoted by means of an arrow, thus, '➽—>'

The methods of indicating the various kinds of wooded lands, such as oak woods, mixed woods, fir plantations, orchards, &c., are more properly subjects for planning, than for field-book keeping, and will be treated of under that head. They may be indicated in character in the 'field-book,' though it is better, in order to avoid mistakes, to make written notes with reference to them, as well.

Adjoining owners' names should always be ascertained and noted.

Plotting a Chain Survey. Scales.

Scales can be obtained of any desired pattern, and to any proportion.

Those for plotting surveys made with Gunter's chain, are usually 12 inches in length, and marked upon one side, in chains and decimals of a chain, and upon the other side, with an equivalent scale of feet. This is useful in case it is necessary for measurements to be scaled or

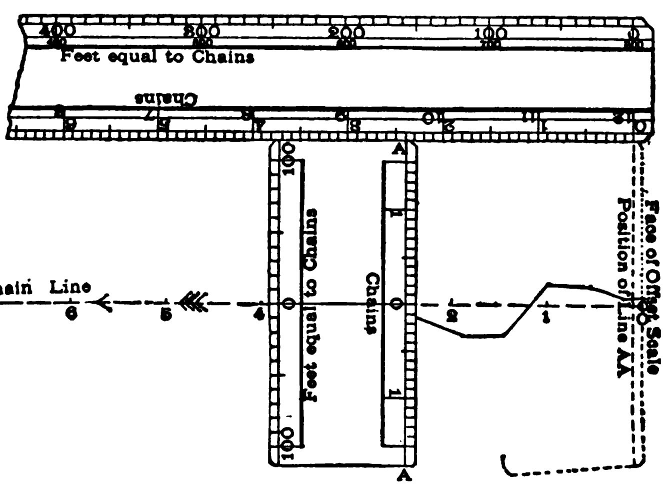


FIG. 12

plotted in feet, as in the case of buildings and building land, the dimensions of which are generally made up in feet.

Offset scales (fig. 12) are similar to the above, but only about 2 or 3 inches in length, and frequently without the 'feet equal' side. It is very desirable that the zero of the offset scale should be at the centre of its length, and not at one end.

The scale to which any given plan should be plotted depends so much upon the object for which the plan is intended, and the size of the piece of ground or estate, that it is impossible to lay down a hard and fast rule.

The following are some of the scales commonly used in England by land surveyors.

List of Scales in common use.

$\frac{1}{792}$	or 1 chain to an inch.	For small estates and single fields.
$\frac{1}{1584}$	" 2 chains "	Ditto and for working surveys.
$\frac{1}{2376}$	" 3 " "	For 1st class tithe maps of England and for large estates.
$\frac{1}{3168}$	" 4 " "	Ditto.
$\frac{1}{3960}$	" 5 " "	Ditto.
$\frac{1}{4752}$	" 6 " "	Ditto.
$\frac{1}{6336}$	" 8 " "	Ditto.
$\frac{1}{1200}$	" 100 feet to an inch.	Building and engineers' plans.
$\frac{1}{2400}$	" 200 " "	Drainage schemes, &c.
$\frac{1}{4800}$	" 400 " "	Ditto.
$\frac{1}{120}$	" 10 " "	Surveys on which working drawings are designed.
$\frac{1}{240}$	" 20 " "	Ditto.
$\frac{1}{360}$	" 30 " "	Ditto.
$\frac{1}{480}$	" 40 " "	Ditto.

Medium Ord. of England. } $\frac{1}{10,560}$ or $\frac{1}{6}$ mile to an inch = 6 inches to a mile

Minimum scale of deposited plans. } $\frac{1}{15,840}$ or $\frac{1}{4}$ " = 4 "

Small Ord. map of England.	$\frac{1}{63,360}$ or 1 mile to an inch	=	1 inch to a mile.
Large Ord. of England.	$\frac{1}{2,500}$ or ·03946 "	=	25,344 inches or 2·112 feet to a mile.
'New' Ord. sc. for towns.	$\frac{1}{500}$ or 41·667 ft. to an inch	=	10·56 ft. to a mile.
'Old' ditto.	$\frac{1}{528}$ " 44 "	=	10 "

'Diagram of Lines' in a Field-Book.

It always well repays the surveyor to make a diagram in the 'field-book' of the main lines run, *vide* F. B. of Yeldon Farm, and to enter upon it their numbers, lengths, and directions. This diagram will be found of great assistance when laying down, and fitting the lines upon the plan, and constant reference to the 'field-notes' is, to a great extent avoided, while plotting and proving the principal triangles.

Paper for Plans.

All original plans should be plotted upon the best hand-made, rough, double elephant drawing paper, mounted upon brown holland, and well seasoned. If the plan is a very large one, it should have a thin paper mounted between the double elephant and the brown holland, in order to give it an extra substance.

Machine-made papers should be avoided, as they are more liable to become distorted by contraction than *hand-made*, and in case of alterations being necessary, are rendered rough and unsightly by erasures, while the abraded surface takes up dirt very rapidly, and cannot be effectually cleaned without injury to the plan.

Laying down Lines on Plan.

The position of the first, or main line, should be carefully chosen, and so ruled in, that the whole plan when completed, will be well placed upon the paper. If it runs true north and south so much the better, but this is not essential for estate purposes.

A steel straight-edge is the most suitable kind of ruler, and may be obtained of any desired length.

Some surveyors use a hard, fine-pointed pencil for ruling in the lines, and afterwards ink them in with faint blue, while others use the eye end of a needle, which makes a fine indented line without scratching the paper.

A fine needle-point should be used for pricking off the *stations*, and *each 10-chain mark.*

This latter is a practice greatly to be commended, as it saves much time in the adjustment of new stations upon any of the main lines, after a possible expansion or contraction of the paper.

No matter how carefully seasoned a piece of paper may be, there will always be some movement of this kind, when once work has been commenced upon it, and many good plans are spoiled by the want of the precaution above mentioned.

It is advisable to lay off the first few main lines a little full in length, if the survey is of considerable size, for owing to the heat of the hands in working upon the paper, slight contraction is sure to occur.

Then again, if the paper be kept in a damp room, expansion may set in, and thus it is constantly varying in size. If however, every 10 chains is marked, no difficulty need be experienced inl plotting the details correctly, or in inserting new lines.

The main line having been laid down, and the stations where the lines of the principal triangles join it, marked off upon it, arcs must be struck to intersect each other, with these stations as centres, and the lengths of the sides of the triangles as radii.

Use of the Beam Compass.

These radii are usually struck by means of a beam compass, i.e. compass points fitted to a rule, or straight-edge, and capable of adjustment for length. The ordinary jointed compasses are not, as a rule, long enough for the purpose.

An excellent substitute for beam compasses can be made in the following manner.

From the edge of the paper upon which you are about to make your plan, or from a similar piece, cut a narrow strip about half an inch in width, and rule a fine line down the centre of it. Commencing about 1 inch from one end, mark off the lengths you are likely to require, pricking each through the paper, so that a fine pencil-point may be pushed through it. Then, with a fine needle passed through the mark at zero, and also inserted into the station upon the main line, and with a fine pencil-point passed through the strip of paper at the length of the radius which it is desired to strike, an arc of a circle of the required radius can be drawn in with accuracy.

The strip of paper described above, should always be kept rolled up in the plan in order that it may expand and contract in the same ratio as the plan itself.

Plotting the main lines on the Plan.

Referring again to the survey of Yeldon Farm, page 15, and to the diagram of lines in the 'field-book,' page 16. Line 1 having been laid down, and the stations upon it marked, it is required to lay down lines 2 and 3.

With the station at 2882 as a centre, and length of line 2, 1622 as a radius, strike an arc in the direction in which line 2 was run, then with 0·00 on line 1 as a centre and the length of so much of line 3 as is necessary for a radius, viz. 2555, strike another arc intersecting the former one, the point of intersection of the two arcs, if properly drawn, will give all that is necessary to complete the triangle. Before, however, this is finally accepted as correct, and pricked off, the length should be carefully tested, and line 10 also (which forms the principal proof-line of this triangle), should be tried, from 1825 on line 1 to this same intersection, and if that also fits, then the point should be carefully pricked off, and lines 2, 3 and 10 ruled in.

Line 10 should then be produced, this production being called line 16 in the 'field-notes,' its length 568, should be pricked off, and line 15 applied, which should then measure exactly 1296 to point 2882 on line 1.

The triangle formed by lines 18, part of line 20, and part of the main line 1, should then be laid down, and proved by means of line 19.

When these lines have been plotted from the *diagram*, the 'field-notes' may be taken up, and all remaining lines laid down in almost numerical order.

If care has been taken with the main line measurements, all the minor ones will fall into place, and will fit, within at all events, the margin allowed for permissible error in chaining.

Should any greater differences than those which are allowable be apparent, an error of some sort should be sought for, and it will generally be found that some mistake has been made in marking off the lengths on one or more lines, or that some line has been ruled to a wrong point. The mistakes thus made in the office, are at least three times as numerous as those made in the field, and in any case, the actual point of error may be so closely located, that it can be readily pitched upon in the field, without re-chaining more than one or two lines or parts of them.

It may not always be possible to remain in the neighbourhood of the survey until all the detail is plotted, but at all events, it is exceedingly unwise to leave it until every line has been laid down and proved to be correct, either upon the plan itself or upon a diagram drawn to scale.

If this be done the surveyor may leave the neighbourhood confident that, at the worst, only a few cases of error in details are possible, and that these will probably be discovered when plotting.

Most surveyors manage to keep their plotting closely up to the field-work. The first wet day affords an opportunity of getting the main lines laid down, after which, it is easy as a rule to keep the plotting well in hand, either by working in the evening, or by rising earlier in the morning.

Plotting details on the Plan.

For plotting the details, an offset scale as already described, page 27, is required.

When the survey lines are laid down and proved, for the whole, or for a portion of the survey, the details can be proceeded with, and here again the precaution taken to prick off each 10 chains upon the main lines, will come in very useful.

Practically, the art of plotting is simply that of reproducing upon paper to a small scale, and in true relative proportion, the actual measurements already taken '*in the field*' and recorded '*in the field-book*.'

As offsets have sometimes been recorded upon one side of the chained line, and sometimes upon the other (as when skew fences or streams have been crossed), or on both sides (as when surveying roads), it will be found more convenient and expeditious to lay the long scale parallel (but not close as is often done) to the line in question, and at such a distance that the zero of the offset scale shall coincide with the survey line.

The scale should then be weighted down at both ends, and the offset scale left free for the work. This method of plotting, especially if the details are very numerous, will be found to afford facilities for ruling in from point to point, and drawing in details, which would not exist were the long scale laid close to the survey line.

It will probably give the beginner some little trouble to get into the way of fixing his scale accurately in position, but with perseverance he cannot fail to become an adept at it, and the ease and expedition with which the work can then be plotted, will soon convince him that the result was worth the trouble.

In the ordinary way the scale is laid close to the line, and the offsets upon one side of it plotted first. The scale is then laid upon the other side of the line, and the remaining offsets plotted. This method, however, does not enable the surveyor to recall his every movement when in the field, with the same facility as the first-mentioned, and consequently, it does not enable him to so easily detect an error in 'plotting' or even in 'booking.' In plotting, the points should be marked off either with a very hard finely pointed pencil or be pricked in with a fine needle.

The '*ruling in*' should be done with a medium pencil, which can be cut to a fine chisel-shaped point, and will admit of the lines drawn being erased with india-rubber when necessary.

The surveyor should acquire the habit of reading his 'field-book' when it lies in any position upon the table, so that the line in the book may always be placed before him in the same direction as the line he is plotting.

This will save much confusion especially in complicated work.

Finishing the Plan, and Title.

After the work is all plotted, and any errors or doubtful points examined and corrected, the next thing is to ink it in.

This must be done with a drawing pen, and Indian ink should always be used, as it is practically everlasting, while other inks sooner or later either fade away or injure the paper. It is usual to show by characteristic signs, all leading features such as woods, plantations, gardens, quarries, sand, gravel or clay pits, tips, spoil banks, railway banks and cuttings, and such like, also to colour all dwelling houses carmine or lake, all other buildings Payne's grey or neutral tint, rivers and other water blue, &c.

For these and the printing, unless a highly ornamental finish is desired, the characteristics published for the 6-inch, 25-inch, and other Ordnance maps of England, may be taken as a guide, except that more detail is put upon these maps than is desirable or necessary for estate plans. In fact the superabundance of trees shown upon the 25-inch Ordnance map is a source of annoyance when tracings are required.

A title of a more or less ornamental character should be printed upon the plan.

Scale and North Point on Plan.

A scale and a North point must also be drawn. The latter should be plotted from the 'field-notes,' and drawn upon the plan in some convenient space off the surveyed work.

Methods of Determining the Direction of True North

The direction of true north, or the meridian line, should be ascertained in the field, before the actual chainage has commenced. It may be found by any of the following methods, with sufficient accuracy for small surveys :—

1. By means of the magnetic compass, the variation of which from true north may be seen by reference to 'Whitaker's Almanac.'

2. By ascertaining true 'local time,' and taking the direction of a shadow cast by a vertical rod at mid-day.

3. By means of a vertical pole and its shadow, as follows (fig. 13). Select

a convenient station upon the main line, where the ground is smooth and level, and plant a pole there firmly and truly vertical.

About one or two hours before noon, mark with a small peg the extreme end of the shadow cast by the pole, and with the foot of the pole as a centre and the distance to the peg as a radius, describe a segment of a circle upon the ground in the direction in which the shadow is travelling.

When the sun begins to decline, watch until the extremity of the shadow again coincides with the circle, and drive another peg at that point. Bisect the distance between the two pegs, and a line through the station and this point, will be in the direction of the true north. This line should be extended to a sufficient distance to enable it to be accurately surveyed.

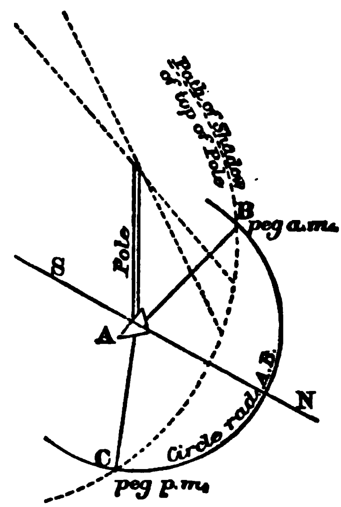


FIG. 13.

In case the sun is likely to be obscured by passing clouds, it is well to take two or more points, at say, 10 minute intervals before noon, and strike two or more corresponding arcs.

All these methods are sufficiently near for chain surveying, but when angular instruments are used, other and more exact methods must be employed.

A Terrier to Accompany the Plan. The 'fields' and 'enclosures' should now be numbered and the area of each calculated. A 'Terrier' is then made, which should exhibit at least the following particulars, in its columns:—

1. The numbers by which the enclosures are distingushed upon the plan.

2. The occupier's name or names (if more than one) in full.

3. A description of each enclosure (i.e.) whether simply a field, a field and pond, a farm-house, a garden, homestead or orchard, &c.

4. The state of cultivation, whether arable or pasture.

5. The area in acres, roods, and perches, or acres and decimals, as may be required.

6. A column of general remarks.

The items in the terrier may be entered either in their numerical order, or in collected form.

It is best to have it in both, especially if the estate is a large one.

The collected terrier is one in which each tenant's holdings have been collected together, and the total quantity of each is carried to a general summary.

This sum total should, of course, agree with the sum total of the numerical terrier.

Many estate agents have their own particular form of 'Terrier,' and may require separate columns for the quantities of arable or pasture land, woods, buildings, &c., in addition to a general column.

Calculation of Areas from a Plan.

The area included under each number must be ascertained, and since when using an ordinary scale to obtain them, all enclosures of whatever shape have to be resolved into triangles, right-angled parallelograms, or trapezoids, a description of how to calculate the areas of these figures will be sufficient.

1. The area of a triangle is found by multiplying its base by half its perpendicular or *vice versâ*. It may also be found from the lengths of the three sides, thus :—

$$\text{Area} = \sqrt{S \times (S - a) \times (S - b) \times (S - c)},$$

a, b and c being the three sides and S their half sum.

Or, putting it into words :—

From half the sum of the sides, subtract each side separately, and multiply together the half sum and the three remainders. The square root of the product will be the area required. This method is rarely used.

2. The area of a right-angled parallelogram is found by multiplying the length by the breadth.

3. The area of a trapezoid is obtained by multiplying the perpendicular distance between the parallel sides, by half the sum of the parallel sides.

Calculation of Areas from the Field-Notes.

The areas of enclosures may be obtained from the 'Field-Notes' direct, without reference to a plotted plan, but when it is desirable to do so the survey lines must be specially arranged for the purpose.

In the case of the 'single field' (fig. 10) illustrated upon page 23, the survey lines were so arranged, thus :—

The line A C formed the common base for the triangles A B C and A D C while B E and F D were purposely set out to form the perpendiculars, and these two triangles embrace the bulk of the area of the field.

The remainder can be dealt with by using the portions of the chained lines lying between each pair of offsets as the length for each trapezoid, and the average length of each pair of offsets as the widths.

Details of Working Out.

Working out of the area of the single field. See table, page 34, which refers to field-book, p. 23.

The result of the following calculation is recorded in square links, the chain used being the ordinary one, of 66 feet. Since there are 100,000 square links in an acre, it is only necessary to mark off, by means of a decimal point, five figures from the right, when the figures to the left of the decimal point represent the acres, and those to the right decimals of an acre.

The quantities are given in these units upon the 25-inch Ordnance maps of England. When an apparent division occurs, such as a road, or pathway the sign ɷ is used to indicate that this is included in the total area given.

If the result is required in acres, roods, and perches, it is necessary to multiply the decimal portion by 4 (the number of roods in an acre), and again cut off five figures, when the figure on the left of the point will be roods, multiply again by 40 (the number of perches in one rood), and again cut off five figures, the result is in perches, and decimals of a pole or perch.

It will also be seen that in this case 20½ chains of fence belong to the field and 23 chains do not, leaving an excess of 2½ chains, of a width of 6 links, to be deducted from the total quantity.

No.	Offsets $(h + h_1)$.	$\frac{h + h_1}{2}$.	Length.	Area.	Remarks.
1	15 / 30	22·5	100	2250	Line A B.
2	30 / 30	30	500	15000	
3	30 / .10	20	300	6000	
4	10 / 0	5	10	50	
5	10 / 40	25	120	3000	Line B C.
6	40 / 40	40	350	14000	
7	40 / 20	30	190	5700	
8	20 / 5	12·5	450	5625	
9	5 / 10	7·5	210	1575	
10	10 / 3	6·5	65	422	
11	5 / 50	27·5	130	3575	Line C D.
12	50 / 50	50	230	11500	
13	50 / 12	31	322	9982	
14	12 / 0	6	5	30	
15	27 / 10	18·5	300	5550	Line D A.
16	10 / 40	25	370	9250	
17	40 / 10	25	170	4250	
18	10 / 20	15	360	5400	
19	20 / 10	15	150	2250	
20	10 / 5	7·5	15	113	
21	743 / 509	626	1670	1,045,420	A B C D, line A C.

Deduction for fences
= 250 × 6
= 1500 sq. links
= ·015 acre
= 2·4 perches.

Final area, 11·494 acres.

1,150,942 sq. links.

11·50942 acres
4
2·33798 roods.
40
13·50720 perches.

Area = 11·509 acres, or 11 acres, 2 roods, 13·5 perches.

If the chain used had been one of 100 feet in length, the result would of course have been in square feet, of which there are 43,560 in an acre.

The foregoing method gives the area with the greatest possible accuracy, but when it is determined to make use of the 'field-notes' for this purpose, great care must be exercised to so arrange the survey lines in the field, that no piece of ground be taken into account twice over, and that no piece be omitted altogether.

Such cases are likely to occur at the corners of fields, unless proper precautions be taken.

The best method of avoiding such double measurements, is to commence each line from a station upon an existing line, and to produce it through the point from which the next line will start, until it strikes the boundary.

Thus line 2 commences from 1670 line 1, and is continued beyond 660 the actual closing point from which the next line starts, until the boundary fence is reached at 687. This enables the surveyor to run an offset to the corner by which to obtain the area of that small piece, which otherwise would be omitted from the calculations. Likewise line 3 commences from 660 line 2 and is run past 1350 the starting point of next line, and lines 4 and 5 are treated in like manner.

When the survey embraces a considerable acreage of land, or even more than a single field, the areas are usually obtained from the plotted plan, and in that case the enclosures have each to be divided up into triangles and trapezoids, &c., and their dimensions scaled from the plan.

Area of a large survey by scale only.

Had the foregoing example of a 'field' formed part of a large survey, the method of obtaining the area would have been practically the same, except that the boundary fences, which form no obstruction to an imaginary line drawn upon the plan, would be equalised, as far as possible, by give and take straight lines, and thus the number of the figures to be worked out would have been reduced to as few as possible, and in this instance might have numbered two large triangles and two small parallelograms.

Area by computing scale.

The method of arriving at the area by means of the 'computing scale' must now be considered.

The 'computing scale' can be obtained to any scale, and to give results in acres and decimals, or in acres, roods, and perches, as desired. It is simply a device by which the number of chains passed over by the sliding vane is recorded, and to this end it is used in conjunction with a piece of transparent paper, ruled with parallel lines at a fixed distance apart (say one chain) as noted on each scale.

It is manifest that as 10 square chains make one acre, a length of 10 chains upon the scale has simply to be divided into 4 parts for roods, and these again into 40 parts to represent perches, or the whole 10 chains can be marked decimally with equal facility.

For small irregular enclosures this system is much quicker than that of dividing each field into imaginary figures, and scaling their dimensions, as already described, indeed the whole operation is simply mechanical—no calculation whatever being necessary.

The scale consists of two parts. The scale itself, and a small sliding frame, carrying a fine wire or hair at right angles to the scale.

To use the scale. Set the centre of the sliding frame to zero, placing the scale upon the paper in the same direction as the lines, and with the wire so adjusted that the portion marked '*a*' in the above diagram, fig. 14, is equal to the portion marked '*b*,' then, holding the scale firmly, slide the frame forward until the wire is in such a position as to make the portion marked '*c*' equal to that marked '*d*,' then lift the scale bodily, taking care not to disturb the sliding frame, and commence upon the next line, making '*e*' equal to '*f*,' then slide the frame on till '*g*' equals '*h*,' and so on until the whole length of the scale is used. If the field is not then completed, make a small mark or dot immediately under the wire, and commence from the right hand and work *backwards* to the left. It will be seen that the scale is figured from left to right upon the upper side, and continued from right to left upon the lower side, for a 3 chain or 25·344-inch scale it is usually 5 acres in length, so that double its length, or 10 acres, can be run off, before making any special note.

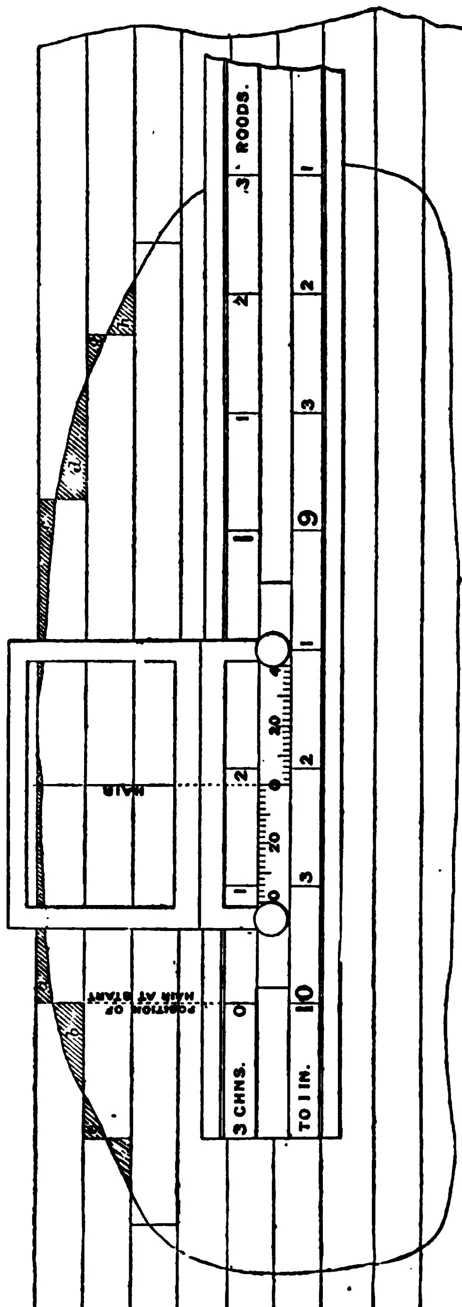


FIG. 14.

The degree of accuracy attainable by the use of this instrument is proportionate to the skill of the operator in the equalisation of the boundaries.

A 'universal computer' is made in which the scales are interchangeable, and its case usually contains scales of 1, 2, 3, 4, 5 and 6 chains to an inch, as well as, 6 inches, and 5 feet to a mile. Ruled transparent papers, or parchments to match, are also obtainable.

It is manifest that any required scale could be fitted to such a sliding frame.

Computing scale 'adapting papers.' If a 'universal computer' be not available, it is useful to know that computing scales to one scale, may be used for plans of other scales, either by correcting the quantities by a table of equivalent areas, or by using papers, the parallel lines of which have been adapted in width to meet the special case.

These 'adapting papers,' unlike those for ordinary scales, cannot be purchased, and must be made by the surveyor himself, in fact it is as well to prepare all one's own papers, as they are easily and quickly made, either on tracing paper or on linen, and when soiled, can be replaced by clean ones.

For an 'adapting paper' the required width for the lines is arrived at in the following manner:—

The number of square chains in one square inch upon the plan, divided by the number of lineal chains to one lineal inch upon the scale, will give the number of lines to be ruled to 1 inch upon the computing paper. Suppose it is required to scale a plan plotted to 3 chains to an inch, with a 4 chain computing scale then $\frac{3 \times 3}{4} = \frac{9}{4} = 2 \cdot 25$ lines per inch, or $22 \cdot 5$ lines to 10 inches, which makes it easy to divide up. Again for a 4-chain scale to be used with a 5-chain plan, $\frac{5 \times 5}{4} = 6 \cdot 25$ lines to an inch.

In the above cases a proportion common to both plan and scale is given, in chains to 1 inch, but where this is not the case, some common denomination must be found. Thus to use a 3-chain scale to scale a $25 \cdot 344$ inches to the mile plan. Here inches per mile is a convenient denomination.

3 chains to 1 inch $= 26 \cdot 6$ inches to a mile, then $\frac{25 \cdot 344 \times 25 \cdot 344}{26 \cdot 6}$ $= 24 \cdot 087$ inches to be divided as if for 1 mile, or 80 chains.

Area by the 'Planimeter.' Another means of arriving at areas, is by the use of Amsler's 'Planimeter,' which in its improved form, may be adjusted for any scale. The result is read off in square inches and decimal parts, or square acres and decimals, as the case may be.

This useful instrument consists of two arms, hinged together, one of which is of fixed length and furnished with a fine needle point for fixing it to the plan, forming a centre round which the whole instrument is free to rotate.

This needle point is kept in position by a small weight placed above it.

The other arm is of variable length, and capable of adjustment to suit any scale.

At its extreme end it is provided with a tracing point, which is free to move in any direction. It also carries a drum or wheel, which is rotated by the contact of its rim with the surface of the plan as it is dragged over it, and also with another wheel which records every complete revolution of the drum.

One complete revolution of the drum is ten whole numbers, of whatever denomination is being used. It is fitted with a vernier which is capable of being read by the eye alone, to two places of decimals, and by the aid of a magnifying glass, to a third place.

When using this instrument, first adjust the variable arm to the mark upon it indicating the requisite scale, and then place the instrument upon the plan in such a position, that while the *needle-point*, with the weight upon it, remains stationary at some spot a convenient distance *outside* the field to be measured, the *tracing point* upon the other arm can be run all round its boundary. The tracer must first be placed at some conspicuous point upon the boundary, and the reading taken, it must then be passed carefully round the entire boundary of the field (in the same direction as that in which the hands of a watch revolve) until the starting point is again reached.

The reading upon the wheel and drum, must then be taken again. The difference of the two readings will be the area of the field.

It is possible, by very delicate handling while holding the tracing point in position upon the boundary of the field with one hand, to turn the drum to zero with the other, so that after passing the tracer round the field, the area can be read off direct.

In case the adjustable arm is not marked for the special scale required, a simple, and probably the surest method of adjustment, is to mark out a square or parallelogram of ten acres, and by trial, adjust the sliding bar until the instrument records exactly one revolution of the drum, after the tracer has been passed round the boundary.

This instrument is extremely sensitive, but if skilfully used, is a great help to those who have a large number of areas to take out.

Methods of making linear measurements with the Steel Tape where special accuracy is required.

Formerly, when great accuracy was required in making linear measurements, such as for base lines for national trigonometrical surveys, it was customary to employ rods made of glass or metal, whose lengths were known at a given temperature, say 60° Fahr., additions or deductions being made for any variation above or below that temperature, or compensated rods made of two metals were sometimes used.

Substitution of Steel Tapes for Rods.

Nowadays, however, steel tapes are taking the place of the above (especially in the United States), it having been found that quite as great a degree of accuracy can be attained by their use, sufficient care being taken in the first instance to ascertain their exact length, and the necessary corrections made for any 'rise' or 'fall' of temperature. Tapes have the advantage of being much more convenient to use than rods.

Steel Tapes.

The section of tapes used in practice varies from '0·1225 × 0·005' to '0·25 × 0·02' of an inch (the latter weighing $\frac{3}{4}$ oz. per yard), and the length from 66 feet to 100 metres. The latter is generally employed for important work, such as measuring a base line for a large trigonometrical survey, and it is divided into 20-metre lengths. For purposes of carriage these tapes are rolled on reels, and loops are formed at each end by annealing and riveting it back on itself, one for securing the rear end to a stake, and the other for attachment to the apparatus by means of which the tension is applied. The lighter section has the advantage of offering less surface to the wind, and for dew to condense on.

'Tension apparatus' for Tapes. The best form of 'tension apparatus' consists of two equal levers attached at right angles with a knife-edge bearing at the elbow. The tape is attached to the vertical arm, and the horizontal arm is provided with a spirit level and weights, or a spring balance, by means of which the proper degree of tension is applied, this should vary from 10 lbs. to 25½ lbs. according to the length and weight of the tape used.

Between the rear end of the tape, and the stake to which it is secured, there should be a slow-motion screw to adjust it in position.

The best method of marking the leading end of the tape, is by means of a vernier fixed on the head of a stake and raised by a screw under the tape. The graduation extends 9 mm. each way, which being divided into ten parts reads to $\frac{1}{10}$ mm. The screw is provided with a hook in the centre of its head to which a plumb-bob may be attached.

The true length of, say a 100 metre tape, may be determined in the following manner. Select a level piece of ground and drive supporting stakes 10 metres apart, the heads being ranged in line and levelled. The tape is then secured in position and the tension applied. Its length can then be measured with a standard bar, the correction for temperature being applied. The temperature of the tape is constantly observed during the operation, which is repeated from 50 to 100 times, the differences being averaged by the 'method of least squares.' By this means the probable error in the length of the tape can be reduced to $\frac{1}{500,000}$ of its total length.

System of Measurement. Where an important 'base line' has to be measured, marking and support stakes are ranged and adjusted beforehand 10 to 20 metres apart to save time when measuring. The measurements should be made at night, so as to be subject to the smallest possible range of temperature. The number of men required is about 12. An observer at each end of the tape, 2 thermometer observers, 1 recorder, 3 stretchers and 5 men to carry lamps, and bring forward the tape after each length is measured.

The rear extremity is first adjusted by bringing the zero exactly over the mark, by means of a slow-motion screw, the vernier reading of the leading end is then noted and the exact distance recorded, the readings of the thermometer are entered and the chain advanced. By this procedure about 2 kilometres may be measured per hour.

To obviate the tendency of the temperature of the tape to lag behind that of the air, the measurement of a line should be made first with a rising and then with a falling thermometer. This is repeated several times in opposite directions, and a mean taken, the range of the differences should not exceed about $\frac{1}{160,000}$ of the entire length.

Corrections for Slope. If the ground is not level the measured length must be corrected and reduced to the true horizontal length.

If L = length as measured

h = difference of level

Then Horizontal length $= \sqrt{L^2 - h^2}$.

Correction for Changes of Temperature.

Changes of temperature are allowed for as follows :—

If A = nominal length of tape.

a = small excess over round numbers.

α = rate of expansion.

t = temperature in degrees C.

Then $L = (A + a) + \alpha t$.

On the survey of the city of Sydney, where a tape 66 feet long, and $\frac{1}{4}$ inch wide, was used, it was found that the rate of expansion was equal to 0·005 inch per 1° Fahr.

Correction to be applied, if one or more supports are omitted.

It sometimes happens that it is convenient to omit one or more supports, in which case the length must be corrected as follows :—

If w = weight per unit length of tape.

r = tension applied.

$a = \dfrac{w}{r}$.

n = number of equidistant supports.

l = length between do.

L = total length of tape under standard tension = $n\,l$.

μ = reciprocal of modulus of elasticity multiplied by sectional area of tape.

Then ny change in length Δ L, due to a change $\Delta\, r$ in tension,

$$= n\,l\,\mu\,\Delta\,r + \frac{l}{12}\,a^2\,n\,l^3\,\frac{\Delta\,r}{r}.$$

Example. If n = 10.

l = 10 metres.

w = 22·32 grammes per metre.

r = $25\frac{1}{2}$ lbs. = $11{,}566\frac{3}{8}$ grammes.

$a^2 = 372 \times 10^{-9}$.

$\mu\,\Delta\,r = 16 \times 10^{-9}$ per gramme unit.

or $= 450 \times 10^{-9}$ per ounce.

Hence

If $\Delta\,r$ = 1 ounce,

$$n\,l\,\mu\,\Delta\,r = 10 \times 10 \times 450 \times 10^{-9} = \cdot 045 \text{ mm.}$$

$$\frac{1}{12}\,a^2\,n\,l^3\,\frac{\Delta\,r}{r} = \frac{1}{12} \times 372 \times 10^{-6} \times 10^3 \times \frac{1}{408} = \cdot 0076 \text{ mm.}$$

$$\therefore\ \Delta\,L = 0{\cdot}0526 \text{ mm.}$$

For 100 m. tape, supported every 20 m.,

$$\Delta\,L = \frac{1}{24} \times 372 \times 10^{-8}\,(5 \times 20^3 - 10 \times 10^3) = 4{\cdot}65 \text{ mm.}$$

For further information on this subject *vide* 'Transactions of the American Society of Civil Engineers,' vol. xxx. page 81.

CHAPTER II.

OPTICS, MAGNETISM, THE SPIRIT BUBBLE, ETC.

General Remarks.

BEFORE describing the various instruments used for taking observations when surveying, it is desirable that the student should have before him a short account of the optical principles involved in their construction. The writer also considers that a few words on magnetic forces, atmospheric pressures, and the spirit bubble, will not be here out of place.

OPTICS.

Reflection.

The course of a ray of light after reflection follows two simple laws, viz. (*vide* fig. 15).

First law :—*The reflected ray lies in the same plane as the incident ray.*

Second law :—*The angle of reflection is equal and opposite to the angle of incidence.*

These laws equally govern the reflection of rays from the faces of glass prisms, with that of rays incident on plane reflecting surfaces.

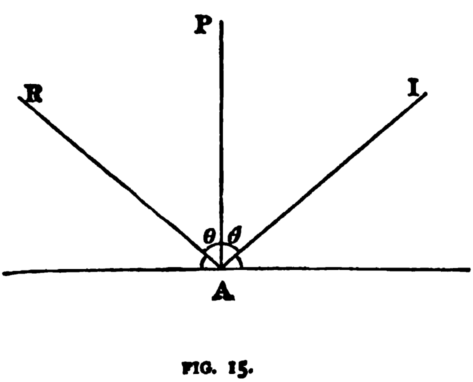


FIG. 15.

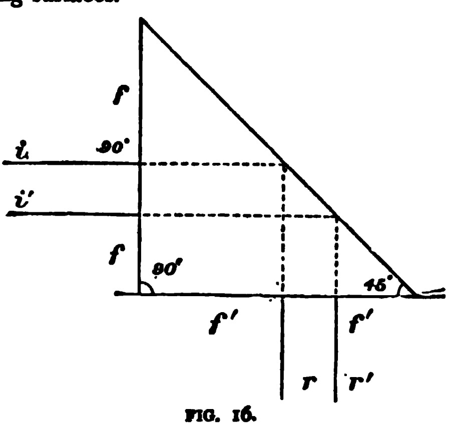


FIG. 16.

With a right-angled prism (fig. 16), the incident ray *i* entering perpendicular to the face *ff*, and meeting the hypothenuse at an angle of 45° (an angle with the normal greater than the critical angle, hereafter described under the head of 'Refraction'), is reflected and does not emerge, but proceeds at right angles to its original course, leaving the prism perpendicular to the face *f'f'*.

Prismatic reflection is used where practicable, since the reflecting surface is covered, and therefore free from dust or dirt.

Deviation of Reflected Rays produced by the Rotation of Mirrors.

Let A B (fig. 17), represent a mirror capable of rotation about an axis through O, and let I O represent an incident ray. In the first position of the mirror the ray will be reflected directly back upon its course, but on being turned through an angle A O A′ = a, to its second position A′ O B′, the incident ray will be reflected in the direction O R. If N O be the normal to the mirror in this position, then the angle of incidence I O N = a' is equal to the angle of reflection N O R = a'' (both being equal to a), hence the deviation of the reflected ray is double the angle I O N = the angle A O A′ through which the mirror has been rotated. It thus appears that, *when a plane mirror is rotated in the plane of incidence, the direction of the reflected ray is changed by double the angle through which the mirror is turned.*

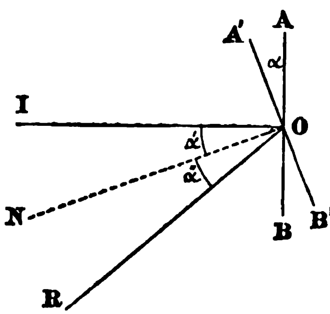


FIG. 17.

Advantage is taken of this principle in the construction of the sextant.

Refraction.

The 'properties of a lens' depend on the fact, that a ray of light passing obliquely, from air into a dense, transparent medium, and conversely, is *bent*, or as it is termed, *refracted*, at a certain angle towards the normal to the surface of the dense medium (in the present instance, *glass*).

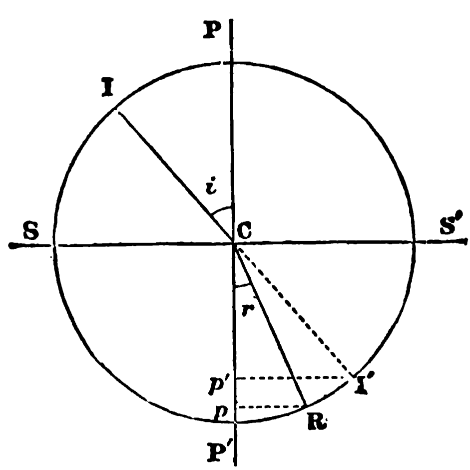


FIG. 18.

The ray on entering, is termed the *incident ray*, and when leaving, the *emergent ray*.

There is no known medium which refracts the component coloured rays of a 'pencil' of white light at a uniform angle, the pencil of white light becoming *dispersed* into rays of all colours, and forming *the rainbow*. In considering 'refraction' in its simplest aspect, it is therefore necessary, to follow the course of *one* of the rays composing a pencil of white light, such as the red, yellow, green, &c., that is, a *monochromatic ray*.

Every transparent medium has a special power of refracting rays, hence different kinds of glass refract in different degrees, a quality made use of in making achromatic lenses and object-glasses.

First law:—*The plane of refraction lies uniformly in that containing the incident ray, and the perpendicular to the plane separating the two media.*

Second law:—*The ratio which the sine of the 'angle of incidence' bears to the sine of the 'angle of refraction,' is constant for any two transparent media.* This ratio is termed the '*index of refraction.*'

The second law of refraction is exemplified by the following diagram (fig. 18).

Let P P′ be a perpendicular to the surface of the plane of the dense medium (glass) with air above it, and S S′ the surface separating these media. All refractions are measured from this perpendicular, or *normal*. The incident ray I C is refracted *toward* this normal to R. Let the angle I C P = i and the angle R C P′ = r, then it is found that $\sin i : \sin r$ is a constant ratio according to the density of the glass, and is usually expressed by the equation $\sin i = \mu \sin r$, μ being termed the '*index of refraction.*' For example, if $\sin i$ = I′p' measures 3 parts whilst $\sin r$ = Rp measures 2, the 'index of refraction' $\mu = \frac{3}{2}$ or 1·5.

This law is the same for an *emergent* ray.

The following table gives the 'indices of refraction' of several substances:—

Indices of Refraction.

Diamond	2·44 to 2·75	Rock salt	1·54
Sapphire	1·79	Alcohol	1·37
Flint glass	1·57 to 1·64	Humours of the eye	1·34
Crown glass	1·53 to 1·56	Pure water	1·34

Air at 0° C. = 32° F. and 760 mm. pressure 1·000294.

Limit of Angle of Refraction. Since the angle of the 'incident' ray with the normal, is always greater than that of the 'refracted' ray, it is obvious that with an emergent ray from the denser medium, there is a certain limiting angle, beyond which the ray will not emerge, but will be internally reflected. With ordinary glass this limiting, or *critical angle*, as it is termed, is 41° 48′ 37″ from the normal. This property of *refraction*, turning, as it were, into *reflection*, is made use of in the construction of several optical instruments.

If x = the critical angle, and μ = the index of refraction, then $\sin x = \frac{1}{\mu}$.

Convex Refraction. Let A B (fig. 19), represent the base of an equilateral prism or axis of a lens, being in the direction of the line joining a luminous point O with the eye. Let a ray a from O be so bent in its course through the prism, that it becomes parallel to the base, and after emerging, reaches the eye at a point in the axis, equidistant with O from the prism. If the rest of the prism be covered up, we should see the image of the luminous point O. If we now assume a prism of similar density but of less angle, whose base is A' B', a ray *b* from O will pass through the prisms parallel to the base, and meet the eye at the same point as the ray *a*. Similarly for other prisms of still lesser angles. Assume now that a half lens, as shown, has its surfaces so ground as to be tangential to the sides of these prisms where the several rays enter them, then a perfect lens would be formed. The refractions above assumed, bend the rays till they are parallel to the bases of the prisms, which could only occur when the eye and object are at equal distances from the lens, this distance being proportionate to the refractive power of the glass used. If the rays were all *parallel* on incidence they would still be collected in one point, but nearer to the lens.

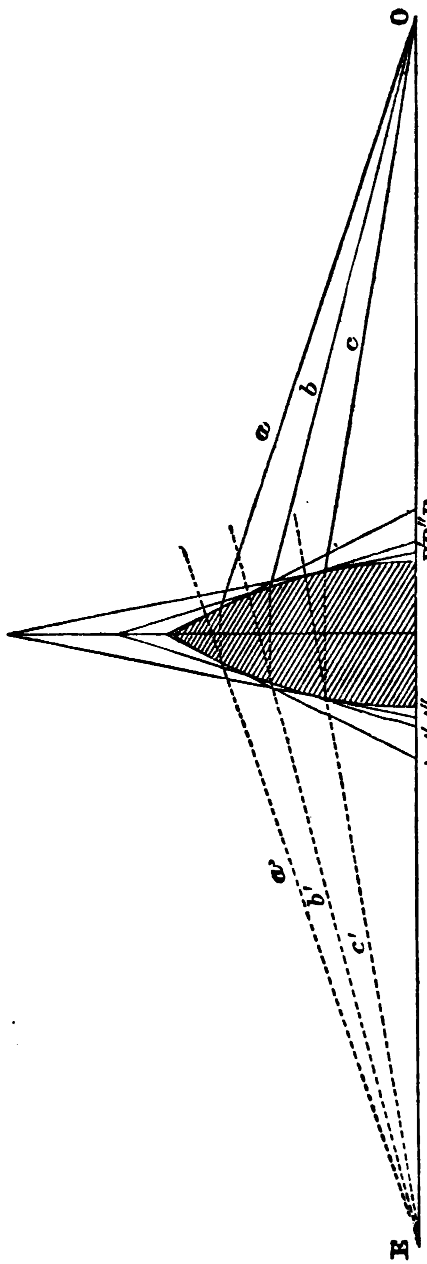


FIG. 19.

Axes and Foci of Lenses.

Principal Axes.—A lens may be regarded as a solid of revolution, the axis of which is termed the 'principal axis.'

Principal Foci of Convex and Concave Lenses.—When rays which were originally parallel to the 'principal axis' pass through a convex lens (fig. 20), the effect of the double refractions they undergo, is to make them converge, approximately (this term is introduced advisedly on account of the effect of aberration), to one point F, which is called the 'principal focus.' The distance F A is called the 'principal focal distance,' or 'focal length' of the lens. In this case the focus is *real*.

When similar rays pass through a concave lens (fig. 21), they diverge from the line of the 'principal axis,' and if produced backwards, would approximately meet in a point which is still called the 'principal focus,' though being imaginary and not *real*, it is termed a *virtual* focus.

Optical Centre of a Lens.—This point lies on the 'principal axis,' and is so situated that every ray whose incident direction is parallel to its emergent direction, must pass through it. In the case of a double convex or double concave lens the 'optical centre' is so situated in the interior, that its distances from the surfaces are directly proportional to the radii of the same. In a plano-convex or plano-concave lens it is situated on the convex or concave surfaces respectively.

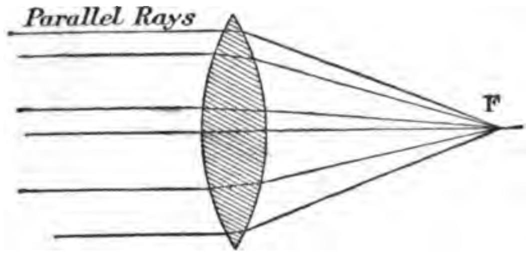


FIG. 20.

In elementary optics it is usual to neglect the thickness of the lens, since we may lay down the proposition that *rays which pass through the centre of a lens undergo no deviation.*

Secondary Axes.—Any straight line through the centre of a lens oblique to the 'principal axis,' is termed a 'secondary axis.' Rays parallel to a 'secondary axis' converge to a point, and for small obliquity from the 'principal axis,' the 'focal distances' are the same.

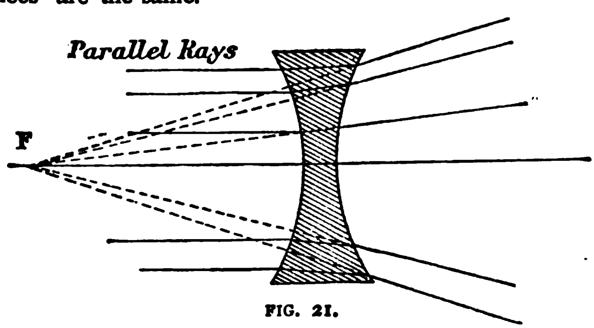


FIG. 21.

Conjugate Foci.—When a luminous point is placed near a lens, but beyond the 'principal focus,' incident rays proceeding from it, converge to a point (approximately) on the other side of the lens. Two points so related are termed 'conjugate foci.'

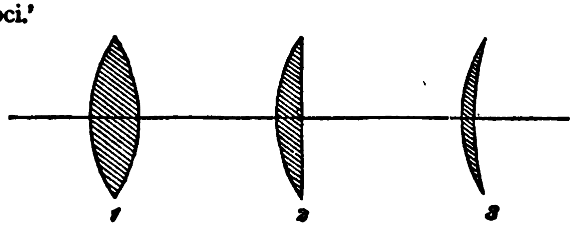


FIG. 22.—CONVERGING LENSES.

1. Double convex. 2. Plano-convex. 3. Concavo-convex.

Lenses.—There are two principal classes of lenses, viz. 'converging' and 'diverging' (*vide* figs. 22 and 22*a*).

Aberration.

Spherical Aberration.—If a lens were ground truly spherical, it would be found that the rays from a luminous point on one side of it, would not converge accurately to one point after passing through it,

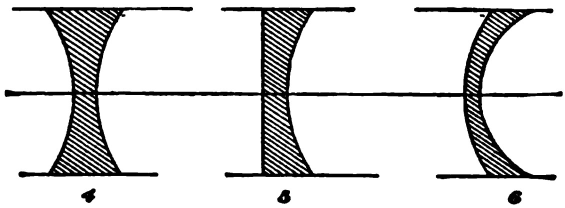


FIG. 22a.—DIVERGING LENSES.
4. Double concave. 5. Plano-concave. 6. Convexo-concave.

the rays traversing the circumferential portion falling short. This defect is termed 'spherical aberration,' and to correct it, lenses have to be finally 'figured' by hand.

Chromatic Aberration.—Since the elementary rays of coloured light which make up white light are unequally refracted, they do not converge to one point after passing through a simple lens, but form a spectrum or series of coloured images, the violet being the nearest and the red the most remote. This source of confusion is termed 'chromatic aberration.'

In order to correct this dispersion, different kinds of glass, viz. flint and crown (which do not possess the same powers of refraction) are combined to

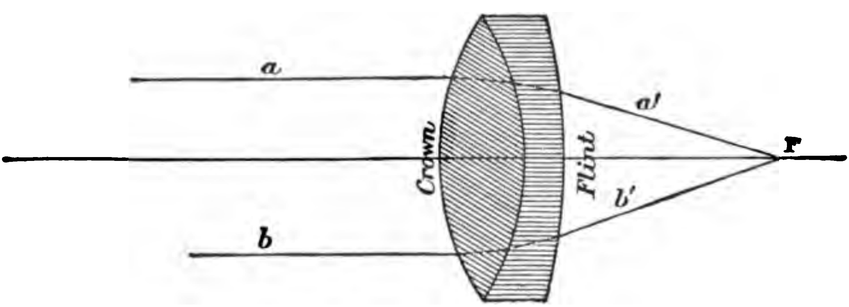


FIG. 23.

form what is termed an 'achromatic lens' (*vide* fig. 23). Here the parallel rays of white light *a b* are converged, and as *a' b'* pass through the principal focus F still as rays of white light.

Pencil of Rays.

A 'pencil' means a solid cone of rays whose vertex is termed the 'focus.' When a pencil of rays undergoes reflection or refraction, the reflected or refracted rays are still usually spoken of as a 'pencil,' even if they no longer pass through one point.

Formation of Images by Lenses.

For purposes of investigation the object placed before a lens must be assumed to be composed of a number of luminous points, which delineate it, so that the effects of the lens on the rays emanating from these points, may be severally discussed. Let A B (fig. 24), be such an image, placed in front of a lens O at a distance greater than that of the principal focus F from its centre. It will be seen that a real image *a b* is formed on the other side of the lens. To determine the position of this image graphically, draw through any point A a ray parallel to the 'principal axis,' which is incident on the lens at A′, and after refraction passes through the 'principal focus' F. From the same point A draw a secondary axis A O, and produce it till it meets the ray A A′ F in '*a*,' which is thus determined as the focus conjugate to A. In like manner the image of any other point B is found

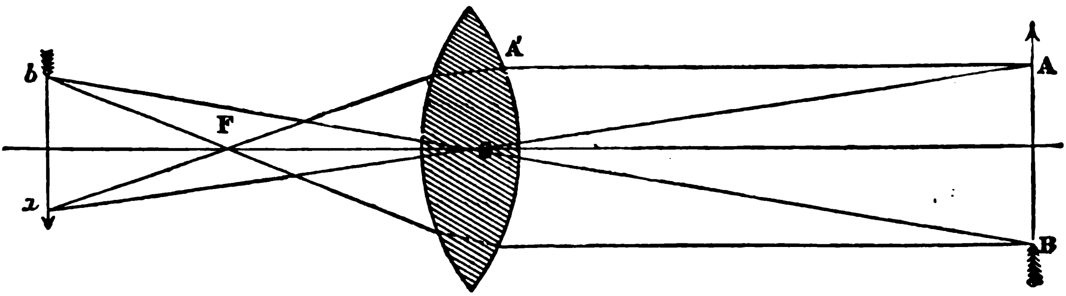


FIG. 24.

to be formed at '*b*.' The line joining *a*, *b*, etc., will be the image of A, B, etc. If the object be placed within the principal focal distance of the lens, a virtual (and not a real) image will be formed (fig. 25). The conjugate axes must be produced backwards to *a* and *b*, and an erect though unreal, *virtual image* (as it is termed) is formed.

Concave lenses invariably form unreal or *virtual* images.

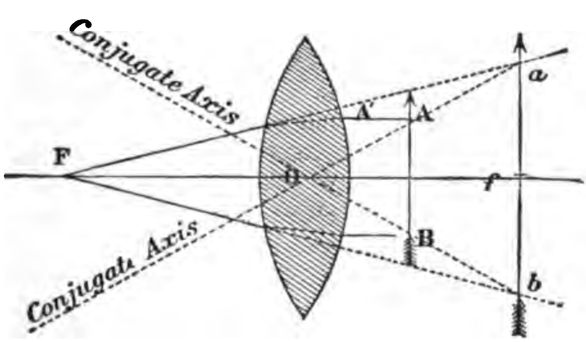


FIG. 25.

The Human Eye as an Optical Instrument. A pencil of rays entering the eye from an external point undergoes a series of refractions in passing through the cornea (in shape like a convex watch-glass), the aqueous humour, and crystalline lens, and finally converges on the retina, forming a real and inverted image there of the object viewed. Either by change in focal length, or in the distance of the retina, the eye adapts itself to distinct vision for varying distances. In looking at distant objects (if our vision be not defective) we experience very little sense of effort. Not so when an object is viewed at distances less than that which gives the most distinct vision (this averages about 8 to 10 inches with a healthy eye), for in this case it soon becomes impossible to get a clear view, and all becomes blurred and indistinct.

Although the image of an object formed on the retina is inverted, the same is erected by an effort of the optic nerves, which is difficult to account for.

Visual Angle and Magnifying Power. The angle which a given distance subtends at the eye is called the 'visual angle.' Two discs of different diameters appear to be the same size if the angles they subtend are the same.

By the 'magnifying power' of an optical instrument is usually meant, the ratio in which such distances are apparently increased, and the instrument is said to magnify *so many diameters*.

Simple Magnifier. A magnifying glass is a convex lens of shorter focal length than the human eye (*vide* fig. 26), and it must be placed at a distance somewhat less than its focal length from the object to be viewed. In the figure *a b* is the object and A B the virtual image seen by the eye at K.

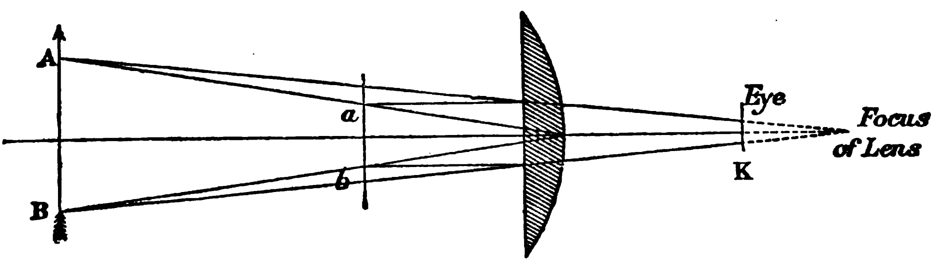


FIG. 26.

Optical Arrangements of the Telescope. The optical arrangements of the earliest form of telescope (Kepler's) are shown in fig. 27. It consisted of two lenses, one (the object-glass) forming a real and inverted image of the object, the other (the eye-piece) acting as a magnifying glass to view the same.

A′ B′ is the virtual image of the object as viewed through the eye-lens. The rays from A and B are slightly divergent on entering the eye, as they converge on the points A′ and B′, and they are brought to a focus on the retina, thus forming distinct images. It must be remembered that it is only parallel and *slightly* divergent rays which can be so focussed by the human eye—a very important consideration when arranging a sequence of lenses so as to produce distinct vision.

Magnification. The angle under which the distance A B would be viewed by the naked eye is obviously $a\,O\,b$, whilst the angle at which it is seen through the telescope is $a\,O'\,b$. The magnification is therefore $\frac{a\,O'\,b}{a\,O\,b}$, which is approximately the same as the ratio $\frac{O\,F}{O'\,F}$. If the eye-piece be adjusted so as to throw the image A′ B′ to infinite distance, F will be the principal focus of both lenses, and the magnification is the ratio of the focal length of the object-glass to that of the eye-piece.

The magnification of a telescope can be directly ascertained by observing with one eye (externally to the instrument) the graduations on a staff, whilst with the other eye the number of graduations which appear to be superimposed or correspond is noted through the telescope.

Best Position of the Eye.

If the telescope be directed to a bright sky, and a piece of paper be held behind the eye-piece, a circular spot of light (which is the image of the aperture which the object-glass fills), will become sharply defined, when the paper is held in a certain position. This is the best position for the eye when observing, since all the rays passing through the object-glass must also pass through this spot. This spot is usually much smaller than the pupil of the eye.

One method of determining the magnifying power of a telescope consists in measuring the diameter of this bright spot, and comparing this measurement with the diameter of the object-glass. An instrument termed a dynameter is used for making this measurement.

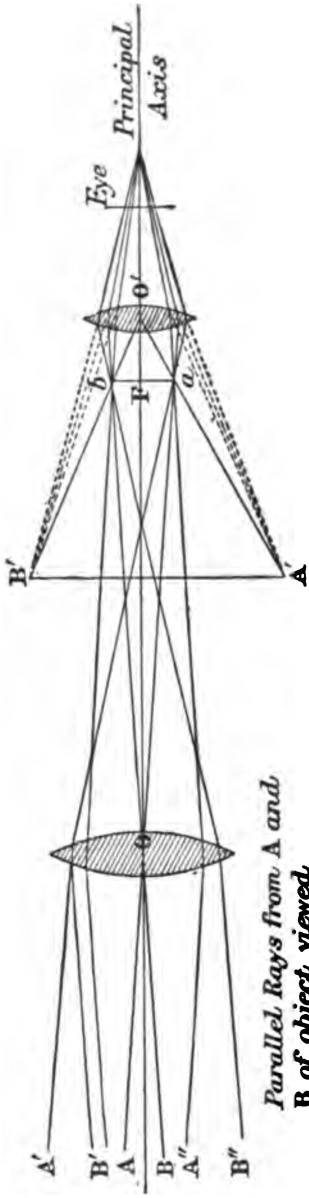


FIG. 27.

Terrestrial Telescopes.

Since the combination of lenses in a telescope as above described, produces an inverted image, erecting eye-pieces are often used for terrestrial telescopes, but as loss of light is necessarily involved owing to the greater number of lenses employed, these eye-pieces are not often used for theodolites, where sharp, distinct vision, is of great importance.

Galilean Telescopes.

This telescope as invented by Galileo, and the earliest of all telescopes, gives erect images with only two lenses. A double-concave lens is used for the eye-piece, and the rays refracted by the object-glass are brought to a virtual focus at a point between the two lenses, a magnified and erect image being formed.

This telescope has the disadvantage of not admitting of the employment of cross-wires, for no real image is formed. Opera-glasses and the telescopes of sextants (in which the wires only define the central portion of the field of view, and are in no way used for purposes of measurement), are constructed on this principle.

Eye-pieces.

1. The Ramsden, or *positive* eye-piece. This consists of two plano-convex lenses, the *convex* surfaces of which are turned to each other (*vide* fig. 28). The rays from the inverted image *a b* examined by this eye-piece enter the eye slightly convergent towards a principal focus at F, and form a distinct, but inverted image of the object, (as viewed at *a b* on the plane of the cross-wires) on the retina at *f*.

Each lens is of focal length 3 *a*, and they are fixed at a distance 2 *a* from each other.

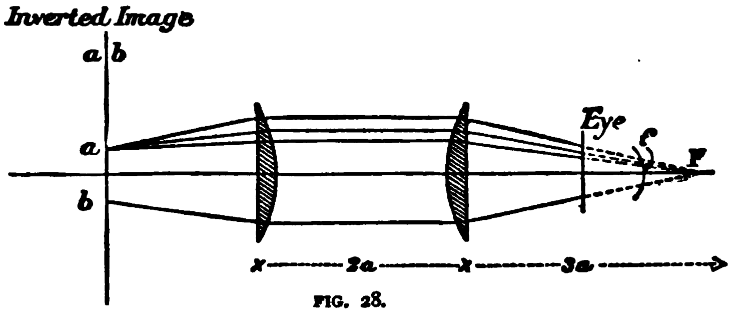


FIG. 28.

2. The Huyghenian, or *negative* eye-piece. This consists of two plano-convex lenses, the convex surfaces of which are turned towards the objective (*vide* fig. 29).

The larger lens (next the objective) has a focal length of 3 *a*, whilst the smaller has one of *a*, bringing the converging rays from the objective to a focus at F′, these being further converged by the eye, forming an image on the retina.

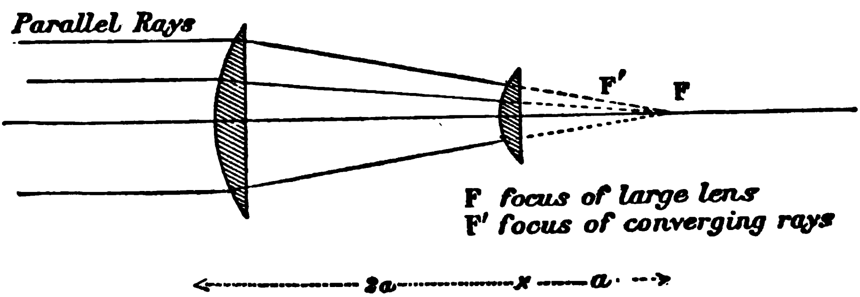


FIG. 29.

The following is a graphic account of the optics of lenses leading up to the construction of a Telescope.

The Optics of Lenses and Telescopes.

A properly formed lens, like the objective of a telescope, has the property that the rays of light emanating from a point in front of it are re-assembled in a point behind it. Suppose that the object-glass O (fig. 30) of a telescope be removed and fixed in a hole in a window-shutter. Let a screen of cardboard be placed vertically on a table inside the darkened room, and let a graduated staff be held vertically outside it at any fixed distance from the lens. By moving the screen to or from the lens, a position will be found at which a complete, and distinct *inverted* picture of the staff and its graduations, will be seen projected upon the paper screen inside the room. If a sheet of sensitive paper, like that used for copying plans, were used as a screen, a photograph of the staff, and of the landscape behind it, might be obtained. The reason of this is, that the cone of rays emanating from the graduation A and falling on the surface of the lens, is again concentrated to a point, or *focus*, at *a* on the screen, similarly the other

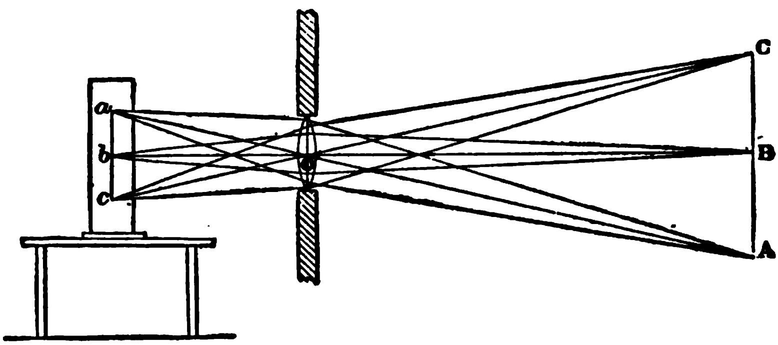


FIG. 30.

graduations B and C are projected at the foci *b* and *c* respectively, and so with all intermediate graduations. With a well-constructed lens the various graduations on a straight staff will all be seen simultaneously, distinctly, and sharply, on a flat screen at some one distance from the *lens*. Again, if O be the centre of the lens, the lines A O *a*, B O *b*, C O *c* will be straight lines so that

A B : A *b* : : O A : O *a* : : O B : O *b* and so on.

A line joining the centres of the two spherical surfaces of a lens, is called the 'Optical Axis,' (*vide* fig. 31).

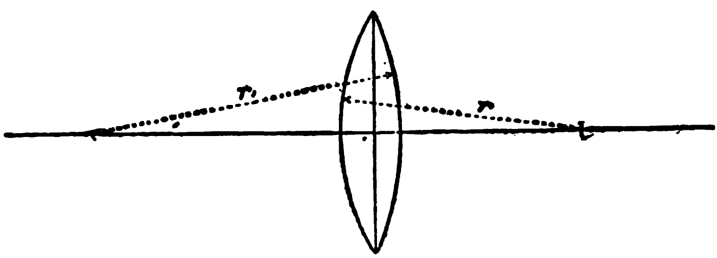


FIG. 31.

Suppose now that the object-glass O were correctly centred in its brass socket, or *cell*, so that the axis of the cylindrical cell coincides exactly with the optical axis of the lens which it contains, then, if the cell were fitted into the hole of the shutter, and the screen were at right angles to the 'optical axis' of the lens, it could be revolved in the hole in the shutter, without causing the image to move, in the slightest degree. If, however the 'optical axis' were inclined to the axis of cell, then when the cell is rotated, the image on the screen would move also. The adjustment of the 'optical axis' to the centre line of the cell is the duty of the instrument maker, and cannot well be altered.

Now suppose that in fig. 30 the 'optical axis' of the lens is horizontal, the plane of the table also horizontal, the plane of the screen vertical, and the staff also vertical.

Having obtained a correct focus (that is to say, a clear and sharp image), let the position of the screen be marked on the table, and let the staff be approached to, or removed from the lens. It will now be found that to obtain a sharp image the screen must be moved also. The shorter the distance from lens to staff, the longer the distance from the lens to the screen. Now make the necessary adjustments of the screen for the staff held at 10, 20, 30, 40, 100, 200, etc., feet from the lens, and after obtaining correct focus for each position of the staff, mark that of the screen on the table. It will be found that the *differences* of the distances of the screen from the lens, are by no means the same for *equal differences* in the distances of the staff. The difference of the screen position for staff-distance of 10 to 20 feet will be large. From 20 to 30 feet less, and so on. Finally at a staff-distance of from 200 to 400 feet or more, according to the form of the lens, the difference for an additional length of 10 feet will become negligible, and the landscape beyond the staff will be equally well

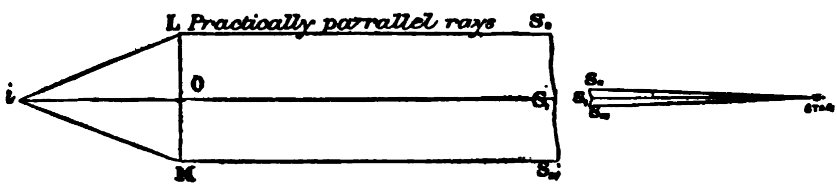


FIG. 32.

defined on the screen, with the image of the staff itself. No further inward movement will produce any perceptible improvement in the sharpness of the picture. Even the image of the sun, or of a star, will be to all intents and purposes as sharp as that of the staff itself. When the screen is in this position, it is said to be at 'solar focus,' and the distance from it to the 'optical centre' of the lens, is its '*solar focal length*.' 'Solar focal length' is attained when the extreme rays of a 'pencil' or cone of rays proceeding from a distant point are *practically* parallel.

Let S (fig. 32) be the distant object, *i* its image, L M the diameter of the lens, and O its centre, then *i* O is the 'solar focal length' and *i* the 'solar focus,' since the extreme rays $S_{,,}$ L and $S_{,,,}$ M, of the *cylinder* of rays, (the object S being so distant that the rays from it are parallel for all practical purposes) impinging on the lens assemble at this point. The distance of the light-emitting point, at or beyond which the sides of the cone or pencil of rays from it, and falling upon the surface of the lens, may be regarded as parallel, depends upon the form, or curvature of the lens, its transverse diameter, and upon the perfection of its figure. The better the lens the more nearly are any one pencil of rays brought together at one point, and therefore the better the lens, the more sharply does it come in and out of the focus. That is to say, with a good lens a very slight movement of the screen from the true focus, in either direction, will cause the image on it to become blurred. With a lens badly corrected for spherical aberration the image is at best *less* sharp, and the screen may be moved each way for a perceptible distance without making it much worse.

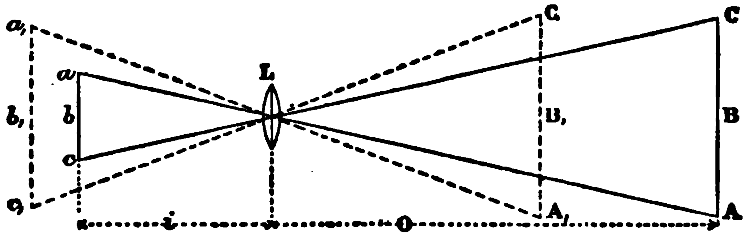


FIG. 33.

Let A B C (fig. 33) be three marks on a staff. Their images will be formed at $a\,b$ and c. For a second position of the staff, the images will be formed at $a_{\prime}\,b_{\prime}$ and $c_{\prime}$.

Let L B the distance from the lens to the staff = o, the outer focal distance, Let Lb the distance from the lens to the screen or image = i, the inner focal distance. Let f = the solar focal length of the lens.

Then generally

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \qquad (1)$$

$$\text{and } \frac{AB}{LB} = \frac{ab}{Lb} \text{ and so on.} \qquad (2)$$

These two equations give complete information as to the geometry of the lens.

Knowing the value of f, we can calculate the inner focal distance i, for different outer distances o;

$$\text{for from (1)} \quad i = \frac{fo}{o-f}.$$

Suppose now that we have a lens so formed that $f = 1$ then for different values of o we can calculate the corresponding values of i and form the following 'Table of differences' (see next page) between the successive positions of the sharp images on the screen.

From this table it appears that with a solar focal length of 1 foot, the screen must be moved nearly half an inch, to focus a staff at 10 and 20 feet respectively, whilst from 400 to 500 the movement of the screen is but ·006 of an inch. Hence, at about 500 feet the lens is practically at solar focus, its distance from the screen being 1·002 foot, or only ·024 of an inch longer than solar focal length, a quantity within the limit of error of a lens.

Let a screen of tracing paper, or of ground glass, be now substituted for that of the cardboard. When correctly focussed, the image will be seen on the *back* of the screen, as a distinct picture, the detail of which can be examined by a strong magnifying glass. Take the eye-piece of a telescope, and fit it on a support K

TABLE OF INNER, FOR CORRESPONDING OUTER FOCAL DISTANCES, WITH SOLAR FOCAL LENGTH = 1.

Outer Focal Distances = *o*.	Inner Focal Distances = *i*.	Differences.
10	1·1111	
20	1·0526	0·0585
30	1·0345	0·0181
40	1·0259	0·0086
50	1·0204	0·0055
100	1·0101	0·0103
200	1·0050	0·0051
300	1·0033	0·0017
400	1·0025	0·0008
500	1·0020	0·0005
1000	1·0001	0·0019

(fig. 34), focus it so that the details of the staff are clearly seen on the screen. Now withdraw the screen, leaving the eye-piece in position, when the magnified image of the staff will be seen more clearly than before. The image is formed in space, and is invisible to the eye when the same is on one side of the principal axis of the object lens. If the eye were placed in the position of the screen and directed towards the lens an inverted image of the staff could be seen. Indeed

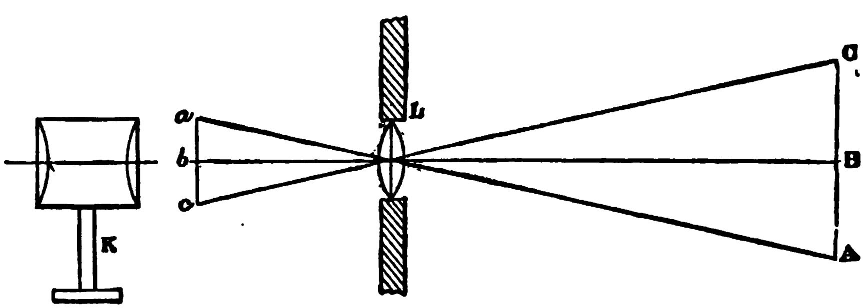


FIG. 34.

the image of the staff would be seen with the eye in any position beyond the focus of the lens, provided that it were placed in the optical axis of the object glass and directed to it. The distance between the eye-piece and the screen will depend upon the eye-sight of the observer. For a short-sighted person, the distance will be less, for a long-sighted person greater, than for one of normal powers of vision.

The distance between the object and the screen, or image, is constant with all sights, for any one distance, as it depends solely on the optical proportion of the lens. For the screen, now substitute a frame carrying a network of fine threads, or a plate of thin glass with lines ruled on it. If the network or plate be placed exactly in the common focus of the two lenses, the threads or lines will be seen magnified, and apparently coinciding with the magnified image of the staff. If this adjustment be properly made, it will be found that if the eye of the observer be moved up or down, right or left, the divisions of the staff will nevertheless appear fixed, with regard to the lines on the plate, or the wires. If the frame carrying the wires be moved slightly from its proper position in the common focus of the object-glass and eye-piece but not so much so as to prevent the wires from being seen, then on moving the eye the wires will apparently move with regard to the image. The immovability of the wires is therefore a test of the correct position of the frame or plate in the common focus of the two lenses.

The object-glass and eye-piece above described, when mounted in a tube, form a telescope. By adding in their common focus a '*diaphragm*,' that is, a small ring carrying fine cross wires, or a plate of glass, with lines engraved thereon, this telescope, when properly adjusted, affords the means of determining a line of sight from the eye to a distant point of observation, with far greater precision than any system of open sights such as those of a compass or of a rifle. The object observed is magnified, and therefore made more distinctly visible. The cross wires are also magnified, and may therefore be made of extreme fineness.

The telescope therefore, in its simplest form, consists of an objective O (fig. 35) and an eye-piece E. The objective is mounted in a cell or ring which screws into a slide tube S, which in its turn fits the main tube T accurately, so that the object-glass may be drawn out or in, with its 'optical axis' always coinciding with, or parallel to, the axis of the main tube. The draw-tube, with the objective attached to it, is moved by means of a rack and pinion with a milled head. At the other end of the tube the diaphragm D is fixed, consisting of a brass ring, across the centre of which wires or spider's threads are

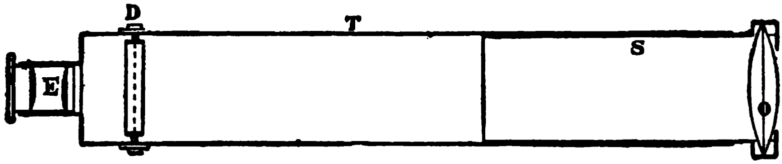


FIG. 35.

stretched, or the cross threads may be replaced with advantage by lines en graved on a glass plate. Beyond the diaphragm a smaller tube receives the eye-piece. The intersection of the cross-wires should be in the 'optical axis' of the objective, and when the lenses are adjusted for varying focuses it should move in that same line. The cross-wires are brought into the 'optical axis,' by means of two or four antagonising screws. The manner of making the adjustments of a telescope with wires is fully described in the chapter on 'Instruments.'

In many instruments the object-glass is screwed into the main tube of the telescope, and the draw-tube is provided at the eye end, carrying both the diaphragm and the eye-piece (*vide* fig. 36). This produces no difference in the manner of adjustment. In any case it is all important that the draw-tube shall

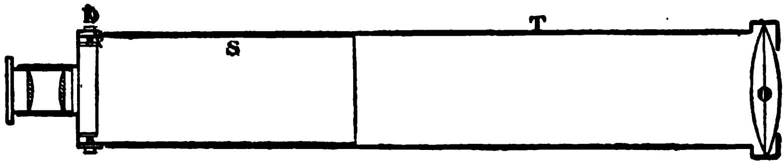


FIG. 36.

be accurately fitted so as to move out and in, strictly parallel to the 'optical axis' of the instrument. As this adjustment is made by the maker, it cannot be altered by the Surveyor if found defective. Any looseness or shake in the draw-tube is fatal to accuracy of observation. When selecting a new instrument, this point should, therefore, be carefully looked to.

The Telescope as a 'distance measurer.'

The telescope, fitted with a special diaphragm and termed a subtense telescope, may be used for measuring distances, with the aid of a graduated staff.

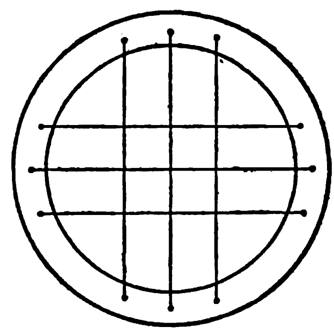


FIG. 37.

In the diaphragm (fig. 37), two horizontal or vertical wires are added, being fixed (or engraved on glass) at certain distances above or below, or to the right and left, of the ordinary cross wires.

Let O (fig. 38) be the object-glass, D the diaphragm, S a graduated staff held at right angles to the optical axis of the telescope. Let *c* be the centre wire, and *a* and *b* two other wires at some fixed distance from it. Let *o* and *i* be the relative distances of the inner and outer foci of the telescope when properly adjusted.

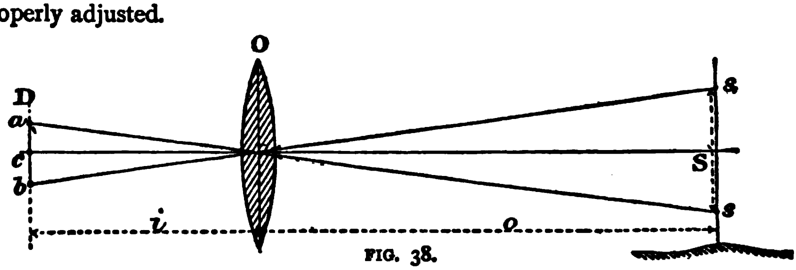


FIG. 38.

Then, when the telescope is properly focussed, an image of the staff will be formed on the plane of the wires. The wire *a* will intersect some division, such as s, whilst the wire *b* will intersect some other graduation s_1. Let $s_1 - s = S$, the distance subtended on the staff.

Then we have the two equations

$$\frac{1}{f} = \frac{1}{i} + \frac{1}{o} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\frac{\Delta}{i} = \frac{S}{o} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Where Δ is the distance between the wires a and b, a constant—f is also a constant, namely, the solar focal length of the object-glass.

Hence

$$o = \frac{Sf}{\Delta} + f. \quad . \quad . \quad . \quad . \quad (3)$$

The wires a and b are in some cases made to be adjustable as to their distance from the central wires, by means of screws. In other cases they are engraved on glass by the instrument maker at the proper distance. It is convenient to make $\frac{f}{\Delta}$ some round number, such as 100 or 200.

Let the wires be so adjusted that $\frac{f}{\Delta} = 100$. Then $o = 100\,S + f$.

If the staff were graduated into feet and hundredths, as usual, then the distance from the 'object-glass' to the staff, would be the length subtended by the wires on the staff multiplied by 100, plus f; but the distance of the staff from the centre of the instrument is usually required, hence a further addition of the distance from the object-glass to the centre of the instrument should be added.

Let d be this distance, then

$$D = \frac{Sf}{\Delta} + d + f$$

$$= C\,S + K.$$

Where C = 100 usually, and K is a constant, which should be given by the instrument maker. If it be not, then it can be obtained with sufficient accuracy by actual measurement of d and f as follows. Set the telescope to solar focus, and then measure f from the diaphragm screws to the middle of the object-glass, and also measure the distance from the latter to the axis of the telescope.

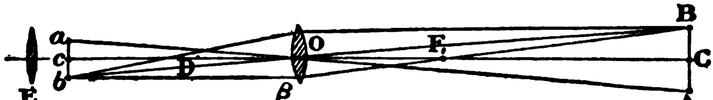


FIG. 39.

A direct geometrical proof of this may also be given as follows.

Let a and b (fig. 39) be the horizontal wires as before, and suppose that when the instrument is focussed on a staff A B, the image of A coincides with the wire,

a, and that of B with the wire b. Then all the rays from B will converge to b; but considering the particular one $\beta\, b$, which, after emerging from the object-glass is parallel to the optical axis, it is clear that in its path it must have passed through F_1, the principal focus of the object-glass, exterior to the telescope.

Now, since when the eye-piece and diaphragm are moved for focussing the staff at different distances, the point b simply travels along the line $b\,\beta$, these rays will always cut the optical axis produced at the same point F_1, and *at the same angle*. Hence, the distance F_1 C of the staff from F_1, is proportional to the length B A intercepted. But we cannot place the vertical axis of the instrument at F_1, because this point is outside the telescope. Hence, we must place it, say, at D, and add the constant $O F_1 + O D$ at each reading. This is the same result as already obtained, since $O F_1 = f$.

To test the accuracy of subtense wires.

To test the accuracy of the subtense wires, if permanently adjusted by the maker, or to adjust them if movable:—

Chain out accurately some distance such as 100 + K feet from the centre of the instrument. Then, if $\frac{f}{\Delta} = C = 100$, the wires should subtend exactly one foot on the staff. If the distance were 200 + K, then they should subtend two feet, and so on.

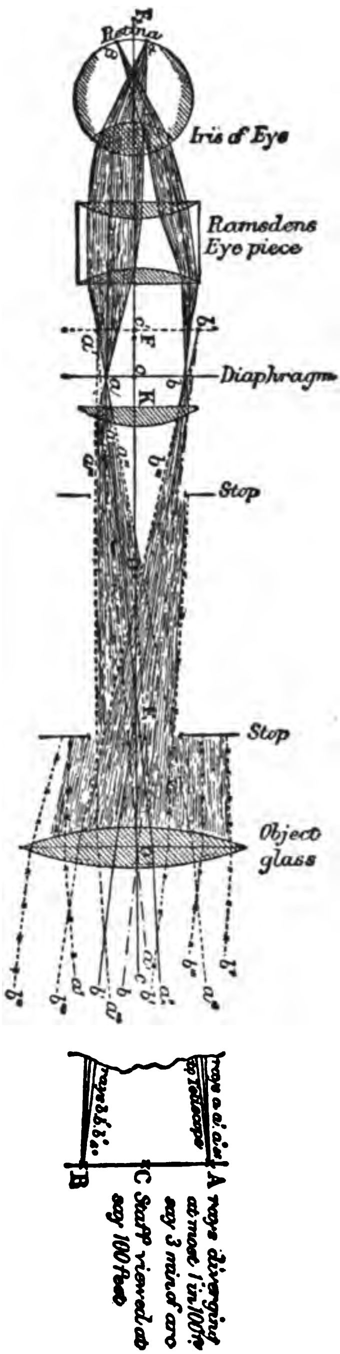
If the wires are movable, then they may be adjusted to give the proper distance subtense on the staff.

The Annalatic Lens discussed.

In order to avoid the addition of the constant K at each reading, a third lens, called an 'annalatic lens,' was introduced by Porro, of Milan.

Fig. 40 shows the arrangement, and the course of rays from a staff A B. O represents the centre of the object-glass, and F is its principal or solar focus. K is the centre of the annalatic lens, and F_1 F_2 are its principal foci.

FIG. 40.



Now let O F $= f$ the solar focal distance of objective.
K F_1 = K $F_2 = f_1$ " " of 'annalatic lens.'
K O $= d =$ distance between optical centres.

Let the instrument be focussed on a staff at A B, so that the horizontal hairs *a c b* coincide with the images of the points A C B on the staff, and suppose that $a_1 c_1 b_1$ represent the images of these same three points which would be formed by the objective alone, without the interposition of the 'annalatic lens.'

Let O $c_1 = w =$ distance of inner focus for object-glass only.
O C $= o =$ " outer " "
K $c = i =$ " image from 'annalatic lens.'
A B $=$ S $=$ length of object, or staff subtended.
$a_1 b_1 = I_1 =$ " image due to objective only.
$a b = I =$ " " actually formed $=$ a constant.

Then, by the formulæ for lenses, we have for the object-glass

$$\frac{1}{o} + \frac{1}{w} = \frac{1}{f} \quad . \quad . \quad . \quad . \quad (1)$$

$$\frac{S}{I_1} = \frac{o}{w}. \quad . \quad . \quad . \quad . \quad (2)$$

Now, without the 'annalatic lens,' the rays of light from C would converge to the point c_1. In consequence of the action of this lens, however, they will converge to a point c, such that c_1 would be the virtual focus of the 'annalatic lens' conjugate to c.

Hence, since K $c_1 = w - d$, we have

$$\frac{1}{w-d} + \frac{1}{f_1} = \frac{1}{i}; \quad . \quad . \quad . \quad . \quad (3)$$

and for the magnitude of the image,

$$\frac{I_1}{I} = \frac{w-d}{i}. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We proceed from these four equations to eliminate the quantities w, I_1, and i, and so to obtain an equation connecting together the quantities o and S. Now, from (2) and (4), by multiplication,

$$\frac{S}{I} = \frac{o\,(w-d)}{w\,.\,i}, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and from (3)

$$\frac{1}{i} = \frac{f_1 + w - d}{f_1\,(w-d)};$$

whence, by multiplication,

$$\frac{S}{I} = \frac{o\,(f_1 + w - d)}{w\,.\,f_1}. \quad . \quad . \quad . \quad . \quad (6)$$

But from (1)

$$w = \frac{of}{o-f};$$

so that, by substitution in (6) and simplifying,

$$\frac{S}{I} = \frac{o(f+f_1-d)+f(d-f_1)}{ff_1}$$

and

$$S\frac{ff_1}{I(f+f_1-d)} = o + \frac{f(d-f_1)}{f+f_1-d}.$$

All the quantities in this equation except S and o are constant. We see, therefore, that, if to the distance o of the staff from the object-glass we *add* the constant quantity $\frac{f(d-f_1)}{f+f_1-d}$ in each case, the result will be a constant multiple of the distance S subtended on the staff, and by suitably arranging the distance I between the hairs we can make the multiplier $\frac{ff_1}{I(f+f_1-d)}$ a convenient round number, such as 100 or 200. Further, by arrangement of the quantities f, d and f_1, the constant $\frac{f(d-f_1)}{f+f_1-d}$ can clearly be made less than the length of the telescope, and if we place the vertical axis of the instrument at that distance from the object-glass, then the distances of the staff from that point will be in direct proportion to the lengths subtended on the staff, which is what was required.

A geometrical proof may also be given as follows:—

Of all the rays which converge at a to form the image of A, consider that which after emerging from the 'annalatic lens,' is parallel to the optical axis, say $a\,a''$. This ray, being parallel to the optical axis of the instrument, will pass through the principal focus F_1 of the 'annalatic lens,' and will cut the optical axis at that point at a constant angle, since a'' is a constant point. Hence, on the other side of the objective they must have always travelled along one line, such that, if produced, it would intersect the optical axis at a constant angle at the point D, which is the virtual focus of the object-glass conjugate to F_1. Hence by similar triangles the distances of the staff from D will be in a constant ratio to the lengths intercepted, which is the same result as already arrived at. The consideration of the one particular ray which is parallel to the axis at $a\,a''$ very much simplifies the geometry of the problem, but, as a matter of fact, we may treat the whole pencil by the formulæ for lenses as already shown. It is easy to show that the distance O D is given by the formula $\frac{f(d-f_1)}{f+f_1-d}$ already found.

The 'annalatic lens' can be moved by a key to adjust the instrument if necessary.

The Micrometer Eye-piece described.

A fixed length on the staff and movable wires are sometimes used. This arrangement is known as the micrometer eye-piece. The staff may be provided with two vanes A B (fig. 41), fixed at some known distance from each other, such as ten or twenty feet. It is also convenient to add a third vane C, exactly intermediate between the two.

The centre wire of the diaphragm (fig. 42) is fixed in the optical axis of the instrument. The other two wires *d* and *e* can be approached to, or withdrawn from, the centre wire of the micrometer by means of two very fine screws *s s*. These screws are provided with a drum-head H H, graduated into 60 or 100 parts on the rim, a vernier being also added. The distance

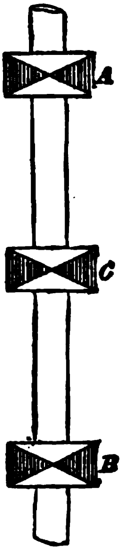


FIG. 41.

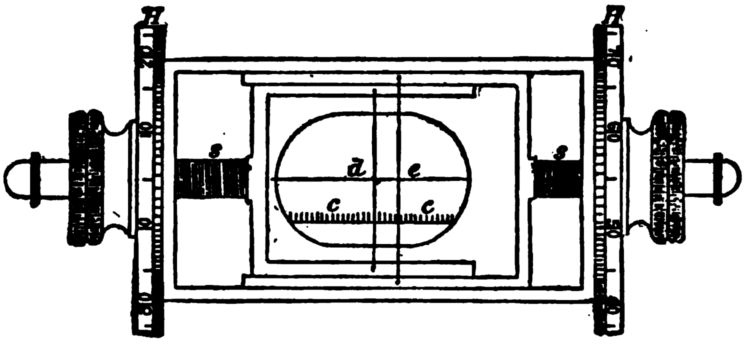


FIG. 42.

between the centre wire and either of the movable wires may be measured by the number of turns, and parts of a turn, which the screw has made in moving the wire from the centre to any given position, provided that the value of one turn is known. The whole turns made are counted by means of the comb *c c*, which is seen in the field of the eye-piece.

The sum of the turns and part of a turn made by the two screws measures the distance Δ between the two movable wires.

Then, in the general equation,

$$D = \frac{Sf}{\Delta} + f + d = \frac{Sf}{\Delta} + K.$$

S, *f*, and *d* are constant, and Δ alone a variable.

Let *t* be the value of one turn of the screw in feet or inches; let *n* equal the sum of the turns and fractions of a turn made by the two screws when the moveable wires subtend the constant distance S on the staff.

Now, an inspection of the formula shows that, to obtain accurate results, it is necessary to know the value of the fraction $\frac{f}{\Delta}$ with great accuracy, though the actual value of both factors in any given unit is unimportant.

K also need not be ascertained with any high degree of accuracy.

We may now write $n\,t$ for Δ. Then

$$D = \frac{S f}{n\,t} + K.$$

The value $\frac{f}{t}$ can be ascertained by experiment in the field. Let this fraction equal C. Then

$$D = \frac{S\,C}{n} + K,$$

or

$$C = \frac{n\,(D - K)}{s}.$$

To ascertain the value of C, chain out with accuracy on level ground some distance L. Adjust the wires to subtend the distance S on the staff, and read the two micrometers. Summing these readings,

$$C = n\,\frac{(D - K)}{S}.$$

Suppose that S, the distance between the centres of the two vanes, were 10 feet, $K = 1 \cdot 5$ feet, and that the staff were held at the distance of 500 feet from the instrument, the sum of the readings of the two screws being $19 \cdot 706$ turns.

$$C = 19 \cdot 706\,\frac{(500 - 1 \cdot 5)}{10}$$

$$= \frac{19 \cdot 706 \times 498 \cdot 5}{10}$$

$$= 960.$$

Then

$$D = \frac{960\,S}{n} + K$$

$$= \frac{9600}{n} + 1 \cdot 5.$$

This would correspond with a micrometer screw having 80 threads to an inch, and a telescope with a focal length of one foot. Such exact numbers will not usually occur in practice. It would also be well to arrange matters so that $C\,S = 10{,}000$. The distance $\frac{C\,S}{n}$ could then be taken by inspection from a table of reciprocals merely by moving the decimal place. This can be done by making S, instead of 10 feet, some other length, so that

$$C\,S' = 10{,}000.$$

Thus in above case

$$S = \frac{10{,}000}{960} = 10 \cdot 47 \text{ feet.}$$

Then, if the vanes of the staff be set $10 \cdot 47$ feet apart, distances would be obtained simply by taking the reciprocal of n and moving the decimal point five figures to the right, adding K to the result.

Index Error of Screws in Micrometer Eye-pieces.

The two micrometer screws should be so adjusted that when the two movable wires are in coincidence with the fixed central wire, both micrometers read zero. Unfortunately there are mechanical difficulties in arranging matters so that the moveable wires can be brought into actual contact with the central wire. Some means must be devised of testing for index error.

In the first place, it is evident that an equal number of turns of each micrometer should subtend an equal number of graduations from the centre wire, on a graduated staff. Or, in other words, if a third vane were fixed equally midway between the two as shown in the sketch (fig. 42), then, if the fixed middle wire is made to bisect the central mark, equal numbers of turns should be required to bisect the upper and lower marks with the movable wires.

But correctness, in this respect, does not prove the absence of index error, but merely that the errors of the two screws are equal. To determine the index error, chain out two distances a and b.

Let $a + K$ and $b + K$ (fig. 43) be the two distances to the staff. Having levelled the instrument, place a staff at A. Turn one of the micrometer screws

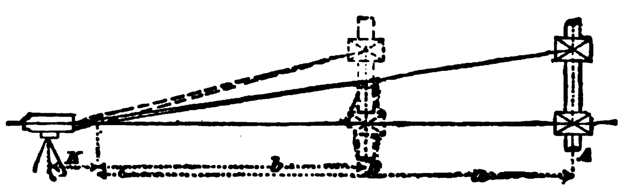


FIG. 43.

until some fixed distance, say five feet, is intercepted on the staff between the middle and one movable wire. Call the number of turns made α. Now move the staff to station B, and move the micrometer wire until the exact same number of feet is again similarly intercepted. Call the number of turns β.

Then, if there were no index error, $\frac{\beta}{a} = \frac{\alpha}{b}$. The number of turns would be inversely as the distances. If there be an index error it will be constant. Let x be the error.

Then

$$\frac{\beta + x}{a} = \frac{\alpha + x}{b},$$

or

$$b\beta + bx = a\alpha + ax$$

hence

$$ax - bx = b\beta - a\alpha$$

and

$$x = \frac{b\beta - a\alpha}{a - b}.$$

It is convenient to make distance $a = 2\,b$. Then

$$x = \beta - 2\,a.$$

Example.—Two distances, K + 250 and K + 500, are chained out from the centre of the instrument.

The reading (a) at the distant station is 9·477.

That (β) at the near station is 19·077.

$$\begin{aligned} x &= 19{\cdot}077 - 2 \times 9{\cdot}477 \\ &= +\,0{\cdot}123, \text{ to be added.} \end{aligned}$$

Again: suppose that the other wire gave

$$\begin{aligned} a &= 9{\cdot}627 \\ \beta &= 19{\cdot}227 \\ x &= 19{\cdot}227 - 2 \times 9{\cdot}627 \\ &= 19{\cdot}227 - 19{\cdot}254 \\ &= -\,0{\cdot}027, \text{ to be deducted.} \end{aligned}$$

Magnetism.

The Magnetic Needle. The magnetic needle forms part of a great many surveying instruments such as the Theodolite, the Level, the Prismatic Compass, &c., and is made in the form best adapted to the use of the instrument in which it is placed.

Magnetism is a *molecular* force which resides in every part of a magnet, and is induced into the fine cast steel, of which magnets are usually formed, (best with 3 per cent. of tungsten) by means of a permanent or electro-magnet.

Natural and Artificial Magnets. Natural magnets, or *lode-stones*, are exceedingly rare, and therefore artificially magnetised pieces of steel are in general use in their stead.

Methods of Magnetisation. The usual process of magnetising a bar consists in rubbing it with an already magnetised bar. Different processes, called single or double touch, &c., have been devised, but since much greater power can be obtained by means of electro-magnetism this method is now almost exclusively employed by the best makers.

Magnetic Needles forming part of Surveying Instruments. Magnetic needles, when forming parts of surveying instruments, are generally made in the form of flat bars with a cup (or cap as it is termed) at the centre, which is balanced on a standing point. The longest section is sometimes placed horizontally and at others vertically, in the former case it is termed a 'broad needle' and in the latter an 'edge-bar needle.' The 'edge-bar needle' is generally used when it is required to read into a fixed scale of divisions, and then the extremities are brought to a fine knife-edge.

The centre or *cap*, usually consists of a hard precious stone, such as an agate, ruby, or sapphire, &c., and this is mounted in a light brass or aluminium cell. The whole is balanced on a hardened steel needle-point.

The needle of a surveying instrument should never be supported upon its centre except for the time it is in use for taking observations, and a means of lifting it off the pivot is always provided.

The Declination of the Needle.

When a magnetic needle is swinging freely, it will seldom point in the direction of true north and south, the deviation varying at different points of the earth's surface. This deviation is termed 'the declination of the needle,' and varies at the same place from year to year, at different times of the year, and to a lesser extent even diurnally. It is therefore very necessary to ascertain the *local* 'declination' of a needle before making use of it for survey purposes. (*Vide* Changes in 'Magnetic Variation,' p. 158.)

Inclination or Dip of the Needle.

The dip of the needle, is the vertical angular difference between the position it would assume *before*, and *after* magnetisation, respectively. This inclination or dip varies in different parts of the globe, and at different times. It vanishes at the equator, and increases until it becomes 90° (the needle being vertical) over either of the magnetic poles.

For surveying purposes the needle is balanced so as to maintain a longitudinal position, a sliding weight or *rider* being sometimes provided for purposes of adjustment. Adjustment can also be made with sealing-wax if required.

Level, or Bubble Tubes.

Level, or Bubble Tubes.

The level, or *bubble* tube is one of the means of applying the force of gravity, so as to ascertain by observation, level lines on the earth's surface, or vertical angular measurements. They are attached to nearly all important surveying instruments.

The glass tubes from which they are made are drawn of as nearly straight and equal bore as possible, but as they become slightly curved and tapering after annealing, portions are cut off having regular longitudinal curvature, and these are finally ground, sealed and divided. During the process of internal grinding the curvature is tested from time to time on a bubble tester. This consists of a bar, some 20 inches long, which rests on two feet at one end, and a micrometer screw with a disc graduated to seconds of arc, whose point forms the support at the other end. The whole stands on a cast-iron plate. The tube is placed on two Y's which can be adjusted for distance to suit the length of the tube.

The ultimate radius of curvature to which the interior of the tube is worked, depends upon the delicacy of the work for which it is intended, and varies from 30 feet to even 1000 feet or more. Level tubes are usually filled with pure alcohol, though for very delicate work sulphuric ether, or chloroform, is used.

Instruments for use abroad should be provided with spare tubes, as a blow or even the heat of the sun's rays may sometimes fracture them.

A good bubble should possess the following qualities :—

1. It must be long enough for a given diameter, to admit of quick displacement of the air bubble, and yet not so long as to admit of excessive elongation of the same in low temperatures.

2. The curve must be such that the sensibility and uniform run of the bubble

will indicate quantities sufficiently minute, and corresponding exactly, with the inclinations read on the graduated limb of the instrument to which it is attached.

3. The opposite ends of the bubble must elongate or contract equally so that the central position remains stationary in all changes of temperature.

Atmospheric Pressure and the Barometer.

Atmospheric Pressure and the Barometer.

It never occurred to any of the philosophers who preceded Galileo to attribute any influence in natural phenomena to the weight of air. In 1650, Otto Guericke, the inventor of the air pump, made decisive experiments by weighing air, and other gases, in a globe of glass. He proved that under the pressure of 760 millimetres (one atmosphere) dry air weighs 1·293 grammes per litre. Speaking generally, a cubic foot of air under ordinary circumstances weighs about an ounce and a quarter. The earth is encircled by a layer of atmosphere from 50 to 100 miles in thickness, and this heavy fluid mass exerts on the surface of all bodies a pressure entirely analogous to that sustained by a body wholly immersed in a liquid.

This pressure is constant in value for the same horizontal layer when the air is in a state of equilibrium, but diminishes as we ascend to higher levels. When inequality in pressure occurs at a given level, wind must ensue. Torricelli proved, by means of a glass tube about 36 inches long, filled with mercury and inverted in a cup containing mercury, that the atmospheric pressure could support a column of about 30 inches, or if water was substituted for mercury a column of 34 feet would be supported, and this is the maximum height to which water can be raised by an ordinary air pump.

The ordinary mercurial barometer is constructed on the principles above indicated, and by its means the atmospheric pressure can be ascertained at any time or place.

As the pressure of the air diminishes as we ascend, and the height of the barometric column becomes less, it is natural to seek in this phenomenon a means of measuring heights. The problem is not so simple though as it would be were the air of uniform density.

Since the boiling point of water varies with the atmospherical pressure, heights can be deduced by this means, this method being called *Hypsometry*. (*Vide* Part II.)

CHAPTER III.

DESCRIPTION AND ADJUSTMENT OF INSTRUMENTS.

1. The Optical Square.

The Optical Square.

This is a reflecting instrument (fig. 44), depending for its action on the well-known law that a ray of light striking a plane mirror, is reflected again from the surface at an angle equal to that of incidence. It will be seen with the aid of a diagram that if two mirrors inclined to one another be employed, the angle between the first incident ray and the same twice reflected, will always be double the angle included between the faces of the mirrors themselves.

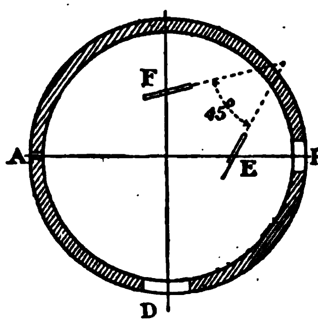


FIG. 44.

The construction of the optical square is as follows. In a cylindrical metal box from $1\frac{1}{2}$ to $2\frac{1}{2}$ inches in diameter, and $\frac{3}{4}$ of an inch deep, having two square or oblong apertures, B and D (cut on the side of its circumference at right angles to each other), and also a small circular eye-hole A (bored diametrically opposite to the aperture at B), are placed two mirrors E and F, inclined to one another at an angle of 45°. The upper half of the mirror E is silvered, while the lower half is of plain glass. This mirror is placed opposite to the eye-hole A and in a line with the aperture B, about midway between the latter and the centre of the instrument, and is inclined to the axis of the instrument drawn through the eye-hole and the centre, at an angle of 120°. The second mirror F is silvered all over, it is situated diametrically opposite to the aperture at D, and at the same distance on the other side of the centre as in the mirror E, and it is inclined to the axis A B at an angle of 165°.

Adjustment of the Optical Square.

To adjust the instrument, set out four poles A A′ B B′ in line (fig. 45). Standing at C, and facing towards A in such manner that the two poles A A′ are seen as one, set out a pole D, and nearer to the instrument a second pole D′, so that D′ D appears in line with A′ A, then turning round to face DD′, again observe the poles at D′D. If they coincide with the poles at B B′ the instrument is in adjustment; if they do not, put up another pole D″ alongside D′ to determine whether the angle obtained is too small or too large. Correct the mirrors accordingly, by slackening the screws which attach the mirror F to the bottom of the case, and moving it in the direction required, again tightening up.

The reason for using two poles in each direction, is that the instrument being carried in the hand, there is no other means of ensuring that it is held by the observer truly in alignment. It is, however, usual to set up a pole at C, as a rough guide as to position.

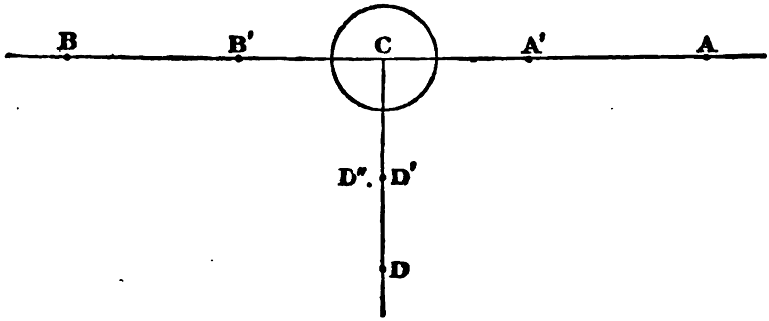


FIG. 45.

Sometimes this instrument is provided with a light tripod stand, so that the vertical axis can be fixed over any given point with considerable accuracy, by means of a plumb-line attached to the under side of the tripod head. For practical purposes, this refinement is seldom necessary, and its use does away with the great advantage of the optical square, namely, its portability.

The instrument can also be obtained with two additional mirrors, arranged so as to reflect both from the right and from the left, the direct vision of the object in front being obtained through a small space left between the two horizon glasses.

2. The Line Ranger.

The Line Ranger. This is also a reflecting instrument (fig. 46). The ray of light passes perpendicularly through one side of a right-angled prism of glass, to its hypothenuse, being reflected therefrom perpendicularly to the other side. The instrument is composed of two such

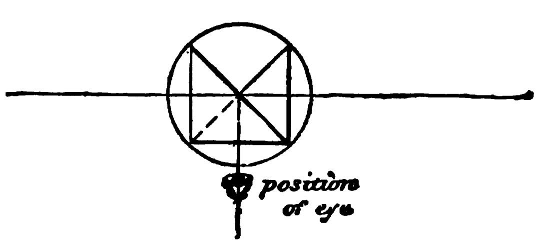


FIG. 46.

prisms, one placed vertically above the other in the positions shown, two of the shorter sides of each right-angled triangle being placed over one another and opposite the eye of the observer, while the other two similar sides are parallel to one another, and the hypothenusal sides are perpendicular to each other.

To adjust the instrument. Fix a pole C in line between two poles A and B (fig. 45), A being to the left of the observer, and B to the right, then turn to the other side of the pole C, having the pole B on the left, and A on the right, if the images of the two poles are still coincident, the instrument is in adjustment.

If the images do not exactly coincide, they can be made to do so by slightly altering the position of the upper prism as regards the lower, by means of screws which are provided at the back of this prism.

A similar adjustment screw is furnished for ensuring that the vertical faces of both prisms are parallel.

The instrument when closed forms a cylinder, about an inch in diameter, and an inch and a half in length.

3. The Box Sextant.

The Box Sextant. This instrument is somewhat similar in construction to the optical square, but whereas in the latter instrument both mirrors are fixed, and only angles of 90° can be measured, in the sextant, one mirror only is fixed, and the angle made between it and the second

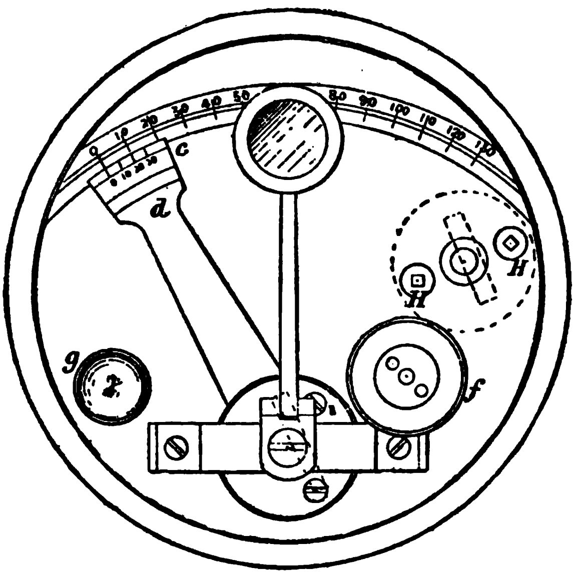


FIG. 47.

mirror can be altered so that any angle from 0° to 110° or thereabouts, can be measured by the observer. Fig. 47 is a plan of the top, and fig. 48 a sectional plan.

The instrument consists of a cylindrical metal box about 3 inches in diameter and ¾ of an inch thick, covered when not in use by a cylindrical metal case which screws on to the lower edge of the instrument, but which, when the instrument is in use, is reversed and screwed on at the back, serving as a handle.

The top of the box, when in use, is formed of a brass plate about $\frac{1}{8}$ of an inch thick, to which the various mirrors, &c., to be described are attached.

In the side, at A, there is an eye-hole pierced in a slide, and in some instruments a small telescope is provided. Opposite to this eye-hole, and on the other side of the box, about $\frac{1}{4}$ of the circumference is cut away.

Between the eye-hole A and this opening, the mirror B is fixed, and is called the 'Horizon Glass.' The lower half of this glass is plain, whilst the upper half its silvered similarly to the corresponding mirror in the optical square.

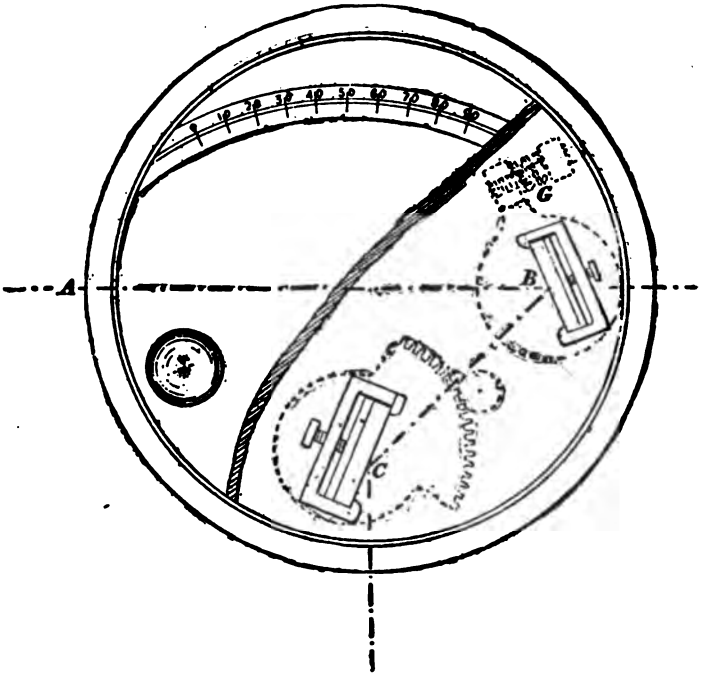


FIG. 48.

In the sextant, the second mirror, or 'Index Glass' C, is so arranged that it can be caused to revolve on its axis by means of a rack to which it is attached, and a toothed wheel which engages in the same. The toothed wheel is connected to the milled head *f* which appears outside the case.

By revolving the 'index glass' C, and consequently altering the angle of 'incidence' and 'reflection,' it is obvious that angles of varying magnitude may be recorded by means of the 'index' arm *d* attached to the axis of the mirror. These angles are read on the graduated arc marked on the outside of the case, and the arm is provided with a vernier reading minutes on the half degree subdivisions of the main arc.

An arrangement is provided by which dark glasses may be interposed between the eye and the mirrors to enable the sun to be observed with safety.

Angles greater than about 110° cannot be observed with the box-sextant, as the angle of incidence becomes so great, that the rays, instead of being *reflected* are *refracted*, and lost in the thickness of the glass.

It is clear that if the instrument is in correct adjustment, and any vertical object be sighted when the index is set to zero, the reflected and direct images will be in the same line. Should they not coincide, but at the same time be parallel to one another, then there exists a certain amount of 'index error,' which can be corrected by causing the horizon glass B to revolve on its axis, by means of the screw shown at G, which works in the projecting arm attached to the base of the glass.

Should the reflected image of a vertical object not appear vertical in the mirror, the defect is due to the fact that the horizon glass is not at right angles to the base plate, and may be remedied by tilting the glass in the direction required by means of the two screws H H provided for the purpose. These screws, as well as that at G, are fitted with square heads, and can be turned with a key *g* which will be found screwed into the case.

4. The Prismatic Compass.

The Prismatic Compass.

The prismatic compass, as most commonly used, consists of a round bronzed box about ¾ of an inch deep and 2½ inches in diameter (figs. 49 and 50), with a pivot in the centre, on which revolves the compass needle B. This needle has attached to its upper side a graduated card or dial C, divided to half degrees, the 180° mark being placed at the north end of the needle, in order to allow of the bearing being correctly read

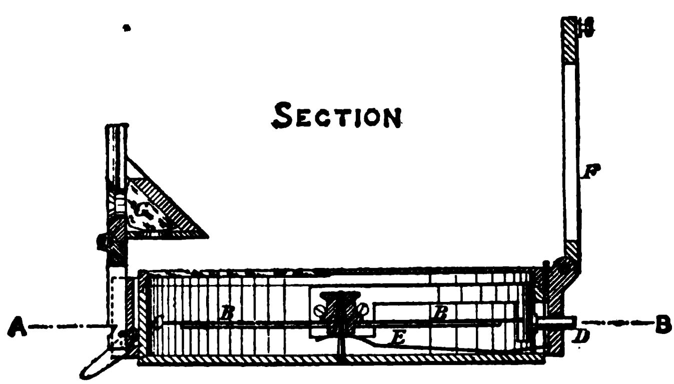


FIG. 49.

at the opposite side of the dial. The case is preferably covered almost entirely on the top, with the exception of a small segmental portion underneath the prism hereafter mentioned. A spring D is provided, which can be pressed against the edge of the dial, by one finger of the hand holding the compass, in order to assist in bringing it to rest.

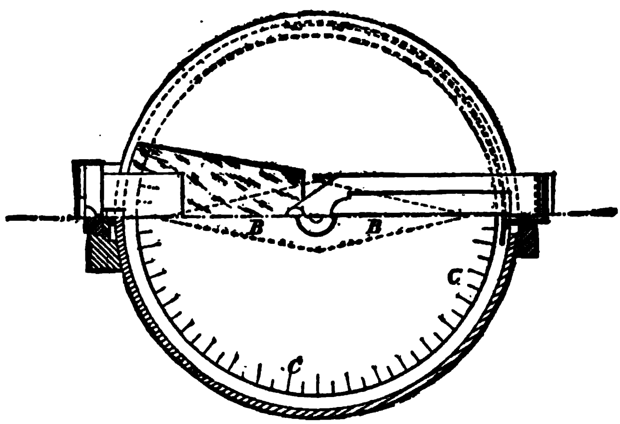
Another lever arrangement E causes the dial to be raised off the pivot, when the sight-vane is closed down upon a small projecting piece of metal, near the hinge.

The sight-vane F is a long slip of metal with the central portion removed so as to form an oblong frame down the centre of which is stretched a wire or hair, or sometimes, a thin strip of the same metal as the frame. When not in use, the sight-vane is folded down on to the top of the case this action causing the lever E to raise the dial off the centre pivot, and so prevent unnecessary wear.

The reading prism G is made convex on two sides at right angles to each other, while the hypothenuse is silvered, so that the divisions on the horizontal dial appear to pass vertically in front of the observer, and at the same time to be slightly magnified. The prism is enclosed for protection in a case, so arranged that it may be hinged back, and secured when not in use.

The case is fitted to a dovetailed slide, working to a limited extent in guides on the compass box, to allow of the figures on the dial being brought into exact focus by different observers.

HALF PLAN OF TOP



HALF SECTION ON A B

FIG. 56.

The hole in the prism case to which the eye is applied, is prolonged upwards by a narrow slit, and the whole is fixed opposite to the hair in the sight-vane.

It may here be mentioned that rings of aluminium, although more accurately divided, and not liable to distortion from atmospheric changes, are often very difficult to read correctly, on account of the bright reflection of the sun in certain positions.

The instrument being held horizontally, the length of the sight-vane enables the bearing of an object to be obtained, although it may be considerably *above* the horizon. The observation of points *below* the horizon is facilitated by a mirror, arranged to slide on the sight-vane.

Coloured glasses are sometimes arranged on a stud at the side of the prism, in such a way that they can be interposed between the eye and the sun when observing the sun's azimuthal bearings.

5. The Level.

The Level. The ordinary surveyor's level consists of a telescope, provided with a diaphragm, and cross hairs or lines, one being horizontal and capable of adjustment, vertically, so as to make the line of collimation at right angles to the vertical axis. A spirit level is attached to the telescope, (or to the 'carrier bar,' as in a late pattern) and the whole is mounted on a tripod stand similar to that of the theodolite.

Levels in common use, are of two patterns, viz. the 'Y' and the 'dumpy.' The telescope of the 'Y' pattern is, in its optical arrangement, similar to that of the 'Y' theodolite, but usually of greater diameter and length, having a higher magnifying power. 14-inch 'Y' and 12-inch 'dumpy' levels are the sizes in most common use, although for work requiring great accuracy, levels with 20-inch telescopes can be obtained.

In the 'Y' pattern the telescope is supported in 'Y's' which are fitted to either end of a straight bar, one being hinged, whilst the other is provided with a screw and nuts, so that the position of the telescope with reference to the vertical axis, may be adjusted within certain limits. In the 'dumpy' pattern (latest) these supports and bar are rigidly fixed to the vertical axis.

The level bubble is attached to the telescope (above or below) by screws and adjusting nuts.

In levels, the diaphragm of the telescope (fig. 51), carrying the cross-hairs, is made in the form of a slide, working in grooves, and capable of adjustment, in a vertical direction only, by two opposing screws, D.

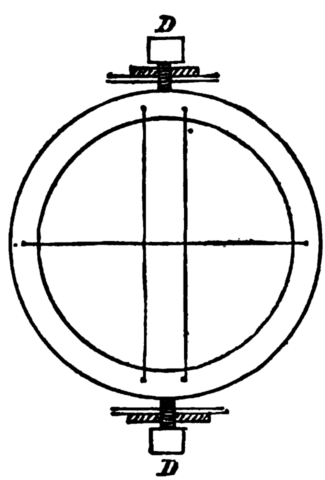


FIG. 51.

The cross-hairs are usually three in number, *one*, horizontal, across the diameter, and *two*, vertical, one on each side of the centre, as shown. The reading on the staff is taken by the central part of the horizontal hair, while the two vertical ones enable the observer to see that the staff is held upright. The flat bar to which the supports of the telescope are fixed, is usually widened in the centre to carry a compass ring immediately over the vertical axis.

A clamp and tangent screw is sometimes fixed to the vertical axis.

The tripod head and stand are similar to those provided for the theodolite with either four antagonising screws, or a tribrach with three screws.

The 'Y' level, although theoretically perfect, is not so much in favour, for general use, as the 'dumpy,' so called from its more compact shape. In this level, the telescope is fixed to the flat bar or limb at right angles to the axis (an adjustment screw being sometimes provided), the optical arrangements being similar to those of the 'Y' pattern. It may here be observed, that in the case of the level, it is not absolutely necessary, although desirable, that the line of collimation should be exactly in the axis of the telescope, all that is required being, that it should be *parallel* to the 'spirit bubble' and *at right angles* to the 'vertical axis.' These conditions can be obtained by means of the slight motion allowed to the diaphragm, without altering the position of the telescope in regard to the vertical axis.

A small mirror is sometimes provided, about 1½ inch long, by ½ an inch wide, attached by a hinge to a spring clip, in such a way, that it may be set over the bubble tube. By this means, the observer can see that the bubble is at the centre of its run, without walking to the side of the instrument, at the time that the reading is taken on the staff.

Improved forms of Levels discussed.

Various improvements have been adopted by different makers, both in the 'dumpy' and 'Y' pattern of instrument, having for their object the reduction of the number of separate parts, and greater strength. In 'Y' levels the loose pin formerly employed for securing the upper half of the 'Y' bearing is sometimes replaced by a spring latch. Glass diaphragms are often used, with hair-lines scratched on them in place of cobwebs, but several must be kept, in case of breakage.

Several patterns of levels have of recent years been introduced, with the object of combining the solidity and compactness of the 'dumpy' with the convenience of adjustment of the 'Y' pattern.

In one of these, named after the inventor 'Cushing's level,' the 'eye-piece' and 'object-glass' are fitted into sockets, and are interchangeable. The 'eye-piece and diaphragm' can also be rotated, so that the line of collimation can be adjusted.

Adjustment of the 'Y' Pattern Level.

1. Parallax.
2. Collimation.
3. Telescope level.
4. Vertical axis of rotation.

1. *The adjustment for parallax.*—That is, to make the focus of the eye-piece, and the image formed by the object-glass, coincide on the plane of the cross-wires. First, adjust the eye-piece till the cross-hairs (illuminated by rays of light reflected from a sheet of white paper held in front of the object-glass) are clearly defined, focus the telescope on some well-defined distant object, when, on moving the eye, slightly, either horizontally or vertically, the wires should appear steadily fixed on the object. Carefully readjust eye-piece till perfect, and all motion is eliminated.

2. *The adjustment for collimation.*—That is, to make the line of collimation (i.e. the line joining the optical centre of the object-glass, and the intersection of the cross-hairs) *coincident with*, or *parallel to*, the axis of rotation of the telescope in the Y's.

Turn the telescope through 180° in its Y's after having sighted the horizontal wire on a well-defined object, when, if the wire has moved from the object it must be brought back, half by the foot-screws, and half by the diaphragm screws. Repeat till perfect.

3. *The adjustment for telescope level.*—That is, to make the telescope level parallel to the axis of rotation in its Y's, and therefore parallel to the line of collimation. Bring the bubble to the centre of its run, by means of the foot-screws. Reverse the telescope end for end, in its Y's, when, if the bubble does not remain in the centre of its run, correct *half* the displacement by the foot-screws, and *half* by the screw at one end of the level. Now rotate the telescope slightly in its Y's, and if the bubble does not remain steady (from the level not being in the same plane as the axis of rotation of the telescope in its Y's) adjust by means of nuts provided for the purpose, which give *lateral* motion to one end of the level. Repeat till perfect.

4. *The adjustment for vertical axis of rotation.*—That is, to make the vertical axis of rotation truly vertical, and to put the line of collimation, and telescope level, at right angles to it. Turn the telescope till it is over two foot-screws and bring the bubble to the centre of its run, by them. Then, turn the telescope through 180°. If the bubble is not still in the centre of its run, correct half the displacement by the foot-screws, and half by the thumb-screw fixed to one of the Y supports. Now turn the telescope through 90° and correct with the third foot-screw (or other pair of foot-screws). Repeat adjustment, till the bubble remains in the centre of its run during a complete revolution of the telescope.

Adjustment of the 'Dumpy' Level.

1. Parallax.
2. Vertical axis of rotation.
3. Collimation.

Adjustments (1) and (2) are similar to those for the 'Y' pattern level.

3. *The adjustment for collimation.*—That is, to set the line of collimation (for *definition*, vide adjustment of Y level), at right angles to the vertical axis of rotation, and therefore parallel to the telescope level. Select a level piece of ground, and drive two pickets about a chain distant, on either side of the instrument. Having carefully levelled the instrument, place a levelling staff on one of the pickets (commence with the lowest picket), and read the staff. Turn the telescope and re-level it, and read the staff now placed on the other picket. Drive this picket till the staff reads as on the first picket. The pickets will now be level. Now set up the level about half a chain beyond either picket, and level it, reading the staff on each picket, and adjusting with the diaphragm screws alone, until the readings on both staves are the same.

If *three* pickets are used, and, after adjusting and levelling the instrument very carefully, the third reading on the most distant staff does not agree with the other two, then it is probable that the tube carrying the eye-piece, does not draw axially with the telescope tube. This defect can only be rectified by the makers.

Adjustment of the Cushing's Level.

1. Parallax.
2. Collimation.
3. To make the line of collimation perpendicular to the vertical axis.
4. To make the level perpendicular to the vertical axis.

1. *The adjustment for parallax.*—As for 'Y' level.

2. *The adjustment for collimation.*—Set the telescope over two foot-screws

and direct the horizontal wire on some well-defined object. Withdraw the fixing screw from the eye-piece carrier, and turn it through 180° in the socket, correcting half the displacement of the horizontal wire by the diaphragm screws, and half by the foot-screws.

3. *To make the line of collimation perpendicular to the vertical axis.*—Direct cross-hairs on an object, as for the last adjustment, then interchange the 'object-glass' with the 'eye-piece.' Correct any displacement of the horizontal wire *half* by the foot-screws, and *half* by the large clamping nut at one end of the horizontal limb.

4. *To make the level perpendicular to the vertical axis.*—Bring the bubble to the centre of its run over two foot-screws. Turn the telescope through 180°, correcting *half* the displacement of the bubble by the level clamping nuts, and *half* by the foot-screws. Now turn the telescope over the third foot-screw (this level being mounted on a tribrach), and correct any displacement of the bubble by that foot-screw alone. The bubble should now remain in the centre of its run during a complete revolution of the telescope.

The Levelling Staff.

The Levelling Staff. The levelling staff in most common use in this country in conjunction with the level, is constructed as follows. An upper solid length slides into a central hollow length, which in turn slides into a lower or bottom length, the whole thus collapsing into a portable form (figs. 52 and 53). These lengths are usually made of well-seasoned mahogany or cedar, fitted and screwed together, and rendered as waterproof as possible. The bottom hollow length is generally about 3¼ inches by 2 inches in cross-section, and 5 feet long, the next length being an easy fit within the first and the third within the second. The positions can be extended so as to make the total length over all 14, 16 or 18 feet as the case may be, the 14-foot staff with a bottom length of 5 feet being most commonly used. Each length when properly drawn out is held in position by a spring catch A, engaging with the brass rim of the next length below. The bottom is furnished with a brass shoe and the top with a brass cap. The face of each length is figured, either in oil colours or on a paper strip, pasted on and varnished over, the painted figures being naturally the more durable. The divisions consist of feet, tenths, and hundredths of a foot. The actual figuring is done in many ways to suit individual taste, but the pattern known as Sopwith's, illustrated in fig. 52, is by far the most generally adopted. In this case the black and white spaces each extend over ·01 of a foot, and are made of different lengths to facilitate reading. The odd tenths only, are numbered in black figures, the top of the figure in every case being in line with the division to which it refers. The figures indicating feet are usually painted red, and are of greater size than the rest. The smaller figures indicating the tenths, are themselves exactly a tenth of a foot high, so that the bottom of figure 3 represents ·2 feet, and so on.

The figure 6 is generally painted with an open loop, whilst the loop of the 9 is filled up, and the figures 10, 12 and 13 on the top length are sometimes replaced by one, two or three dots respectively (or are painted in Roman numerals), this length being too narrow for the figures to be read distinctly.

In every case the foot or other unit is subdivided decimally. Metres and centimetres are used only, in countries in which the metrical system prevails.

In Russia the sagene of seven English feet is subdivided in one thousand (1000) parts.

The telescopic staff is no doubt convenient for travelling by train or cab, but in the writer's opinion is open to many disadvantages.

The woodwork is apt to swell and warp, with wet, so as to make it difficult to draw out.

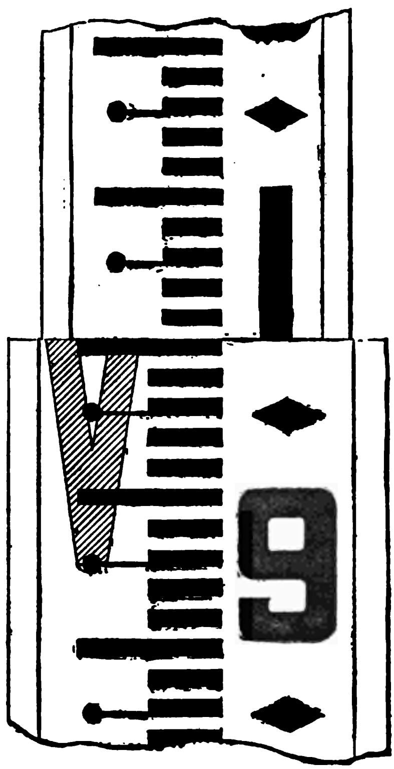
It cannot be safely immersed in water, or used as a sounding rod.

Errors may be introduced by the failure to extend the staff fully.

The surveyor may direct the staff-holder to pull out the top joint, but if the staff is stiff, he may fail to do so completely, and a serious error may be introduced.

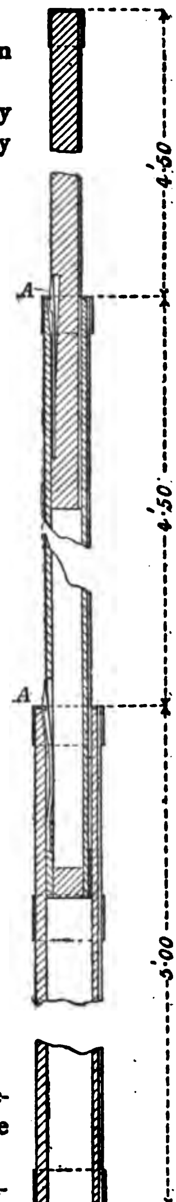
Finally, the length, fourteen feet, is too great for general use, especially on hilly ground.

The average height of the levelling instrument above the ground, is about five feet.



ELEVATION.

FIG. 52.



SECTION.

FIG. 53.

There is naturally a tendency to reduce to a minimum the number of positions of the instrument, consequently there is a temptation, when levelling up hill, to make the length of the 'back-sight' always greater than the 'fore-sight.'

If, therefore, the instrument be not perfectly adjusted in collimation, there is an error, always in the same direction, and more cumulative. This error may amount, in the end, to a serious amount; and it will not be detected by levelling back, for, when levelling down hill the tendency will be to make the 'fore-sights' longer than the 'back-sights,' which tends to produce an error in the *same* direction, making, in both cases, the apparent height of the hill *too great*.

Again, the longer the staff, the greater is the error caused by any given deviation from the perpendicular position.

The error from this cause is one of excess, but the greater the reading of the staff, the greater is the error of excess produced by any given deviation from the perpendicular.

Consequently, when levelling up hill, there is a constant tendency to make the 'back readings' too high, by an amount that exceeds the probable error of the 'fore-sight' readings which are low.

When going down hill, this error is in the opposite direction.

The writer, therefore, prefers a staff ten feet long only, and this length prevents great difference in the length of the 'fore-' and 'back-sights,' when levelling up or down hill.

For use in cases where much travelling by rail or carriage is not required, the writer considers that the staff should be in one single length like the Ordnance survey ten-foot pattern. The staff should consist of a strip of some well-seasoned light wood, about 3 inches wide, and ⅝ inch or ¼ inch thick. A strip of wood cut out to give hand-holes, for the staff-holder, screwed to the back, serves as a stiffening rib.

To each side of the staff small fillets of wood are nailed, projecting about ¼ inch beyond the graduated face of the staff.

By lashing together two staves, face to face, the graduations are fully protected when travelling.

When any extensive work is in progress, the writer has found it convenient to make a pair of staves of this kind, and to keep them on the work.

If the staves are shod at both ends, and made exactly ten feet long, out to out, they will be handy for setting out masonry. The writer also prefers a rounded end to the usual square one, as being more certainly placed in the same position when turned round for observation of a 'back-' after a 'fore-sight.' The ordinary square shoe, when placed on a sloping surface, such as a slanting rock, rests on one corner; and when the staff is turned round, on a change in the position of the instrument, there is a tendency to slip or work down the slope.

The rounded end obviates this tendency to a great extent.

For travelling, by rail or steamer, the writer prefers a folding staff ten feet long, with a very stout brass hinge in the middle of its length.

When folded, the graduated faces should be inside, and therefore protected.

The staff may be made 2½ inches wide, and about ¾ inch thick.

The writer disapproves of the ordinary method of numbering the levelling staff, for the following reasons:—

A glance at fig. 54, will show that sometimes the most prominent figure in the middle of the field has to be recorded, and sometimes the said figure diminished by unity.

Thus, if the horizontal wire cuts the staff as at *a*, the figures 3 and 9, so to speak, stare the observer in the face.

Neither of them must be inscribed in his book, for the true reading is 2·85. Nor does the writer find it is essential to practical accuracy that the staff should be subdivided to single hundredths of a foot.

By dividing into tenths and half-tenths, the hundredths can be estimated, with all necessary accuracy, by the eye. Indeed, the writer believes that by graduation in this manner, there is less liability to serious error, than with the fully-divided staff.

With the latter, especially in the case of beginners, there is a tendency to devote too much attention to the hundredths, so causing errors in the tenths, or even in the whole foot.

The writer has found that the method of graduation, shown in fig. 55, proves convenient.

The feet are marked by large diamonds, half red and half black — red below, and black above.

The tenths are marked by black diamonds, and the half-tenths by small black squares, respectively.

The figures indicating the feet are clearly marked twice on each successive foot, one black and the other red. Thus, taking the part between the third and fourth foot-divisions of the staff, it is obvious that any reading between these points, is three feet and some decimal of a foot.

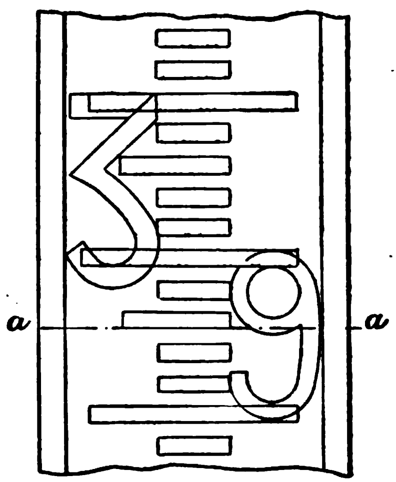


FIG. 54.

Each figure is two-tenths of a foot in height.

The black three, corresponding to the black half of the lozenge, marking the third foot division, extends from 3·10 to 3·30.

The red three, extends from 3·70 to 3·90.

One foot-numeral is therefore always in view, and the numeral which is nearest to the cross-wire is that to be inscribed in the level-book.

The black and red numerals also assist in enumerating the tenths. Thus

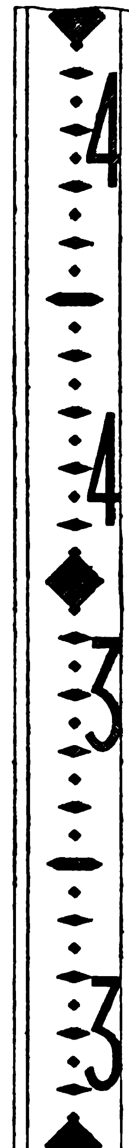


FIG. 55.

Bottom of black numeral	0·10
Middle of black numeral	0·20
Top of black numeral	0·30
Intermediate mark	0·40

A long mark	0·50
Intermediate	0·60
Bottom of red numeral	0·70
Middle of red numeral	0·80
Top of red numeral	0·90
Division of red and black lozenge . . .	1·00

The only case in which there can be any hesitation as to the proper foot-numeral to inscribe, is when the wire falls exactly on the division between the red and black halves of the large lozenge which indicates the exact termination of the foot.

Then the upper or black figure is to be inscribed.

If the wire falls on the black half of the said lozenge, then the black numeral is the proper one; if on the red, then the red one.

The diamonds and lozenges being 0·02 in height, afford the means of estimating the hundredths; 0·02, 0·03, 0·07 and 0·8 alone require to be estimated by eye. The writer is quite aware, that good work can be done with the ordinary staves, but having used a staff of the pattern just described for many years, and having found that persons, not previously conversant with levelling, learned to read it more quickly than the ordinary staff, he has considered it worthy of description.

Staves can also be obtained 12 feet long in solid wood, made with a hinge in the centre, and fitted with a stiffening arrangement at the back, to keep the two portions in a straight line when opened out for use. This pattern of staff has the advantages that the face is the same width from top to bottom, and consequently can be read with equal facility at any part, and that the whole arrangement will stand more rough usage than any telescopic staff. On the other hand it is somewhat heavy and hardly long enough for use in very hilly country.

For Ordnance survey work a 10-foot solid staff, figured on both sides, is used—one side commences at zero at the bottom, and the other at, say 1·5 foot—readings are taken front and back, and the staff can be reversed end for end, and read front and back again.

Perpendicularity of Staff.

It is obviously essential that the staff be held in a vertical position.

The usual plan of attaining this, is to cause the staff-holder to stand to attention, heels together, with the heel of the staff between his toes, and clasping it between the palms of his hands at the height of his face, with the staff touching his nose.

In this way a fair approximation to verticality can be attained.

It is, however, difficult to induce staff-men to assume this severe attitude.

In windy weather, or on rough ground, it is impossible for them to do so. The writer, therefore, recommends the use of some appliance which will enable the staff-holder to hold the staff perpendicular when standing in any attitude.

One way of so doing, is to attach a small circular bubble-level to the back of the staff, by means of a bracket.

The staff-holder then brings the bubble to the middle of the glass-plate, when, if the level is properly adjusted, the staff will be vertical. The drawback to this arrangement is, that circular levels are very apt to leak, the ether in them consequently evaporating.

The level, with its supporting bracket, forms a projection from the staff, and is therefore liable to injury in transport.

The simplest plan is to attach a small pendulum of brass or iron (fig. 56) to the back of the staff, thus:—

The point below the pendulum bob, fits loosely into a ring connected to the staff.

The staff-holder has merely to hold the staff so that the pendulum point swings about the middle of the ring, when perpendicularity, in all directions, is secured.

The writer has not found any difficulty in inducing staff-holders, even wild and uneducated men, to pay attention to the pendulum.

It certainly adds to the comfort of the leveller, by obviating the necessity for continued shouting and signalling to the staff-holder, which is not conducive to that repose of mind which is necessary to accurate work.

Fig. 57 shows another form of pendulum.

In this pattern, the pendulum is suspended by a stirrup in the middle of its length. A small cup in the stirrup rests on a point on a bent rod, screwed into the staff.

The upper point of the pendulum plays freely in a ring, also screwed into the staff at a point conveniently below the eye of the observer.

The merit of this arrangement is, that the staff-holder can see whether the point is in the centre of the ring, with greater ease than in the first-mentioned plan.

The period of oscillation of the compound pendulum, is also longer than that of the simple pendulum of equal length.

It is not usual to provide levelling-staves with levels, or pendulums, as good work can undoubtedly be performed without them.

One simple plan for ensuring verticality when reading, is to cause the staff-holder to swing the staff through a small angle on either side of the vertical, towards the observer.

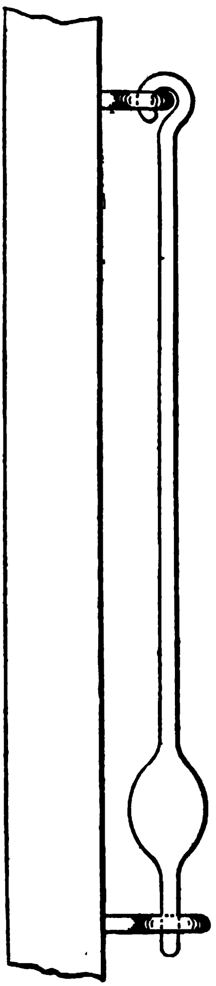


FIG. 56.

It is evident that the lowest reading observed is that corresponding to the vertical position of the staff (fig. 58). The observer can see, by the vertical wire of the telescope, whether the staff is held perpendicularly, in the plane of the instrument.

Care must be taken to see that the staff is swung on both sides of the vertical. The drawback to this plan is, that the staff-holder is apt to swing the staff laterally as well as to and from the observer, in a manner that is annoying. On the whole, the writer believes, as the result of experience in many countries, and with many

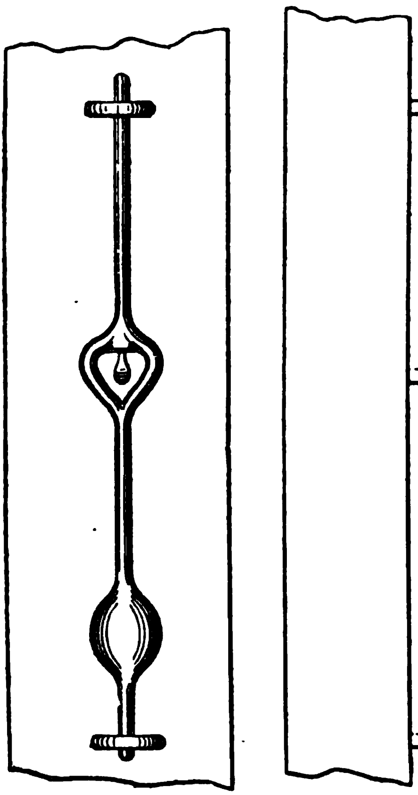


FIG. 57.

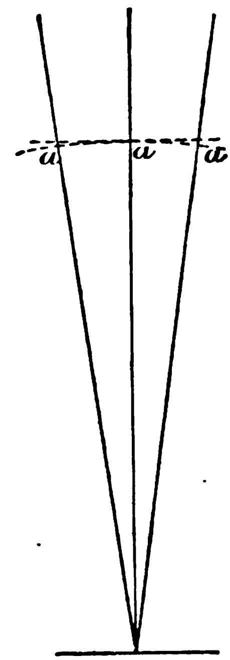


FIG. 58.

different races of men, that the pendulum is a valuable addition to the levelling-staff, tending to accuracy and expedition.

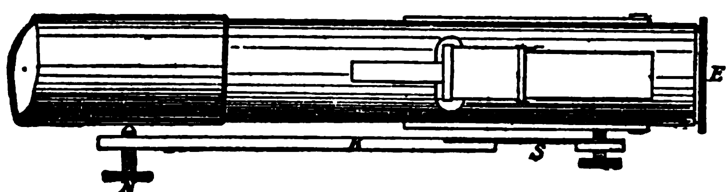
6. Reflecting and Water Levels.

Reflecting Levels. There are several forms of 'reflecting levels' and 'clinometers,' the following being the most useful.

Abney's Level. This level, invented by Captain Abney, consists of a tube in which (at the end farthest from the eye) is placed a metal mirror (filling half the tube) at an angle of 45° with the line of vision. The lower edge of the mirror, which is carried in a metal frame fitting into the end of the outer tube, is placed exactly in the line of sight. A bubble fixed to a carrier, capable of rotation, is reflected in the mirror, when in the centre of its run. The bubble carrier has a graduated arc, and is either moved direct by its rim, or by a pin and ratchet movement. The bubble, when reflected, and seen through the eye-hole, is bisected by the horizontal edge of the mirror, and coincides with the object viewed. The inclination of the line of sight can be read off on an arm attached to the bubble axis, by means of a vernier. This vernier subdivides the degree into six parts, thus reading to 10 minutes. The rates of inclination such as 1 to 1, 1½ to 1, etc., are also sometimes marked. The higher fractions, such as 1 in 6, &c., may be marked twice over, so as to enable the batter of a wall or chimney to be taken, with the assistance of a straight-edge against which to hold the tube, the level in the latter case being at zero when at right angles to its former and normal position. In some patterns the mirror edge is vertical, and a horizontal hair defines the position of the bubble when in the centre of its run.

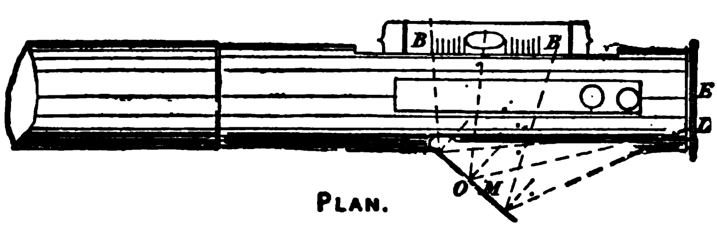
Wagner's Pocket Level.

This instrument is an amplification of the ordinary pocket level, by the addition of a telescope. In the Wagner's level (figs. 59 and 60) the level-bubble B is placed on one side of the telescope, the tube of which is cut away opposite



ELEVATION.

FIG. 59.



PLAN.

FIG. 60.

to it on both sides. On the opposite side of the telescope-tube, a mirror M is fixed, at an angle of about 35° with the axis. This reflects the rays of light from the bubble and renders it visible to the observer's eye, when in position for looking through the telescope. The side of the lens of the eye-piece E, which is nearest to the eye, is cut away, (fig. 61) on the side on which the mirror is fixed. A second lens L, of somewhat longer focal length, is similarly cut and fitted at the side of the eye-piece cell. The focal length of this second lens is such that the level-tube, as seen in the mirror, is in distinct focus. Its optical axis L O, moreover, is inclined to that of the telescope so that after reflection it cuts the middle point of the level tube. The result is that when looking through the telescope, a magnified image of the bubble is distinctly seen.

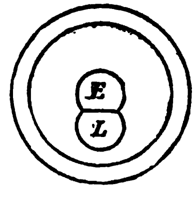


FIG. 61.

at one side of the field (fig. 62). The telescope is attached to a bar K at the eye-piece end, by a spring S. At the object end it rests on a fine point of a micrometer screw N, passing through a nut in the bar. It is therefore only necessary to direct the telescope to the staff, bring it approximately horizontal, and then, by means of the micrometer-screw, raise or lower the object-end of the telescope until the bubble is brought to the centre of its run, and read the position of the cross-wire on the staff. The smaller sizes of these instruments may be used supported on a light rod, cut to the height of the observer's eye, and provided below with a cross-piece or foot. The vertical rod is steadied with a second stick clasped to the upright rod with one hand, whilst the other is free to adjust the micrometer screw. It is preferable to mount the Wagner level on a tripod stand, which may be made very light.

The tripod carries at its head a ball-and-socket joint, which serves both as an axis of horizontal, and of vertical motion. The ball can be clamped into any position, by tightening the screws which hold down the cap, which keeps the ball in its seat. The observer simply adjusts the ball-and-socket joint to easy friction, turns the telescope round until the vertical wire corresponds with the staff. Then

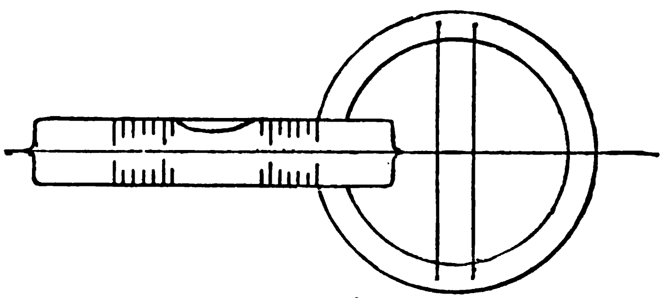


FIG. 62.

he elevates or depresses the telescope, until he sees the bubble somewhere near its central position. He then clamps the ball-and-socket by means of any one of its screws, and finally brings the bubble to the centre of its run by means of the micrometer-screw, and reads the staff. With a little practice, this can be done with great expedition. The ball-and-socket joint gives great range, both in altitude and in azimuth. It is only necessary to plant the legs in a stable position; and then a reading can be obtained in a few seconds, without the necessity for any temporary adjustment. The accuracy of the results attainable with this little instrument is surprising. The writer has more than once checked levels, taken with it, with those obtained by means of a 12-inch ordinary level. On one occasion a circuit was levelled simultaneously with a Wagner level having about a 4-inch focal length, and with a good 12-inch Dumpy level. Each reading was taken with both levels, and a separate book kept. The result was that, in no case did the difference in any one level, exceed 0·03 of a foot, and the final closing error, on completing the circuit, was in favour of the Wagner's level. There is no doubt, therefore, that these small and cheap levels, are capable of rendering excellent service, especially in rough and mountainous country. The optical qualities of the telescope are excellent, and the workmanship of the instrument generally is very good. In a bright light, it is quite practicable with the instrument of 4½ inches focal length, to bisect the hundredths of a foot on an ordinary staff, at a distance of 150 feet. With the larger sizes, having greater magnifying power, it is possible to obtain equal precision, at greater distances.

This level cannot be held to replace the ordinary pattern for general engineering purposes. It is not well suited for giving a series of spot levels, such as those of pegs, etc., from one position of the instrument. Such levels on even ground would be more conveniently, though not more accurately determined, with the ordinary level. For preliminary work, especially in mountainous regions, and for taking sections in which minute detail is not required, and where fore and back sights will, for the most part suffice, it is an excellent instrument.

For these levels, the bubble tubes are ground so as to be "reversible," that is to say, to be curved to equal radii in any longitudinal section (*vide* fig. 63). Consequently, when supported in a pair of Y's and levelled, the tube may be turned up-side down, without altering the position of the bubble. This materially facilitates adjustment. To make the level tube parallel to the line of collimation, take a reading with the level, say to the left, and the bubble in the centre of its run. Then, detach the telescope, and re-attach it with the level-tube right, again bring the bubble to the centre of its run and read again. If the two readings are the same, the collimation is correct. If not adjust to the mean of the two readings by the screws of the level supports, or by moving the diaphragm, preferably the former.

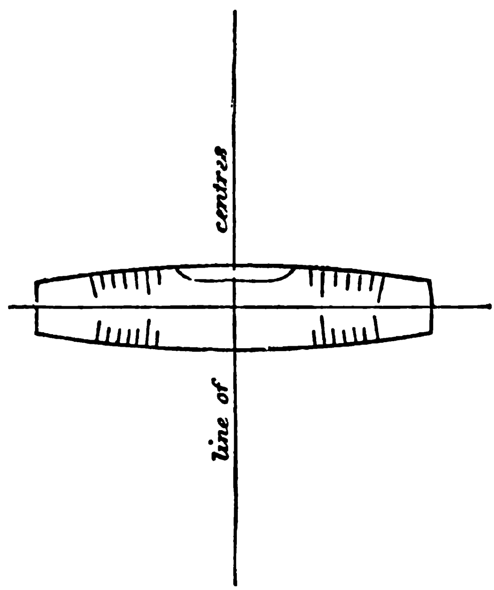


FIG. 63.

As reversible level-tubes are not easily procurable, it is well, when ordering for abroad, to provide a spare level-tube.

The bubble and mirror are enclosed in a casing (not shown in the figures), in the side of which is a slot for admitting light to the bubble-tube.

Pendulum Reflecting Level.

Another very portable form of reflecting level, (which, however, can only be used to trace a horizontal line from the observer's eye,) consists of a piece of glass about 1 inch square, half of which is silvered, and half plain, slung by a universal joint from a ring of convenient size, to slip over the thumb. The bottom of the frame, (*weighing about half a pound*) in which the glass is fitted, is provided with a bar, which, by a screw or by filing, can be adjusted so as to make the mirror hang vertically. The instrument being held at arm's length, with the help of a staff or other support, is moved gently up or down until the pupil of the eye as seen in the mirror is exactly bisected by the edge of the silvering. Any object observed through the clear glass at the same height as the reflected eye, will be level with it.

Other and more complicated forms of this instrument are in use, by which objects may be noted at different rates of inclination above or below the observer. They are all constructed with some form of adjustable balance at the bottom of the frame so that the glass may be tilted out of the perpendicular, to the required degree, the balance being marked with the slopes, 1 in 20, &c., for the convenient adjustment of the movable portion.

Barker's Clinometer. Barker's clinometer consists of a prismatic compass with a supplementary dial, weighted at one side so as to hang with its zero line horizontal, when the case is held vertically. The object is viewed through the slit over the prism and the sight-vane, (as for compass readings), and the angle of inclination is read through the prism in the same manner as the divisions on the compass card.

The clinometer dial is not a complete circle, but has a segment omitted, and a stop is fitted so that it may be fixed in such a position that an unobstructed view of the compass card, below the prism, may be obtained when the instrument is to be used for taking bearings. The case is usually furnished with a flat on one side, in order that it may be placed on a straight-edge, the inclination being read off on another part of the weighted disc, against a line marked on an opening in the box.

Rule and Bubble Level. Another form of clinometer is made in box-wood, somewhat after the manner of a folding rule, but much shorter, and of larger section.

Level bubbles are let into the surfaces of the two arms, the top arm being fitted with a sight and cross-wires at the ends. It is used in conjunction with a straight-edge, the lower arm resting thereon while the upper is opened up till the bubble is in the centre of its run. The inclination will be found in degrees marked on the hinge of the rule.

Water Level. The old form of this level, consisted of two vials fitted to the ends of a tube, and partly filled with water, the level sight being taken over the surfaces of the water as seen in the vials. This form of level is used to a limited extent in rural districts on the Continent. It is found useful sometimes when levelling through close buildings or towns, or for drain laying. Browne's water level is a convenient form for such work.

7. The Theodolite.

The Theodolite. The theodolite is the most perfect portable instrument at present in use for observing angles in azimuth or altitude. It is commonly made in either of the following three forms, viz. the 'Y' pattern, the 'Everest,' or the more modern 'Transit.'

The chief differences between the types, (to be described in detail hereafter), consist, in that the 'Y' instrument is provided with a semicircle only, for reading angles in altitude, the 'Transit' is fitted with a complete circle, while in the 'Everest,' there are two segments of circles for these readings and the telescope cannot be transited, (that is turned completely over on the trunnions which support the centre of the telescope) so as to read the same line either backwards or forwards.

Description of parts, common to Theodolites.

The theodolite (in the 4 foot-screw pattern) is attached to the screw of the tripod head by the lower of what are termed the parallel plates. These plates P P (fig. 64) form in reality a ball-and-socket joint, the motion of which is limited by the foot-screws S S. These are placed diametrically opposite to one another, in such a manner, that by slackening one and tightening the opposite screw, the plates may be placed at a varying angle relatively to one other, (within certain limits), and locked fast in that position. Care must be taken in setting up the tripod, that the lower plate is approximately level, in order that when the upper plate has been levelled by the screws, the latter may be nearly at right angles to the lower plate, obviating any tendency in the screws to slip sideways and jam.

In the centre of the upper parallel plate is formed a conical bearing, to receive the hollow axis of the 'lower limb.' Round the edge of the 'limb' or plate attached to this axis, is a ring *r r* of silver divided into degrees and half degrees in the smaller instruments, or to 20′ or 10′, in 6-inch to 12-inch instruments. The diameter of the plate is used to indicate the size of the instrument.

Within the hollow axis of the 'lower limb' revolves the solid cone attached to the under side of the 'upper limb.' The 'upper limb' carries the supports *t t* of the telescope, (with a magnetic compass arranged between them), two spirit levels *l* at right angles to one another, and two verniers *v v*, fixed opposite to each other, and reading on the degrees divided on the adjacent rim of the 'lower plate.' In the ordinary 5-inch instrument with the lower limb divided to ½ degrees, the verniers are arranged to read down to single minutes. A low-power magnifier, fitted to a sliding piece in the undercut groove beneath the 'lower limb,' is provided for reading the verniers.

A clamp *c* fitted to a slide in the same groove, carries a screw *d* working in a nut fixed to the 'upper plate,' in such manner, that when the clamp is tightened on to the 'lower plate,' the only relative motion possible between the two plates, is that due to the motion of the screw when revolved. This is termed a 'slow motion screw.'

A clamp *e*, similar in principle, is applied to the 'lower limb' and the 'upper parallel plate,' so that either the 'upper and lower limbs,' or the 'lower limb' and the 'parallel plate,' or both pairs, may be clamped together. On the 'upper limb,' or 'vernier plate' are fixed the two side frames which support the axis of the telescope. Each frame is fitted with a V shaped bearing at the top, with movable caps, the V block on one side being made adjustable in height. Inside each cap a small piece of cork is fixed so as to hold down the axis of the telescope with a firm but slightly elastic pressure.

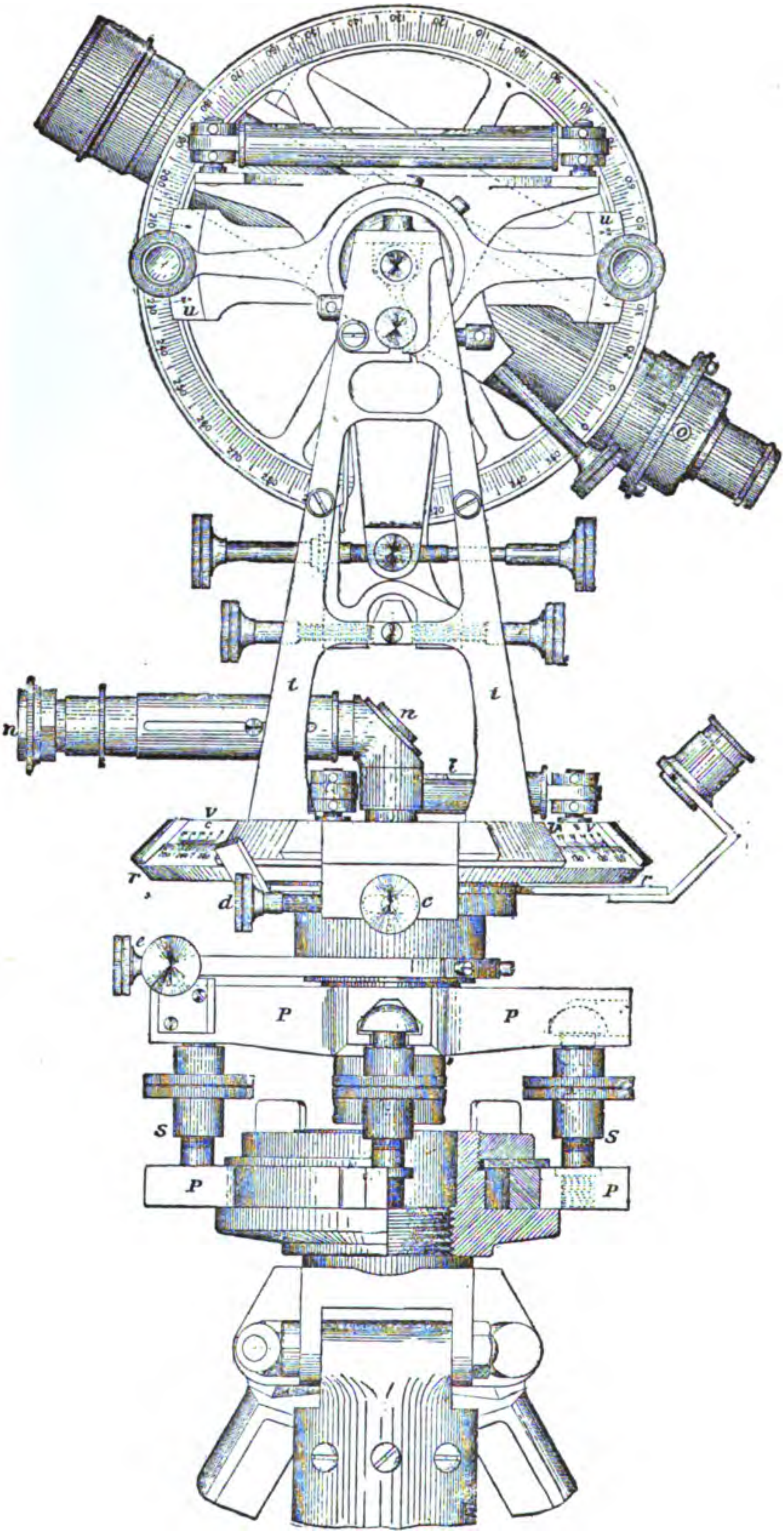


FIG. 64.

One of the spirit levels for levelling the 'upper plate' is commonly fixed on the side of one of these frames, the other being attached to the plate direct. In the centre of the 'upper plate' between the frames is fixed a magnetic compass, with a needle about 2½ inches long, the circle being divided to half degrees. It will be observed that the circle divisions are figured from right to left, contrary to the figuring on the limb, to allow for the fact that if the telescope be directed along a line having a 'bearing' of say 90°, the compass will be revolved round the stationary needle, from left to right.

In the 3 foot-screw pattern, a 'carrier,' with three cups or V slots (and a securing plate, with three holes in it to clip the feet of the screws), is screwed on to the tripod head, and the screws rest in the bottoms of these cups or on the V slots, but are not screwed down.

The Transit Theodolite.

In the transit instrument, (fig. 64) the telescope is fixed at right angles to the horizontal axis, which is supported in the bearings above mentioned on the 'A' side supports or frames. The vertical circle or limb is attached immovably to the telescope, being a flat circle inlaid with a ring of silver, and divided, each way, from the horizontal or zero line, to 90°. Two verniers, *u u*, reading to from 1′ to 10″, are carried on an arm to read in this circle. A clamp and tangent screw are fitted to clamp the vernier arm to the circle, and give slow motion.

The vernier arm is T shaped, the dropping leg being provided at the lower end with a jaw and two opposing 'clip screws,' by means of which it may be adjusted to a piece of metal projecting from either A support. When properly adjusted the verniers should read zero, when the line of collimation of the telescope is horizontal. A microscope is fitted to each vernier in the same manner as for the horizontal limbs. On the top of the 'vernier arm' a spirit level is provided of greater size and therefore of superior accuracy, to the small ones on the upper plate.

The telescope consists of two tubes, an inner and outer, sliding one within the other, actuated by a rack and pinion for focussing purposes. It is more convenient, in use, when the object-glass end moves away from the observer, than when, as in some instruments, the eye-piece end moves towards him. The object-glass is commonly about one inch in diameter, made up of a double convex lens outside, and a concavo-convex lens inside, of different qualities of glass, for a correction of the dispersion which would otherwise occur. Having passed through the object-glass, the rays are brought to a focus at a point close to the other end of the telescope where the 'diaphragm' is placed, and they are picked up by a pair of plano-convex lenses, fixed together in the eye-piece, with their convex faces opposed to one another. The eye-piece lenses are so arranged, that by a small sliding adjustment they may be made to focus on the diaphragm, and to pick up the image from the object-glass, at the same time, with that of two or more 'cross-hairs' stretched across an opening in the diaphragm.

The diaphragm consists of a brass ring, connected to the telescope by two pairs of opposing screws *o*, which pass through slotted holes in the tube, so as to permit of slight adjustment in either a vertical or horizontal direction. The 'cross-hairs' are as a matter of fact, fine spiders' webs attached by a varnish composition, or lines scribed on a glass disc. The use of an eye-piece as above described, produces an inverted image of the object viewed. An erect image may be obtained by the use of the erecting eye-piece, usually supplied with the instrument, but the disadvantage of the loss of light due to the use of the two additional lenses necessary, is generally considered to be greater than the slight inconvenience caused by the inversion of the image. In the body of the telescope two 'rings' or 'stops' are fixed, to cut off the outside rays which are more or less coloured, and to prevent reflections from the interior of the telescope tube. A sun-shade is usually provided for solar observations, with a cap to protect the object-glass from damage when not in use.

The tripod stand to which the instrument is attached when in use, is either made with camera legs, or sectional in shape so as to form a cylinder when closed together. The legs are fastened by bolts and screws to the under side of the tripod head. The tripod head is finished at the top by a large coarse-threaded screw, working into a female thread, cut in the under side of the 'lower parallel plate' or 'tribrach.' Underneath, in the centre, is a small hook from which a plumb-bob is suspended to enable the theodolite to be set up exactly over a given point. This instrument is fitted with a diagonal eye-piece for use when making observations at high altitudes, and also with dark glasses. A striding level is supplied to record the inclination of the horizontal axis of rotation, when desirable.*

The horizontal axis is usually perforated so that the wires can be illuminated at night, by means of a lamp. This can also be effected by a bead suspended in front of the object-glass, or a slip of white paper $\frac{1}{8}$ inch wide (fixed at 45° across the object-glass), rays of light being thrown across them from a lamp held in the hand.

The 'Y' Theodolite. The 'Y' theodolite is similar in construction to the transit as far as concerns the stand and work below the 'A' side supports. The vertical arc is little more than half a complete circle, the balance being cut off and replaced by a flat plate, carrying at either end a bearing or 'Y' to carry the telescope, from which the instrument takes its name. The telescope, which is similar in its optical arrangements to that of the transit, is provided with two turned collars, which fit the bearings, so that the tube may be revolved on its longitudinal axis, or when the caps of the bearings are removed, the telescope may be lifted out and replaced end for end. The spirit-level is fixed to the telescope so as to be underneath, and out of the way, when in position for use. No adjustment is provided in the bearing of the horizontal axis, as in the case of the transit, this adjustment not being of such importance, since the telescope cannot be used to observe angles of great elevation or depression. Only one vernier is fitted to the vertical arc, fixed to one side support, and in some instruments only one level is provided on the upper plate, at right angles to the telescope, levelling in the other direction being accomplished by setting the vertical arc to zero and using the level attached to the telescope or by turning the plate through 90°. A clamp and tangent screw, are provided to the vertical arc. The 'A' side supports are made lower, than in the case of the transit, as there is no need to provide room for transiting the telescope.

* *Vide* Appendix B.

Late improvements and additions.*

It is to be observed that the instruments above described, are of the form most generally used, and hitherto the stock article sold by the ordinary dealer. Of late years, however, considerable attention has been given, by the best makers, to improvement in certain details of construction, the principle of the instrument remaining the same.

1. Various forms of shifting heads can be obtained, to render easier the operation of centering the instrument over a station point. Some of these arrangements are adapted to the 'parallel plate' form, and others to the 'three-screw' arrangements. A useful accessory to the shifting head, where very accurate work is desired, is a small diagonal telescope *n n* with two cross-wires, so constructed that it may be inserted into the inner vertical axis which is made hollow for the purpose. By this means the theodolite can be set up with ease over a mark, not less than 9 inches or more than some 10 feet beneath it and scribed on the head of a copper nail driven into the station peg. When it is desired to use the plumb-bob the hole in the axis is filled by a long turned plug, with an eye or hook at its lower end.

2. A very desirable addition to the tangent screw is a reaction spring, so arranged that the nut shall be always pressed against the screw in one direction, preventing the annoyance of 'back slip' when the threads have become worn from use.

3. Greater accuracy in reading 'magnetic bearings' can be obtained, by the use of a needle of considerably greater length than can be got into the circular box between the standards. Such a needle is usually mounted in a long narrow box, giving a range of motion over a small arc of about 20° at each end, the box being fitted either on the top of the telescope, or underneath the 'lower plate.' The 'upper plate' vernier being set to zero, at the same time that the needle points to the north, if the 'lower plate' be clamped and the 'upper' one released, the bearing of any point to which the telescope may be directed, will be indicated on the divided circle on the lower limb.

The compass needle is sometimes suspended in a small telescope tube, with the end of the needle turned up at right angles, so as to pass to and fro in front of a glass divided scale, the telescope being fixed below the 'lower limb' in the same manner as the trough compass above mentioned.

4. If observations have to be taken at night, or in tunnels, &c., it is necessary to have some means of illuminating the cross-hairs in a slight degree, but not to an extent sufficient to interfere with the view of the distant object. This is accomplished by piercing one end of the axis of the telescope, so as to admit of the rays from a very small lamp placed on a bracket, passing to its centre. The light from the lamp is then reflected towards the eye end, by means of a small mirror placed exactly in the axis of the telescope, giving just sufficient illumination to enable the hairs to be seen. The lamp is balanced by a weight attached to the opposite support.

5. The chief source of weakness in the ordinary instrument is the webbed diaphragm, the cobwebs being of such slight texture that they are readily broken by the least touch, or by the wind or rain, if the eye-piece be momentarily

* *Vide* Appendix A for account of micrometers to read hor. and vert. arcs.

taken out for cleaning, or other purposes. This difficulty is overcome by using discs of parallel worked glass upon which can be ruled lines of the desired fineness. Small points of platinum alloy are sometimes used, attached to the top, bottom, and side of the hole in the diaphragm plate, so that while the tip of the horizontal pointer is at the exact centre of the space, the top and bottom points indicate the direction of a vertical line passing through the same centre. These, however, do not give such a fine definition as lines ruled on glass.

6. Instruments in which the vertical circle bubble is fixed to the vernier arm, will be found much more convenient in use, than those in which it is attached to the telescope tube. In the former case the level is always in its proper position, and when correctly adjusted, indicates when the zero line of the verniers is horizontal, whether the telescope be so or not, while in the latter case this fact cannot be ascertained, without first bringing the telescope back to zero and clamping it there. The level adjustment is also greatly simplified, when the level is attached to the vernier arm.

7. The weight of the instrument is a matter of great importance where it has to be carried for long distances, and where rapidity of observation is requisite, and this may be reduced at least 30 per cent. by the use of aluminium in the construction of parts which are not subject to great wear.

Adjustment of the Transit Theodolite.

1. Parallax.
2. Vertical axis of rotation.*
3. Horizontal collimation.
4. Vertical circle or 'zero of altitude.'

1. *Adjustment for parallax*, as for the Level telescope (p. 74).

2. *Adjustment of vertical axis of rotation*, i.e. to make the vertical axis of rotation truly vertical, and set the levels of the horizontal plate at right angles to it. Turn the telescope till the vertical circle level (attached to the T carrier) is parallel to two foot-screws, and by them bring the bubble to the centre of its run. Turn the telescope and upper horizontal plate through 180°, and if the bubble leaves the centre of its run, bring it back *half* by the foot-screws, and *half* by the T antagonising screw. Then, turn the telescope through 90° over the third foot-screw (or other pair of foot-screws), and by their motion *alone* bring the bubble back to the centre of its run. Repeat till perfect. Lastly, adjust the levels on the horizontal plate so that their bubbles may be in the centres of their runs.

3. *Adjustment for horizontal collimation*, i.e. to set the line of collimation (for definition *vide* 'The Level') at right angles to the horizontal axis of rotation which rests on the trunnions. It must be noted that these instruments have no adjustment screws for moving the diaphragm *vertically*. Intersect some distant well-defined object with the cross-hairs, and clamp the horizontal plate. Reverse the telescope in its bearings, and if the object be not still intersected by the cross-hairs, correct *half* by the tangent screw of the horizontal plate and *half* by the screws that move the diaphragm. Repeat till perfect.

4. *Adjustment of the vertical circle or 'zero of altitude*, i.e. to make the vernier read zero on the vertical arc when its bubble is at the centre of its run. Bring the vertical circle bubble to the centre of its run, by means of the T

* *Vide* Appendix B for 'Adjustment of Hor. Axis of Rotation.'

carrier antagonising screws. Intersect a well-defined object with the horizontal wire, and read one vernier. Reverse the telescope in its bearings, and re-level the bubble by the antagonising screws. Again, intersect the same object with the horizontal wire, and read the same vernier. Set the vernier to read the *mean* of these readings, on the arc. Now bring back the horizontal cross-hair (which will have left the object), by means of the antagonising screws, and adjust the bubble, which will now have left the centre of its run. This is done by means of the capstan-headed screws at one end of the bubble tube. Repeat till perfect.

In lieu of adjusting the bubble tube, the error may be noted and applied as an '*index correction*' to all vertical angles.

Adjustments (3) and (4) cannot be made absolutely perfect, but the effects of residual error may be eliminated by pairing observations with the telescope reversed.

The Adjustment of the 'Everest' Theodolite.

The adjustments of this pattern are exactly similar to those above described for the 'Transit Theodolite.'

Adjustment of the 'Y' Theodolite.

1. Parallax.
2. Collimation.
3. Telescope level.
4. Vertical axis of rotation.
5. Vertical circle or 'zero of altitude.'

1. *Adjustment for parallax*, as for the Level telescope (p. 74).

2. *Adjustment for collimation.*—(For definition *vide* 'The Level.') Turn the telescope by a rotary motion in the Y's until one pair of diaphragm screws is vertical, and sight the cross-wires on some well-defined object. Now turn the telescope half round in the Y's and if the intersection of the wires has moved *vertically* from the object, bring it back *half* by the vertical tangent screws, and *half* by the *now* vertical pair of diaphragm screws. Now rotate the telescope through 90°, and repeat the process with the other pair of diaphragm screws, *now* vertical. Repeat until the intersection of the cross-wires remains fixed on an object during the whole of a revolution of the telescope in the Y's.

3. *Adjustment of telescope level*, i.e. to make the telescope level parallel to the axis of revolution of the telescope in the Y's, and therefore parallel to the line of collimation. Bring the bubble of the telescope level to the centre of its run and clamp the vertical arc. Reverse the telescope end for end in the Y's, and if the bubble does not still remain in the centre, correct *half* the displacement by the vertical tangent screws, and *half* by the screw at one end of the level. Now rotate the level *slightly* in the Y's, and if the bubble moves, place the level in a truly vertical plane by means of the lateral screws provided at the other end of the level. Repeat till perfect.

4. *Adjustment of vertical axis of rotation*, i.e. to make the vertical axis of rotation truly vertical, and to set the line of collimation and the levels of the 'horizontal plate' at right angles to it. Turn the telescope till it is exactly over two opposite foot-screws, and bring the bubble of the telescope level to the centre of its run, by the vertical arc tangent screw. Now clamp the 'lower plate,' and

turn the 'upper plate' through 180° till the telescope is again over the same pair of foot-screws. If the bubble be not still in the centre of its run, correct *half* the displacement by the vertical tangent screws, and *half* by the foot-screws. Now turn the telescope and 'upper plate' through 90° over the other foot-screw (or pair of foot-screws) and bring the bubble to the centre of its run with this screw (or pair of screws) *alone*. The vertical axis of rotation will now be nearly vertical. Repeat till perfect. Adjust the levels on the horizontal plates entirely by their own adjusting screws.

5. *Adjustment of vertical arc or 'Zero of Altitude,'* i.e. to make the vernier of the vertical arc read zero when the vertical axis of rotation is truly vertical. If after performing the fourth adjustment, the vernier of the vertical arc does not read zero, when the telescope bubble is in the centre of its run, then make it do so by adjusting the vernier plate, by means of the small screws by which it is fixed. In lieu of adjusting the vernier plates, the reading of the vernier may be noted and applied as an '*index correction*' to all vertical angles.

CHAPTER IV.

TRAVERSE SURVEYING.

Remarks on Neglecting the 'Figure of the Earth.'

In the preceding chapters on chain surveying, it has been tacitly assumed, that for all practical purposes, corrections necessary when the spherical form of the earth is taken into account, may be neglected, and the area surveyed has been treated as a plane surface. This assumption may be safely made when the area surveyed is small. Indeed, the only way in which the surface of a sphere can be depicted correctly on flat paper, is to divide the spherical surface into limited areas, so small, that each may be treated as a plane. In discussing the geometrical problems involved in traverse surveying, the same assumption will be made as heretofore, and the spherical form of the earth will be neglected so far as field work and plotting are concerned. The errors due to this assumption will be far less than those which are unavoidable in ordinary chainage, and angular measurement. It will be seen hereafter, however, that when astronomical observations are applied to determine the true north and south line, at various points of a traverse, the convergence of the earth's meridians must not be neglected. Simple rules for computing this convergence will be given later on, after the practical portion of traverse surveying has been described.

Definition of the Term 'Traverse Surveying.'

The expression 'traverse' was doubtless originally borrowed from the art of navigation, in which it signifies 'the method by which the mariner determines the change in the position of his ship,' after sailing in various directions, or on different 'courses,' for various distances, as estimated by the log. In the first instance, it was doubtless applied to surveys made with the compass, when the surveyor observed with the compass, the 'bearing' of the line, from one station-point to the next, and measured with the chain, the intervening distance. In this way he determined directly the relative positions of a number of station-points, delineating a road or boundary, and without the necessity for measuring any subsidiary lines, merely for the purpose of fixing the relative inclination of the several station-lines to each other, as above described under chain surveying. In this manner he can prepare a plan of a forest without the labour and expense of chaining lines across it, or of a lake, across which chaining would be impracticable, or he can survey a road without trespassing on adjacent property. In short, he determines the path which he traverses on land, exactly as the mariner determines the path traversed by the ship on the ocean.

The compass not being susceptible of great accuracy in angular measurement, the theodolite is often employed in lieu thereof, and instead of measuring the magnetic bearings of the several lines, their inclinations to each other or to a standard line of direction are measured, the standard line of direction being usually a true north and south line.

Traverse surveying, therefore, may be defined as 'a method by which each survey point is determined by one angular and one linear measurement,' instead of by two or more linear measurements as in chain surveying.

Properties of Polygons and Check on Angular Measurement.

Let A B C D E (fig. 65) be any polygon. It is clear that if all the sides A B, B C, &c., as well as all the angles A B C, B C D, C D E, D E A and E A B, are measured, complete data exist for delineating the polygon on paper, and moreover, it is possible to check the accuracy of the work. Again, if the position of any point such as A, is already determined and fixed on the plan, and the angle which A B makes with some fixed line, such as the true north and south line S A N drawn through A, is also known, then the whole polygon can be placed in its proper position with regard to the remainder of the survey. One might proceed to lay off the angle N A B, and set off the measured distance A B, then lay off the angle A B C and set off B C, and so on. If the work were accurately performed, both in the field and in the office, then on completing the polygon the starting point A would be reached, and the polygon would 'close.' This procedure would be both cumbrous and inaccurate. If the polygon did not 'close,' absolutely correctly, there would be nothing to show whether the inaccuracy was due to faulty measurements in the field, or inaccuracy in drawing.

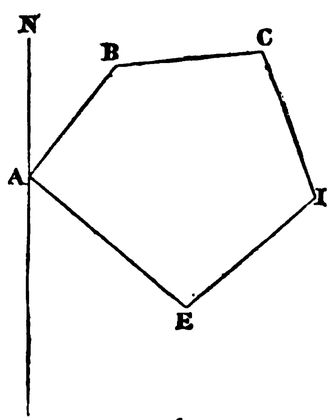


FIG. 65.

The accuracy of the angular measurements can be tested however by the following precept.

'*The sum of the interior angles of a polygon, added to four right angles, is equal to twice as many right angles as the figure has sides.*'

As an exterior angle is the difference between the interior angle and 360°, it matters not whether the 'exterior' or 'interior' angles are measured and recorded.

If 'exterior' angles have been recorded, then the precept assumes the following form.

'*The sum of the exterior angles of a polygon, diminished by four right angles, is equal to twice as many right angles as the figure has sides.*'

The surveyor having measured all the 'interior' or 'exterior' angles of a polygon, is at once in a position to test the accuracy of this portion of his work, by simple addition. The conditions under which it is expedient to measure 'exterior' and 'interior' angles respectively, and the limit of permissible error in summation will be discussed later on.

Rectangular Co-ordinates or Lat. and Dep.

In the methods of surveying hitherto described, the various points have been laid down on paper, by the intersection of the lines which form the network of triangles into which the survey is divided, or of arcs struck with these lines as radii. Minor points, however, have been fixed by 'offsets' at right angles to the principal survey lines. The method of plotting a polygon on paper with protractor and scale, is at all times one of indifferent accuracy, and when the survey is so extensive that it cannot be plotted on a single sheet of paper of reasonable size, some more scientific method, obviating the use of the protractor, must be introduced.

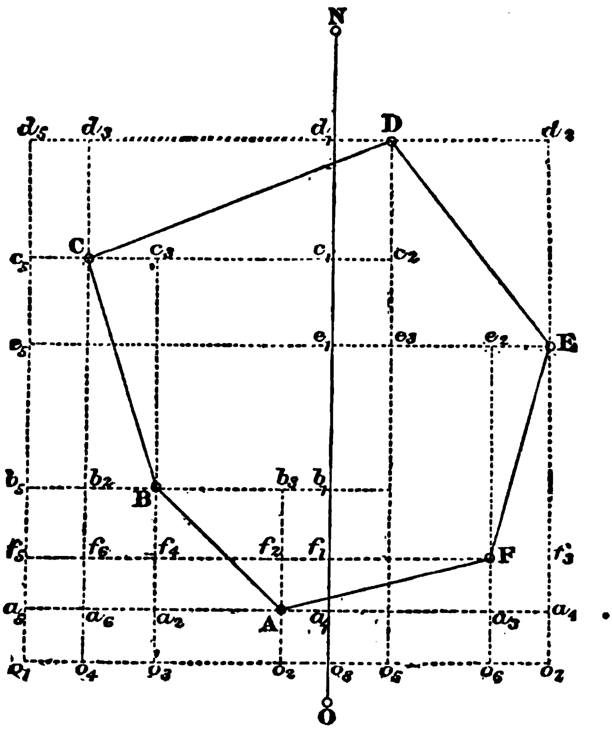


FIG. 66.

To this end, the method of 'plotting by co-ordinates' is adopted in extensive surveys.

This system will be readily understood by the consideration of a method of surveying plots, practised in some countries.

A straight line O N (fig. 66) is ranged through the middle of the plot, joining some already determined points. Perpendiculars are laid off by a cross-staff or optical square from this 'primitive line of direction' to the corners of the plot A B C D E F. The intersections of these perpendiculars with O N are marked at $a_1 f_1 b_1 e_1 c_1$ and d_1 and the distances $O a_1$, $O f_1$, $O b_1$, $O e_1$, $O c_1$, $O d_1$, are successively measured. Then the perpendiculars a_1 A, f_1 F, b_1 B, e_1 E, c_1 C, d_1 D, are measured. Full data for plotting the survey are thus available. Each point, D for example, may be plotted independently by laying off O d_1 along O N and d_1 D at right angles to O N, and so on, with the other points on O N. The sides of the figure are then laid down by joining the several points. If the actual lengths of the sides A B, B C, &c., from point to point were measured, a check on the accuracy of the work could be obtained by comparing the lengths of the sides scaled off the plan, with the measurements made in the field, or more accurately, by computing the distances between the points from the original data, and comparing measured distances.

Prolong the several perpendiculars A a_1, B b_1, &c. (as shown in fig. 66) and draw lines parallel to O N through the angles of the polygon A, B, C, D, E, F. Then all the triangles A a_2 B, B b_2 C, &c., are right-angled, and are further equal to the complementary triangles A b_3 B, B c_3 C, &c., consequently by the property of right-angled triangles,

$$A B = \sqrt{A a_2^2 + a_2 B^2} \text{ or } \sqrt{A\ {}^2 + b_3 B^2},$$

$$B C = \sqrt{B b_2^2 + b_2 C^2} \text{ or } \sqrt{B c_3^2 + c_3 C^2}.$$

The line O N instead of being a line joining two known points, may be a true north and south line, running through one known point such as O, and in what follows '*the primitive line of direction*' will be assumed to be '*a true north and south line.*' It is desirable, but not necessary that this should be the case, the nomenclature being thereby simplified.

The 'Meridian' of a Traverse Survey.

The 'primitive line of direction' will be called the 'meridian.'

'Origin.'

The 'one known point' will be called the 'origin.' The distance measured along the meridian between the 'origin,' and the intersection of a perpendicular drawn from any survey point, will be called the 'meridional distance' of that point.

'Meridional Distance.'*

'Perpendicular.'

The length of the 'perpendicular' from any survey point to the 'meridian' will be called the 'perpendicular' of that point.

'Co-ordinates.'

The 'meridional distance' and the 'perpendicular,' will be called the 'co-ordinates' of that point.

Thus O a_1 is the 'meridional distance' of A. Its 'perpendicular' is A a_1. O b_1 is the 'meridional distance,' and b_1 B the 'perpendicular' of B and so on.

'Difference of Latitude.'

The distance from point to point, measured along or parallel to the meridian, is called the 'difference of latitude' of these two points. Thus, $a_1\, b_1 = A\, b_3 = a_2$ B is the 'difference of latitude' between A and B. Similarly, $b_1\, c_1 = B\, c_3 = b_2$ C is the 'difference of latitude' between B and C, and so forth for the remaining points.

'Departure.'

The distance from point to point in a direction perpendicular to the meridian is called the 'departure' of these points. Thus the 'departure' of B from A is A $a_2 = b_3$ B, that of C from B is B $b_2 = C\, c_3$, and so forth for the remaining points.

'Northings' and 'Southings.' The 'differences of latitude' are called 'northings' or 'southings,' according to the direction in which they are taken. Thus if we proceed round the polygon A B C D E F in the alphabetical order of the points,

The 'difference of latitude'	A to B	is a	'northing.'
" "	B " C	"	'northing.'
" "	C " D	"	'northing.'
" "	D " E	"	'southing.'
" "	E " F	"	'southing.'
" "	F " A	"	'southing.'

Proceeding round in the opposite direction, A F E D C B A those lines which were 'northings' in the first case, will be 'southings' and *vice versâ*. For example A to F would now be a 'northing,' &c.

'Eastings' and 'Westings.' Similarly with the 'departures.' Proceeding in alphabetical order as before:

'Departure'	A to B	is a	'westing.'
"	B " C	"	'westing.'
"	C " D	"	'easting.'
"	D " E	"	'easting.'
"	E " F	"	'westing.'
"	F " A	"	'westing.'

Taking the points in inverse alphabetical order the 'eastings' and 'westings' will be reversed as before.

It is clear that if any figure be traversed, returning to the starting point. *The sum of the 'northings' is equal to the sum of the 'southings,' and the sum of the 'eastings' is equal to the sum of the 'westings.'*

This precept affords a check on the lineal measurement of a polygon, no matter how the 'differences of latitude' and 'departures' have been obtained, whether by chain measurement or otherwise.

The 'Bearing of a Line.' The angle which any line makes with the 'selected meridian' or with a line parallel thereunto is called the 'bearing' of that line. The 'bearing' of A B is the angle b_3 A B, of B C the angle C_3 B C, and of C D the angle d_3 C D, and so on. The 'bearing of a line' must not be confused with the 'azimuth of a line,' since the latter is the angle which a line makes with a 'true meridian' passing through a point in it.

The length of a line and its 'bearing' being known the 'difference of latitude' of its extremities, can be calculated trigonometrically. Thus in the figure, the triangle A b_3 B is a right-angled triangle, the angle at b_3 being the right angle.

Then

$$A b_3 = a_2 B \text{ 'diff. lat.' between A and B} = AB \times \cos \angle b_3 AB$$
$$= AB \times \cos \text{ 'bearing,'}$$

and

$$b_3 B = \text{'departure' B from A} = AB \times \sin \angle b_3 AB$$
$$= AB \times \sin \text{ 'bearing.'}$$

Similarly,

$$b_2 C = B c_3 = \text{'diff. lat.' between B and C} = BC \times \cos \angle c_3 BC$$
$$= BC \times \cos \text{'bearing,'}$$

and $c_3 C = B b_2 =$ 'departure.'

The method of determining 'bearings' will be described after their practical notation has been discussed.

From the above it is evident, that there are complete checks on the accuracy, of both the linear and angular measurements, of a survey of any close polygon, which may be summarised as follows.

The sum of the 'interior' angles augmented by four right angles is equal to twice as many right angles as the figure has sides.

The sum of the 'exterior' angles diminished by four right angles is equal to twice as many right angles as the figure has sides.

The sum of the 'differences of latitude' being 'northings,' is equal to the sum of those which are 'southings.' The sum of the 'departures' being 'eastings' is equal to the sum of those which are 'westings.' Or more briefly,

The sum of the 'northings' is equal to the sum of the 'southings,' and the sum of the 'eastings' to the sum of the 'westings.'

Systems of Notation Discussed.

Before proceeding to discuss the method by which the 'bearings' of lines may be observed directly, or deduced from angles, it is desirable to define the notation used in writing them down. Two systems of notation are in vogue, namely, the 'whole circle,' and the 'four quadrant' or 'reduced bearing,' notation.

The surveyor treats *angles* and *trigonometrical functions* somewhat differently

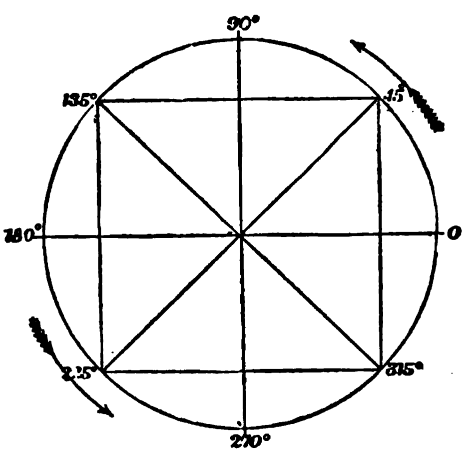


FIG. 67.

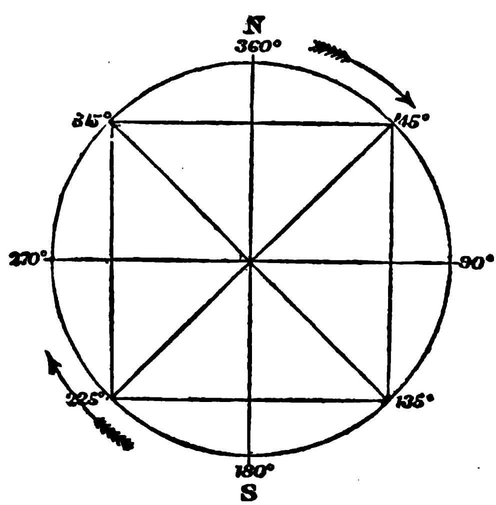


FIG. 68.

from the manner in which they are dealt with in text-books of trigonometry. In trigonometrical works, *angles* and *trigonometrical functions* are measured round from the right to the left, or in the direction opposite to the hands of the clock (fig. 67). When surveying on the other hand, they are assumed to flow from zero or 360° commencing at the top or north of the circle, round with the hands of the clock (fig. 68) and this is the manner in which modern theodolites are graduated. This alteration makes no difference in the value of the various trigonometrical functions, such as sine, cosine, &c.

'Whole-circle' Method.

In the 'whole circle' method, 'bearings' commence at zero, or 360°, at the north, and are measured 'clockwise' round the whole circle thus.

Cardinal Points.		Whole Circle Bearings.
N.	=	Zero or 360°
N.E.	=	45°
E.	=	90°
S.E.	=	135°
S.	=	180°
S.W.	=	225°
W.	=	270°
N.W.	=	315°

This system has the merit, that it coincides with the graduation of the modern theodolite and compass, and that the angle suffices, in itself, to determine the 'bearing,' without reference to the cardinal points of the compass. Moreover, any *correction* has the same *sign*, throughout the entire circle. Thus, suppose that the 'bearings' of a polygon were observed with a prismatic compass, graduated from 0 to 360°, as usual. Suppose (*vide* fig. 69) that the magnetic north was 18° west of true north, then the *magnetic* 'bearings' would be reduced to *true* 'bearings,' merely by deducting 18° from every observed 'bearing.' If the deviation were 18° to east, then the correction would be added throughout. Moreover, the 'whole circle bearings' may be easily computed, from observed 'interior' or 'exterior' angles, by a simple rule to be given hereafter, and common to all cases. This is the system, therefore, that should be adopted universally.

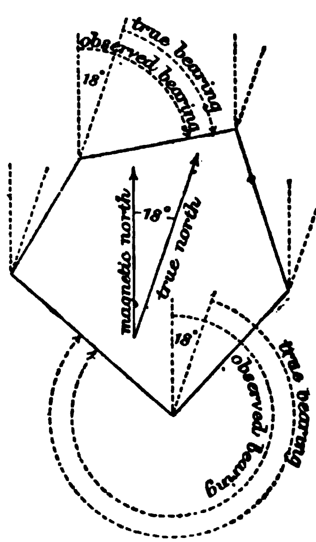


FIG. 69.

Unfortunately, most trigonometrical tables only give the functions of angles from 0 to 90°. (Shortrede's trigonometrical tables, and his traverse-tables are, as far as the writer knows, the only tables which have arguments from zero to 360°, and unfortunately they are scarce.) Consequently, with ordinary tables, to obtain the sine or cosine of an angle greater than 90°, a preliminary calculation has to be made as follows.

Referring to the diagram (fig. 70) lay off N O A = γ = any angle less than 90°. Make < D O N = < N O A, produce D O and A O to B and C respectively. Then < C O S and < S O B are also equal to γ. Join D A and C B, cutting N S in E and F respectively.

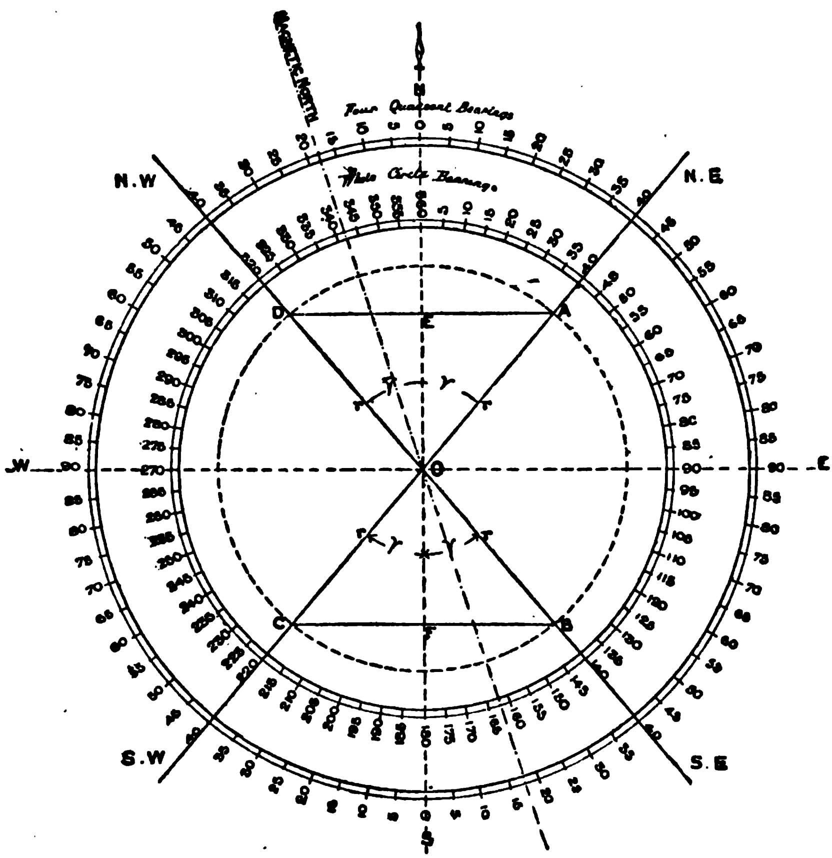


FIG. 70.

Then D E = E A = C F = F B = $r \times \sin\gamma$, (where r = O A = the radius of the circle), and O E = O F = $r \cos\gamma$, *and therefore* $\cos\gamma = \cos(180 - \gamma)$ $= \cos(180 + \gamma) = \cos(360 - \gamma)$ and $\sin\gamma = \sin(180° - \gamma) = \sin(180° + \gamma)$ $= (\sin 360 - \gamma)$.

Consequently, there are four 'bearings,' on the whole circle system, whose *sines* and *cosines* have the same *numerical* value.

< N O A = γ	Cos < N O A northing, and sin easting.
< N O B = $180 - \gamma$.	Cos < N O B southing, and sin easting.
< N O C = $180 + \gamma$.	Cos < S O C southing, and sin westing.
< N O D = $360 - \gamma$.	Cos < S O D northing, and sin westing.

Hence, when ordinary trigonometrical tables are used, a calculation has to be made as follows, before the *sine* and *cosine* can be taken out from the tables.

For 'bearings' less than 90° enter tables, with 'bearing' direct.							
„ ditto greater than 90° but less than 180°, with 180° – 'bearing.'							
„ ditto	„	180°	„	„	270,	„	'bearing' – 180°.
„ ditto	„	270	„	„	360,	„	360 – 'bearing.'

It is necessary to compute these auxiliary angles and to record them, before using the tables.

'Four-Quadrant System.'

To obviate this clerical labour the 'Four-quadrant system' of recording 'bearings' has been introduced, or at least has remained in use, notwithstanding its defects. In this system the graduations of the circle are numbered from north to east, from north to west, from south to east, and from south to west:—

Thus the bearing of

A	from	O	is called	γ N.E.
B	„	O	„	γ S.E.
C	„	O	„	γ S.W.
D	„	O	„	γ N.W.

'Reduced Bearings.'

'Bearings' thus written will be called 'reduced bearings,' which may be computed from the 'ordinary' or 'whole circle bearings,' by the following rules.

1. 'Whole circle bearing' not greater than 90°.
   The angle stands, and is entered as N.E.
2. 'Whole circle bearing' greater than 90° but less than 180°.
   Deduct 'bearing' from 180° and enter remainder as S.E.
3. 'Whole circle bearing' greater than 180° but less than 270°.
   Deduct 180° from 'bearing' and enter remainder as S.W.
4. 'Whole circle bearing' greater than 270° but less than 360°.
   Deduct 'bearing' from 360° and enter remainder as N.W.

The *sines* and *cosines* of the 'reduced bearings,' thus found, may be looked out direct, from the ordinary trigonometrical tables. This is perhaps the sole merit of the 'four-quadrant system.'

It has the following serious disadvantages. (1) In addition to the numbers representing the angles, the cardinal points of the compass have to be entered. (2) Any constant correction has opposite signs, in the first and third, and second and fourth quadrants, respectively. For example, supposing the magnetic bearings of O A, O B, had been observed by means of a compass (Fig 70), graduated (as is usual on board ship) into four quadrants. Supposing that the *true* north was 18° east of the *magnetic* north, or in other words, the 'declination' of the needle were 18° west, then, to find the true bearing from the magnetic bearing,

Deduct 18° from bearing of O A.
Add „ to bearing of O B.
Deduct „ from bearing of O C.
Add „ to bearing of O D.

From the above it will be seen, that the correction changes sign in successive quadrants, and errors are, therefore, more probable than in the case of 'whole circle bearings,' where the correction has the same sign throughout the circle.

Fixing in space, or in reference to Adjacent Polygon.

Assuming that the angles of a polygon and the length of its sides have been measured, its form and size is determined, but its position in space or connection with other polygons has not been fixed. If, however, the position of one point be known, and the angle which the line from that point to any other point in the polygon makes with the meridian, or with the side of any other polygon, be also known, then it is clear that all the points may be fixed in their true position with reference to the meridian, or that other polygon. For example, in the polygon A B C D E (fig. 71), let A be the known point, and the line N_1 S_1 the 'meridian,' or line parallel to the 'meridian,' then the 'bearing' of the line A B is known, and is represented by the angle N_1 A B. The 'bearings' of all the sides may be computed from this known 'bearing,' and from the observed 'interior' or 'exterior' angles.

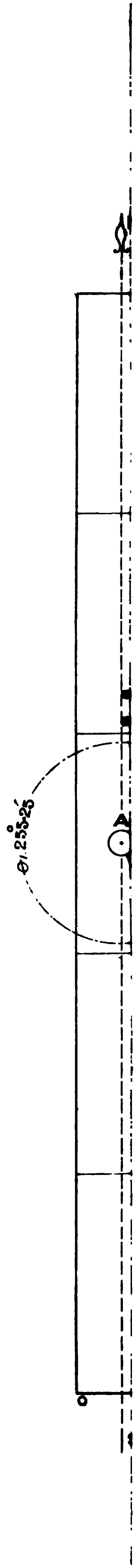
It is to be clearly understood, that as far as the plain geometry of the problem is concerned, it is a matter of indifference whether the line N_1 A S_1 is a 'terrestrial meridian,' (that is a true north and south line through A), or a line parallel to a 'terrestrial meridian,' or any selected 'meridian' being a line between A and some other fixed point. If, in any exceptional case, it be convenient to adopt some other line than the true north, the reasoning which follows remains unchanged. Through B, C, D, and E, draw N_2, S_2, N_3, S_3, N_4, S_4, N_5, S_5, all parallel to N_1 S_1. Assume that β_1 or the angle N_1 A B, the 'bearing' of A B, is given, and that the interior angles ϕ_1, ϕ_2, ϕ_3, ϕ_4, and ϕ_5, have been measured. The 'bearings' of the successive lines can now be calculated as follows. N_2 S_2 is parallel to N_1 S_1, therefore, $\angle$ N_1 A B = $\angle$ A B S_2. Deduct this from the 'interior' $\angle$ A B C = ϕ_2, the $\angle$ S_2 B C remains. Deducting $\angle$ S_2 B C from 180°, the $\angle$ N_2 B C = β_2 or the 'bearing' of the line B C remains. Proceeding in a similar manner, the $\angle$ N_3 C D = β_3, $\angle$ N_4 D E = β_4, $\angle$ N_5 E A = β_5 or the 'whole circle bearings' of the lines B C, C D, D E and E A, may be successively computed. Finally, by using the last 'interior' angle ϕ_1, the primitive $\angle$ N_1 A B or β_1 the 'bearing' of A B, will be obtained, provided that the interior angles ϕ_1, &c., sum correctly, and that the arithmetical operations have been correctly performed. In every case, the 'whole circle bearing' is measured from the north, round in the direction of the hands of the clock, as shown by the dotted arcs β_1 β_2 &c. The 'reduced bearings' are measured *with* the hands of the clock when in the first and third, or dexter quadrants, *against* them in the second and fourth, and are marked γ_1, γ_2 &c. Through A draw A *b* perpendicular to N_1 S_1 and therefore to N_2 S_2 &c. Similarly draw perpendiculars B *c*, C *d*, D *e*, and A *a*.

Then

$$b\,B = A\,B \cos \gamma_1 = \text{'difference of latitude' between A and B.}$$

And

$$A\,b = A\,B \sin \gamma_1 = \text{'departure' of B from A.}$$



θ1.255°25′
A

Similarly,

$$c\,C = B\,C \cos \gamma_2 = \text{'difference of latitude' between B and C.}$$

And

$$B\,c = B\,C \sin \gamma_2 = \text{'departure' of C from B.}$$
$$D\,d = C\,D \cos \gamma_3 = \text{'difference of latitude' between B and C.}$$
$$C\,d = C\,D \sin \gamma_3 = \text{'departure' of D from C.}$$

And so on for the remaining points.

Now in the present case the

Diff. lat.	*b* B is a '*northing.*'	Dept.	C *d* is a 'westing.'
Dept.	A *b* is an '*easting.*'	Diff. lat.	*e* E is a 'southing.'
Diff. lat.	*c* C is a 'southing.'	Dept.	D *e* is a 'westing.'
Dept.	B *c* is an 'easting.'	Diff. lat.	E *a* is a 'northing.'
Diff. lat.	*d* D is a 'southing.'	Dept.	*a* A is a 'westing.'

Now as the polygon is a closed one, evidently by the rule already laid down,

	'Northings'	=	'southings'
	b B, E *a*	=	*c* C, *d* D, *e* E
and	Eastings	=	westings
	A *b*, B *c*	=	*c* D, D *e*, *a* A

Moreover, the accuracy of the angular measurements may be checked independently, by the preceding precept. If, therefore, the angles be correct, or, if their error be inappreciable, then, if the 'northings' and 'southings,' 'eastings' and 'westings,' do not check, error is due to inaccuracy in linear measurement.

The calculation of 'differences of latitude' and 'departures' greatly facilitates plotting, and obviates the use of the protractor, and the points can be projected with scale, T-square, and set-square. The work can also be plotted more accurately than with any protractor. Thus, the position of A and the north and south line S_1 A N_1, being fixed on the paper, the draftsman draws the line A *a b x* perpendicular to $N_1 S_1$. He lays off A *b* equal to the 'departure' of B from A. Through *b* he draws N_2 B S_2 parallel to N_1 A S_1 and lays off *b* B equal to 'difference latitude,' thus fixing the point B. If his work be correct, then the distance A B as scaled on plan, must be equal to the measured distance A B. To project the point C he lays off A *x* (easting) = A *b*, B *c* (eastings). Draws N_3 C S_3 and sets off C *x* = *b* B (northing), *c* C (southing), and so on.

Routine in measurement of Angles, and their reduction to Bearings.

From the above it is easy to see that 'bearings' may be computed from angles, but it is well to adopt a routine in the measurement of angles, and in their reduction to 'bearings,' so as to avoid the necessity for thinking out each case, and to avoid as far as possible, the process of subtraction.

Referring again to the diagram (fig. 71), it will be seen that with a theodolite graduated as usual, if the observer plants his instrument at A, and with the vernier set at zero, directs the telescope to E, and after clamping the lower plate, turns to intersect B, the angle recorded will be the '*exterior*' angle of the polygon. Similarly, if he sets up the instrument at B, and with the index at zero, directs the telescope at A, and then turns to C, he will also record an '*exterior*' angle, and so on at the other station-points.

If, on the other hand, he placed his instrument at A, and with the index at zero, pointed the telescope to B, and then turned it to E, he would record the '*interior*' angle of the polygon. Similarly at E, F, and other points.

Hence it is apparent that if the telescope is directed with index at zero, to the *back* station first, and then turned to the *forward* station, '*exterior*' angles will be measured, when proceeding round the polygon *with the sun* or the *hands of the clock*, and '*interior*' when proceeding *against the sun* or the *hands of the clock.*

If the student will work out the 'bearings' from angles in any polygon, in different ways, he will find that addition or subtraction will be involved, according to the direction in which the computation is made, and according as 'exterior' or 'interior' angles are used. It is expedient to avoid, as far as possible, *subtraction* in all calculations, but especially when dealing with angular measure. He will also find that by adhering to the following rules, subtraction of minutes and seconds, at least, may be avoided, and the desired result will be attained, irrespective of the fact that the angles measured are 'exterior' or 'interior' angles.

With the index set at zero, direct the telescope to the back station first, then turn it to the forward station.

Tabulate the angles, in the order in which they are observed.

Then compute 'bearings' as follows.

To the known bearing, add the angle between the side to which it refers, and the next side.

If the sum be less than 180°, *add* 180° *to it.*

If the sum be more than 180°, *deduct* 180° *from it, and if the difference be still more than* 360°, *deduct* 360° *from it.*

The result will be the bearing of the new side.

Thus, referring to diagram. If the surveyor worked in the direction A, B, C, &c., that is, with the sun, he would record '*exterior*' angles, θ_1, θ_2, θ_3, &c. Assuming β_1 the 'bearing' of A B known. Then as $N_1 S_1$ is parallel to $N_2 S_2$, the $\angle N_1 A B = \angle S_2 B A$. Adding the 'exterior' angle θ_2, we have the angle S (A N_2) C. Deducting 180° we have N_2 B C the desired 'bearing.'

If he proceeded in the direction A, E, D, &c., or against the sun, he would record 'interior' angles, but the method of calculation would be precisely the same.

Every line has two 'bearings,' according to the extremity at which the observer s supposed to be stationed. Thus, in the diagram (fig. 71), the bearing of B from A is N_1 A B_1, but the bearing of A from B is the angle N_2 B A measured round in the direction of the hands of the clock, or 180° + N_1 A B, which may be called the '*back bearing*' of the line A B. Hence if the polygon were observed and computed in the direction A, E, D, &c., or against the sun, then the 'bearing' of A from B, or 180° + N_1 A B, must be taken as the 'primitive bearing.'

Measuring Bearings directly in the Field.

'Bearings' may be measured directly in the field. Suppose that the surveyor were to place the theodolite at A (fig. 72). The direction of the 'prime meridian' A N_1 is known either by astronomical observation, or by the known 'bearing' of A from some distant object, and consequently the known 'back bearing' of that object from A.

Now with the horizontal limb and vernier plate clamped to zero, bring the telescope in the plane of the meridian, with the object-glass to the north. Then clamp the horizontal limb, and unclamp the vernier plate directing the telescope to the point B, and bisecting it with the cross hairs. The 'bearing' of the line A B will now be read off by the vernier (originally at zero).

Keeping the vernier plate and horizontal limb clamped together, but loosening the lower clamp, let the instrument be removed and set up at B. Then without changing the position of the vernier let the telescope be directed to A, and the lower clamp made fast.

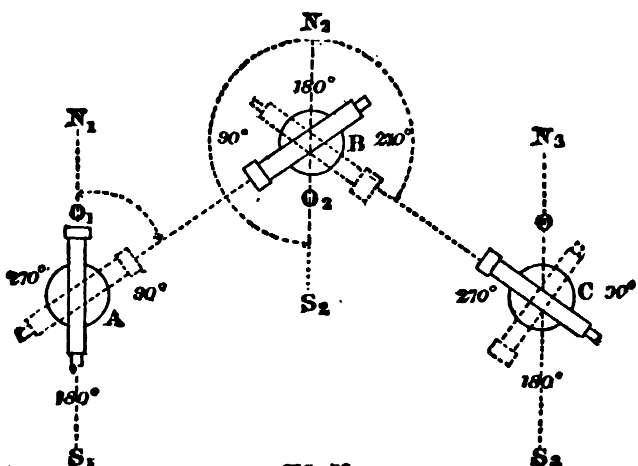


FIG. 72.

Now since $\angle$ O_1 A B is = $\angle$ O_2 B A, the zero line of the instrument S_2 N_2 is parallel to N_1 S_1, so that releasing the vernier plate, and turning the telescope till the vernier again reads zero, the axis of the telescope will be parallel to the original meridian N_1 S_1, but the telescope will now point to the south. If it be further turned, until the cross wires bisect the point C, the bearing of C from B augmented by 180° will be read off.

Proceeding in the same way at the point C, the object-glass of the telescope, when the vernier is at zero, will point to the north again, and the 'true bearing' of C D will be read off. Thus the 'meridian' is carried forward mechanically, the 'bearings' being, at alternate stations, the '*true bearing*' and the '*bearing augmented by* 180°.' When the starting point A is reached, and the instrument set up at it, then, on setting the index to zero, the telescope should be again in the plane of the meridian, the object-glass pointing either north or south, according as the number of sides is odd or even. On turning the telescope to B, and intersecting the point the original bearing of A B or the bearing A B ± 180° should be again observed.

Hitherto, the problem has been discussed on the supposition that the theodolite has but one index or vernier. Most theodolites have two, placed at 180° apart, and marked A and B, some have three spaced 120° apart. With the two-vernier theodolite, therefore, the surveyor has only to read the degrees, alternately, on A and B vernier. A glance at the compass-needle, indeed a little common sense, will leave no doubt as to the proper angle to be recorded.

If a three-vernier theodolite be used, greater care is necessary. The *same* vernier must *always* be used for reading the degrees, and 180° must be added or subtracted as the case requires. If, by accident one of the other verniers is used, then the bearing will be in error by 60°, or 120°. This great error will not be detected on closing, since the *reading only* is incorrect and the *actual angle* between the axis of the telescope and the 360° – 180° line on the limb, is mechanically, and correctly, fixed. When, therefore, the instrument is set up at the following station, it will be correctly oriented in the meridian, and provided that the proper vernier be now used, the next 'bearing' will be correctly recorded. The recorded 'bearings' will give no clue to the error, which will only be discovered after computation or plotting. Even after it is discovered, the identification of the erroneous 'bearing' is not easy, the best plan being to take the 'bearings' again with the compass. When a three-vernier theodolite is used, the direct measurement of 'bearings' is dangerous, in the hands of one not thoroughly experienced.

Relative Merits of 'Direct-Bearing Method,' and computation from 'Included Angles.'

The relative merits of 'direct bearing' measurement, and of computation of 'bearings' from 'included angles,' may be summarised thus.

1. The effect of '*defective centering*,' and '*incorrect bisection of object*,' is common to both. In one case it is detected by summation, in the other, by the difference between the 'primitive bearing' and 'the second determination of the bearing' of the same line, after completing the polygon.

2. The 'direct-bearing method,' eliminates *graduation* and *reading-errors* to a great extent. The parallelism of the successive 'meridians,' is secured mechanically, so that an error in reading a 'bearing,' only affects the side to which it refers, and is not carried forward to the other sides.

3. When 'included angles' are measured, *all errors* are carried forward, so that the *final closing error* in summation, includes *all errors*.

From the above it is evident that the *closing error* in the 'direct method' may be anticipated to be less, than in the case of 'included angles.' The fact that an error in reading one bearing does not affect the closure and so disclose itself is, however, a defect of the 'direct measurement' system. Let us suppose that an error of 30 mins., or 1°, or even more, has been made in reading off a 'bearing.' A prudent surveyor would of course verify the reading of the verniers, when he has set up at a new station, and bisected the back station, to see that the plates have not slipped, or that he has not moved them by accidentally touching the wrong tangent screw, but his mind, being more occupied with the minutes and seconds than with the degrees, it is quite possible (especially when tired after a long day's work) that he might overlook an error of 30 mins., or a whole degree or more, whilst the odd minutes and seconds, were correctly entered. This error will not be detected in closing the angular work, but will affect the result of the final calculation of ordinates. Now if the side which the error affects is short, the effect on the summation of the 'northings,' 'southings,' 'eastings,' and 'westings,' may be within the limits of permissible errors, and will therefore be distributed over the several points, and otherwise good work, will be more or less vitiated, on account of an error in recording the angular measurement of one side only.

If the side affected be long, then there will be a large *closing error*, in the summation of 'northings,' 'southings,' 'eastings' and 'westings.' As the angles closed correctly, they will not be suspected, and *the error* will probably be attributed to defective linear measurement, and time may be lost in re-measuring the lines.

This cannot take place when 'included angles' are measured. Their summation includes *all errors* in angular measurement, from whatever cause.

Direct measurement of 'bearings' gives great facility for checking the work at intermediate points, during its progress. If, when observing at A (fig. 71), the 'bearing' of A E were read, then on reaching E, the 'bearing' of A from E would be read, and should be the same as that originally observed from A + or − 180° as the case may be.

From the above it will be seen, that in the hands of a competent surveyor, the 'direct method' is convenient, and may be safely employed. If, on the other hand, as often happens in extensive surveys, the work has to be deputed to less responsible and competent persons, then the measurement of included angles is the most reliable, on account of the complete check which it affords. Finally, if a high degree of accuracy in angular measurements is required, then the 'included angle' system is to be preferred, on account of the difficulty of 'reiteration' or 'repetition' (*vide* p. 191) with the 'direct method.'

Entries in the Field-Book.

The linear measurements with their offsets should be recorded, exactly as in chain-surveying. The angles, or bearings, if measured directly, may be noted in the margin of the book against the sides to which they refer.

Tabular Form of Angles and Distances.

Points.	Observed Angles.	Bearings.	Distances.
A to B		52° 54′	6768·0
	249° 7′		
B to C		..	4949·8
	263° 38′		
C to D		..	5199·8
	236° 47′		
D to E		..	5742·0
	255° 3′		
E to A		..	4314·1
	255° 25′		
A to B		..	6768·0
	1260° 0′		

It is, however, advisable to enter in the field-book, on a separate page and in a tabular form, the angles and total length of the station-lines, so that the angles may be added up as soon as the whole are measured. The surveyor will then have all the information required for computation under his eye, without having to lose time in turning over the leaves of a field-book containing much detail distributed over a considerable space.

The preceding table is a convenient form.

Computation of 'Latitudes' and 'Departures.'

'Traverse Sheet.'

The surveyor having completed his outdoor work, next proceeds to compute the 'latitudes' and 'departures.' To this end he enters the data from his field-book, in a tabular form, called the 'traverse sheet.' Table A, shows such a form. The figures therein, refer to the ideal polygon shown in Fig. 71. It is here assumed that he has worked round the polygon, in the direction of the hands of the clock, and consequently has measured *exterior* angles.

In the first column he enters the reference letters of the several points in proper order.

In the second column, he enters the exterior angles of the polygon, proceeding round as with the hands of the clock, each entry being made opposite the letter indicating the point at which it was measured.

In the third column, he enters the bearing of point B from point A, (which is assumed to be given), inscribing it opposite the letter A, and thus indicating the point of origin of the line A B. He then computes the bearing of C from B, and enters it on the line opposite B. Proceeding in this way he ultimately computes, and enters against E, the bearing of E A, completing the polygon. Lastly as a check, he applies to the bearing of E A, the angle measured at A, and if the summation of the angles is correct, he will find that, unless an arithmetical error has occurred, the original bearing of B from A will be obtained. If, however, the summation be incorrect, he would obtain a value of the bearing of A B, differing from the original, or given value of the same, by an amount equal to the error in summation.

Computation of 'Reduced Bearings.'

The 'whole circle bearings' being correctly calculated, the 'reduced bearings' are computed by the rules above given, and each bearing, whether 'whole circle' or 'reduced,' is entered opposite to the letter indicating the starting point of the line to which it refers. Thus the bearing A B is entered opposite A, of B C opposite B, &c. The points of the compass are entered in column five, the 'distances' in column six, each opposite the starting point of the line, thus the distance A B is inscribed opposite A, and so on, and lastly the sides A B, B C, &c., in column seven.

The traverse sheet is now 'set up.' The next step is to compute the 'latitudes' and 'departures,' by multiplying the natural cosine of the 'reduced bearing,' by the distance, for 'latitude,' and the natural sine by the same, for 'departure.' The products are then entered in the columns provided for the purpose. Each 'latitude' or 'departure' is entered against the distance to which it refers. The 'difference of latitude' and 'departure' from A to B come opposite the letter A.

TABLE A, referring to Fig. 71.

Points. 1	Exterior Angles. $\theta_1 \theta_2 \theta_3 \theta_4 \theta_5$. 2 °	′	Whole Circle Bearings. $\beta^1 \beta^2 \beta^3 \beta^4 \beta^5$ 3 °	′	Reduced Bearings. $\gamma^1 \gamma^2 \gamma^3 \gamma^4 \gamma^5$. 4 °	′	Quadrant. 5	Distances, or Lengths of Sides. 6	Sides. 7	Differences of Latitude. N	Line on Plan.	S	Line on Plan.	Departures. E Meridional.	Line on Plan.	W	Line on Plan.	Co-ordinates. N Meridional Differences.	E Perpendiculars.	Points.
A	255	25	52	54	52	54	N.E.	6768·0	A B	4082·5	*b* B	..	..	5398·1	A *b*	..	..	5000·0	400·0	A
B	249	07	122	01	57	59	S.E.	4949·8	B C	..	..	2624·2	*c* C	4196·9	B *c*	..	..	9082·5	5798·1	B
C	263	38	205	39	25	39	S.W.	5199·8	C D	..	..	4687·4	D *d*	..	..	2250·9	C *d*	6458·3	9095·0	C
D	236	47	262	26	82	26	S.W.	5742·0	D E	..	..	0756·1	*e* E	..	..	5692·0	D *e*	1770·9	7744·1	D
E	255	03	337	29	22	31	N.W.	4314·1	E A	3985·2	E *a*	..	..	..	..	1652·1	*a* A	1014·8	2052·1	E
	1260	00								8067·7	..	8067·7	..	9595·0	..	9595·0	..	..	..	
Less	−360	00																		
	900	00																		

TABLE B, referring to Fig. 71.

Points. 1	Interior Angles. 2 °	′	Whole Circle Bearings. 3 °	′	Quadrant Bearings. 4 °	′	Quadrant. 5	Distances, or Lengths of Sides. 6	Sides. 7	Differences of Latitude. N	Line on Plan.	S	Line on Plan.	Departures. E	Line on Plan.	W	Line on Plan.	Co-ordinates. N Meridional Differences.	E Perpendiculars.	Points.
B	110	53	..	..	52	54	S.W.	6768·0	B A	..	..	4082·5	B *b*	..	..	3398·1	*b* A	0917·5	4201·9	B
A	104	35	157	29	22	31	S.E.	4314·1	A E	..	..	3985·2	*a* E	1652·1	A *a*	..	..	5000·0	9600·0	A
E	104	57	82	26	82	26	N.E.	5742·0	E D	0756·1	E *e*	..	..	5692·0	*e* D	..	..	8985·2	7947·9	E
D	123	13	25	39	25	39	N.E.	5199·8	D C	4687·4	D *d*	..	..	2250·9	*d* C	..	..	8229·1	2255·9	D
C	96	22	302	01	57	59	N.W.	4949·8	C B	2624·2	C *c*	..	..	..	..	4196·9	*c* B	3441·7	0005·0	C
	540	00	..	..	..	..	..	..	..	8067·7	..	8067·7	..	9595·0	...	9595·0	..	..	..	

Finally, the 'northings,' 'southings,' 'eastings,' and 'westings,' are added, and as the polygon in question is an ideally perfect one, the sum of the 'northings' is exactly equal to the sum of the 'southings,' that of the 'eastings' to that of the 'westings,' but this absolute accuracy in summation cannot be expected in practice. The 'angles' of the polygon shown in fig. 71 and the 'bearings,' &c., entered in Tables A and B, were obtained by calculation, from assumed positions of the points.

If a table of natural sines be used, the process known as 'lightening' or 'reduced' multiplication will be found quick and convenient, as the number of figures is thereby greatly lessened.

The 'latitudes' and 'departures' may also be computed by logarithms. Unless, however, the sides are very long and the highest accuracy is required, little or no time is saved by their use. An example of a convenient form of logarithmic computation will be given in the chapter on 'Minor Triangulation.'

Traverse Tables. 'Traverse tables' are simply tables of natural sines and cosines, multiplied by numbers from 1 to 10, as in Boileau's tables, or from 1 to 100 as in those by Shortrede. The computation is thus reduced to addition. Since full directions for use are given in the introductions to these tables, it is unnecessary to describe them here.

Table B, refers to the same polygon, but assumes that the surveyor started from B and proceeded round the polygon, against the hands of the clock, measuring therefore '*interior*' angles. It will be seen that the 'whole circle bearings' in Table B differ by 180° from those in Table A, as calculated by general precept given above. In other words, the 'fore-bearings' in one, are the 'back-bearings' of the other. The *angles* of the 'reduced bearings' in both are identical, but the cardinal letters are inverted, N becoming S, E becoming W, and *vice versâ* The numerical values of the 'latitudes' and 'departures' are the same, but their names are reversed. The summation is of course the same in both instances.

Computation of Co-ordinates. Having computed the 'latitudes' and 'departures,' it will facilitate plotting to proceed a step further, and compute the 'co-ordinates' of the several points, from a common point of 'origin.' In other words, the 'meridional differences and perpendiculars' as already defined.

In Table A, it is assumed that the point A is known and fixed. We may therefore compute the distances *north* of B, C, D, and E, or *south*, *east*, or *west*, of A.

It is, however, convenient to assume a point of 'origin,' such that all the points of the polygon fall in one quadrant, and, therefore, that the co-ordinates are either all 'northings' or 'southings,' 'eastings' or 'westings,' as the case may be.

In fig. 71 and Table A, it has consequently been assumed that the point of 'origin' O is 5000 feet south, and 400 feet west of A. In other words, that the 'meridional distance' of A is 5000 feet '*north*' and its 'perpendicular,' 400 feet '*east*.' These numbers are inscribed opposite A in the column of co-ordinates. The 'meridional distance' of B is obtained by *adding* the 'northing' from A to B (which is inscribed against A), to the 'meridional distance' of A. The sum is inscribed in the line opposite B. The 'meridional distance' of C is obtained by *deducting* the 'southing' C to B, from the co-ordinate of B, already found, and so on, adding or deducting as the case may be, until the 'meridional distance' of the last point C is determined and inscribed opposite to that letter. Applying to this the 'difference of latitude' C to A (also inscribed against C), the original 'meridional distance' of A will be found, if the arithmetic be correct.

If the summation of the 'northings,' 'southings,' 'eastings,' and 'westings,' be not correct, the second value of the co-ordinates of the starting point, would differ from the first, by the error in summation. Similarly, for the perpendiculars.

In Table B, the co-ordinates of the same points are all computed as 'southings' and 'westings.' In this case, the origin P (fig. 71) is supposed to be 4201·9 east, and 917·5 north of B. Or in other words, the 'meridional distance' of B is 917·5 south, and its 'perpendicular' 4201·9 west.

Plotting of Co-ordinates.

The operation of plotting a polygon, once the co-ordinates of the points in it have been computed, is simple. For example, to plot the polygon shown in fig. 71. Assuming any convenient position for O. Draw a line through it to represent the meridian, or a line parallel to the same. Along this line set off O *a*, the 'meridional distance' of A. Through the point '*a*' draw a line at right angles to the meridian, and make it equal to the 'perpendicular' of A. The point A will thus be plotted. In the same manner for the other points. The check on the accuracy of the plotting is that the distances between the points on the paper, as plotted, should agree very nearly with the actual measured distances as recorded on the traverse sheet.

The expression 'very nearly,' is used advisedly. Hitherto we have been considering an absolutely perfect polygon, in which the error is 'nil.' In practice, the 'latitudes' and 'departures' will rarely sum correctly. They must be corrected before plotting, indeed before the computation of the co-ordinates. If incorrect, the primitive starting point would not be obtained as described. Now the correction of the 'latitudes' and 'departures' alters the length of the line joining the points to which they refer. Consequently, in a *corrected polygon*, the distances between the points on the paper will not coincide mathematically with the distances measured in the field. If the work be good, however, the difference in the length of any one side should be so small as to be inappreciable with scale and compass. In practice, the check is complete.

Division of paper into squares.

If the points to be plotted are numerous, it will be convenient to commence by dividing the sheet of paper, on which the survey is to be plotted, into squares. The sides of these squares should represent to the scale of the plan, a round number of feet, links, or such other unit, as may be used. They should not exceed 6 inches on the side, so that each point may be plotted, within the appropriate square, by means of a scale of convenient length, or within the span of a pair of ordinary dividers. For example, if the survey were to be plotted to the scale of $\frac{1}{2500}$, the squares might be 0·2 feet, each representing 500 feet.

These squares should be set off with all possible accuracy, not trusting to any set square, parallel ruler, or T-square, but using only the beam compass or trammel, and a good straight-edge. The right angle should be set out by intersecting arcs. The number of squares in length and breadth should, if practicable, be a power of 2, 4, 8, 16, 32, &c. By setting off the full length, from a standard scale, the intermediate points may be obtained by continual bisection (the most rapid and accurate method of subdivision).

The squares should be drawn at first not in pencil, but in *faint* carmine red lines, the pen being more accurate than the pencil. They should be carefully checked by stepping, and also by diagonals. The diagonal to the sides of three and four squares, should be exactly equal to five divisions. When the squares are drawn correctly they may be inked in with Indian ink, any erroneous red lines being easily effaced either with the ink-eraser, or with a wash of 'chloride of lime.' Finally, the points may be plotted by means of a short scale, each within the square in which it occurs.

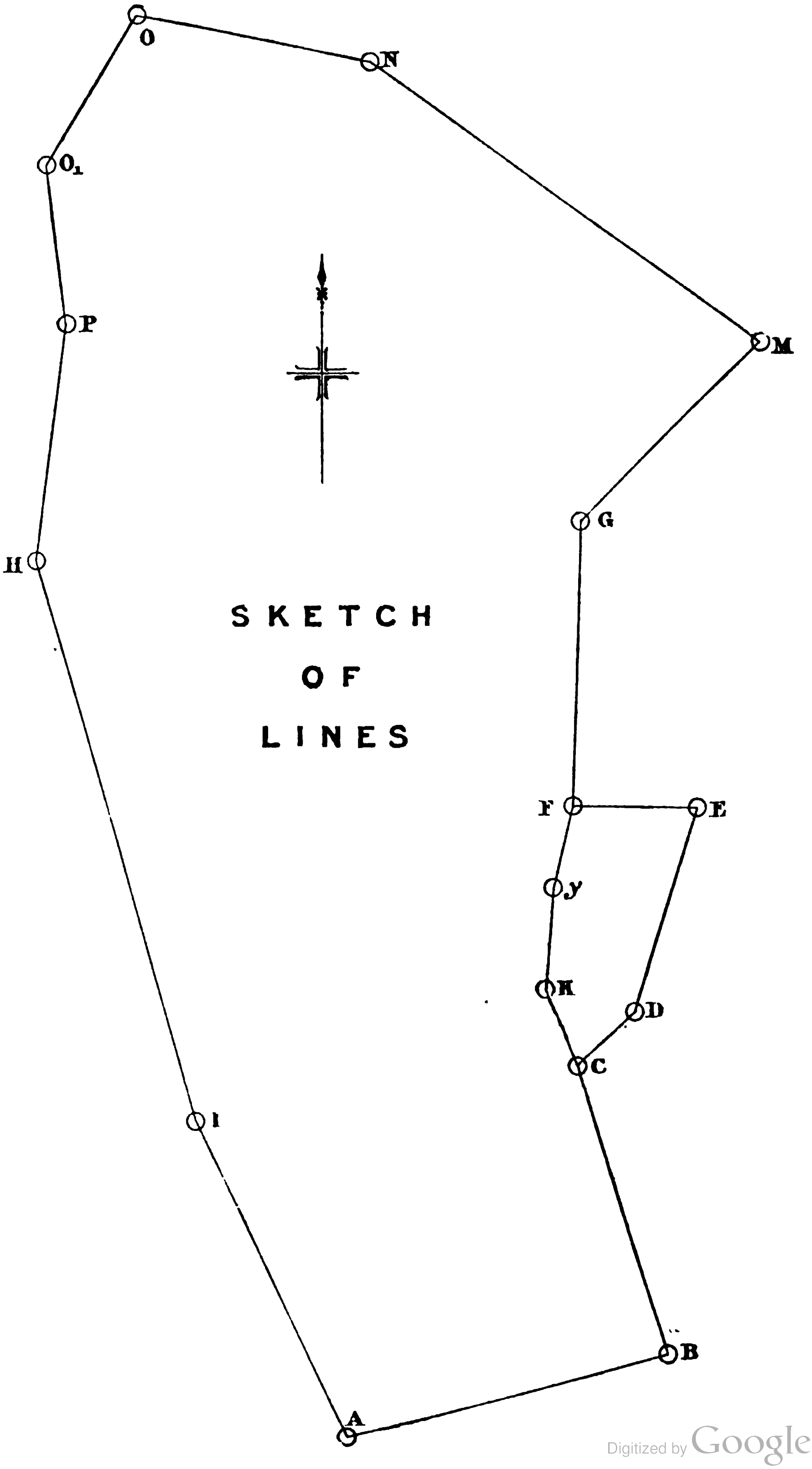
An example of an actual traverse survey, with its traverse sheets, is here given (*vide* plate 2), and the 'field-book' for it on pages 115 to 138. The student is recommended to plot this example himself, taking a new 'assumed bearing,' say 80°, for the line H I and recomputing the co-ordinates. (For the corrections, *vide* 'Adjustment of Final Errors,' p. 151.)

FIELD-BOOK

FOR

TRAVERSE SURVEY.

FIELD-BOOK OF TRAVERSE SURVEY.



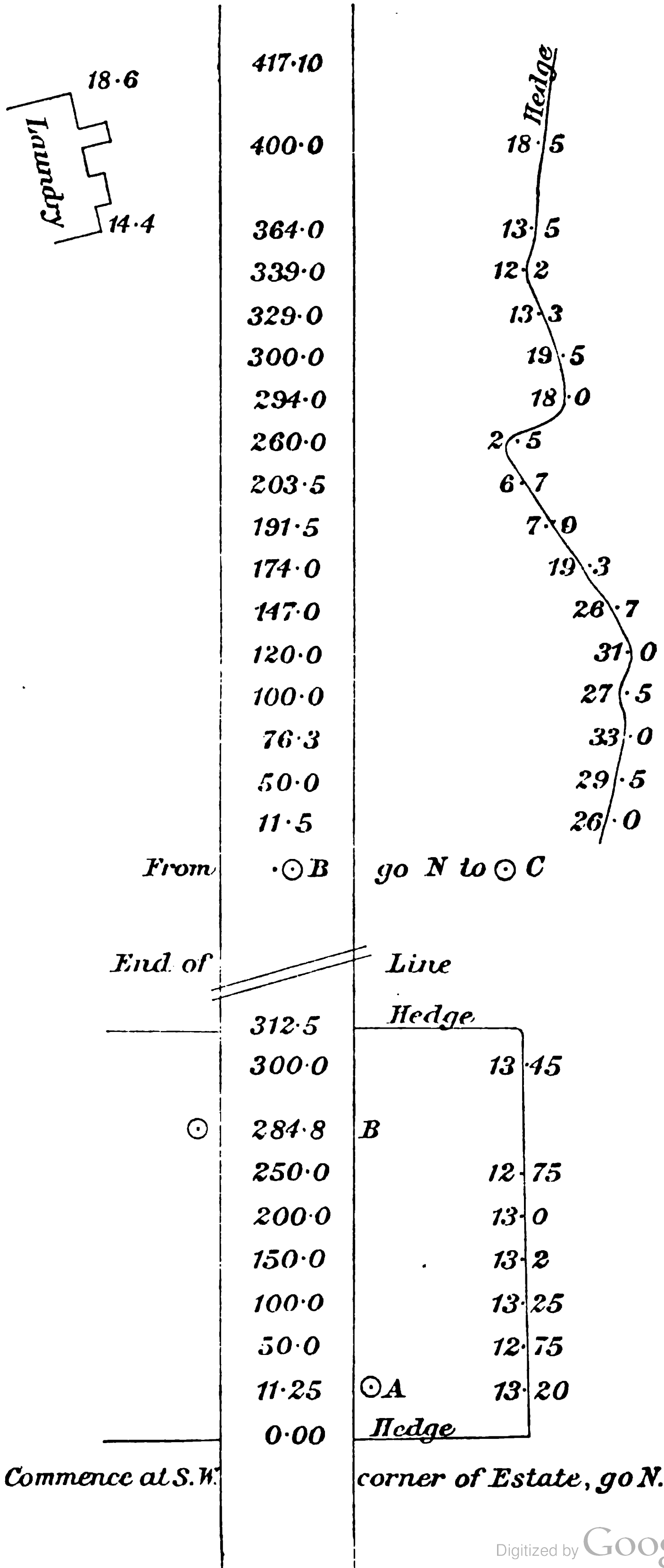
ANGLES AND DISTANCES

MAIN TRAVERSE

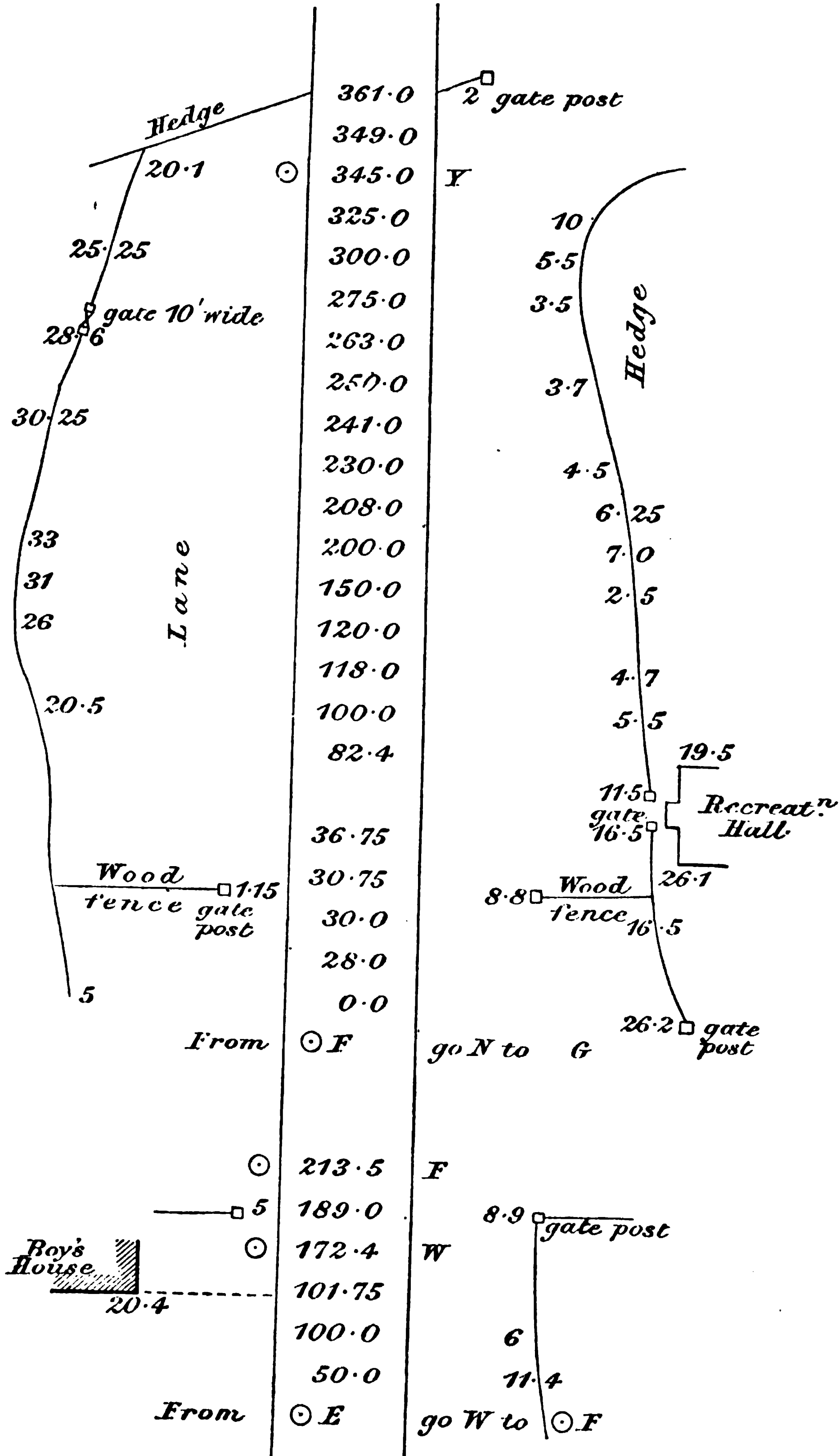
POINT	OBSERVED ANGLE	WHOLE CIRCLE BEARING	DISTANCE
H-I		164°·27′	
I	172°·24′		617·8
A	85°·12′		273·5
B	99°·49′		711·8
C	239°·18′		87·7
D	149°·53′		433·0
E	81°·55′		213·5
F	271°·39′		593·0
G	225°·50′		419·7
M	81°·39′		604·3
N	152°·31′		374·5
O	110°·23′		196·0
O_1	140°·36′		219·7
P	184°· 1′		491·0
H	164°·45′		1274·0
I	1259°·55′		

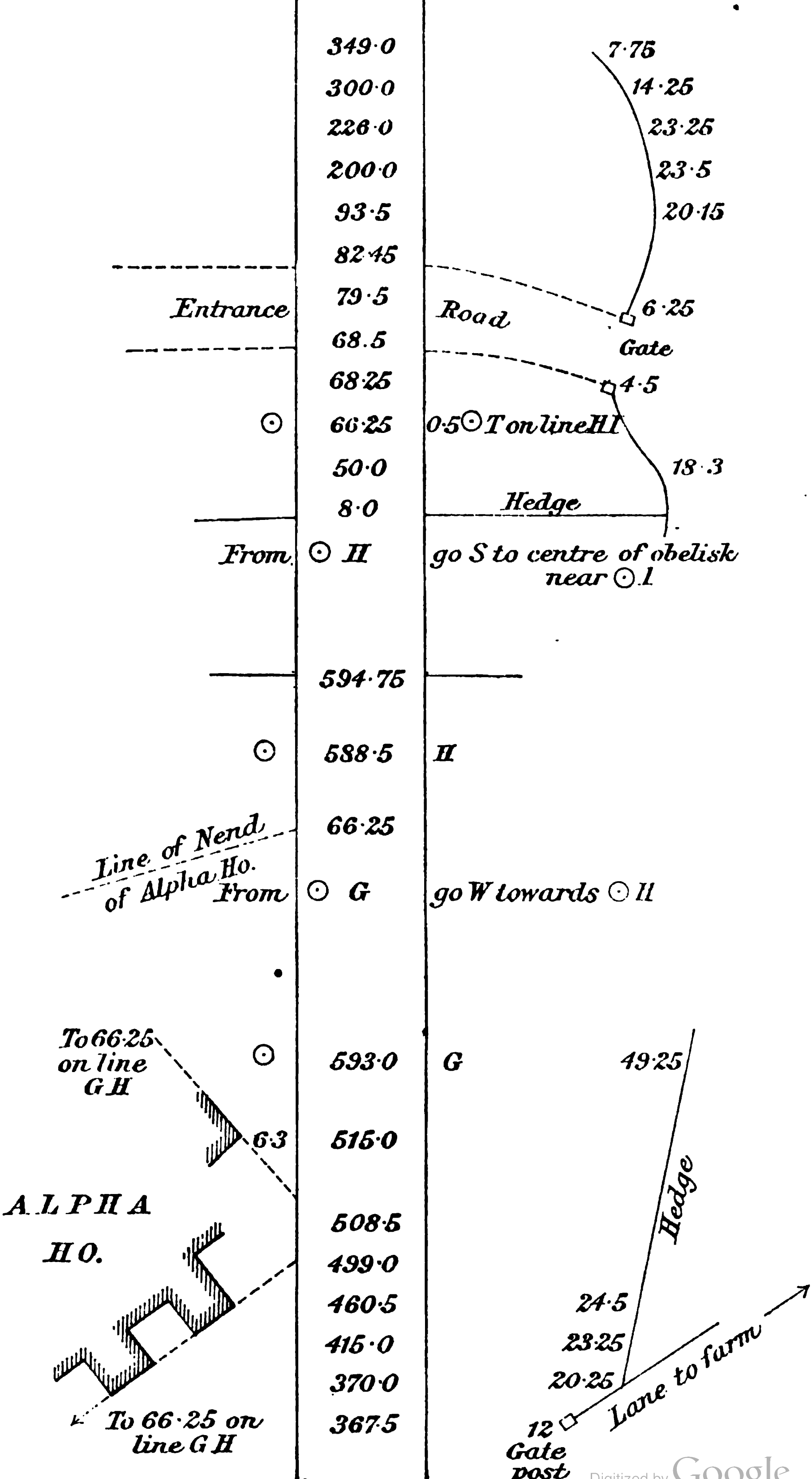
SUBSIDIARY TRAVERSE

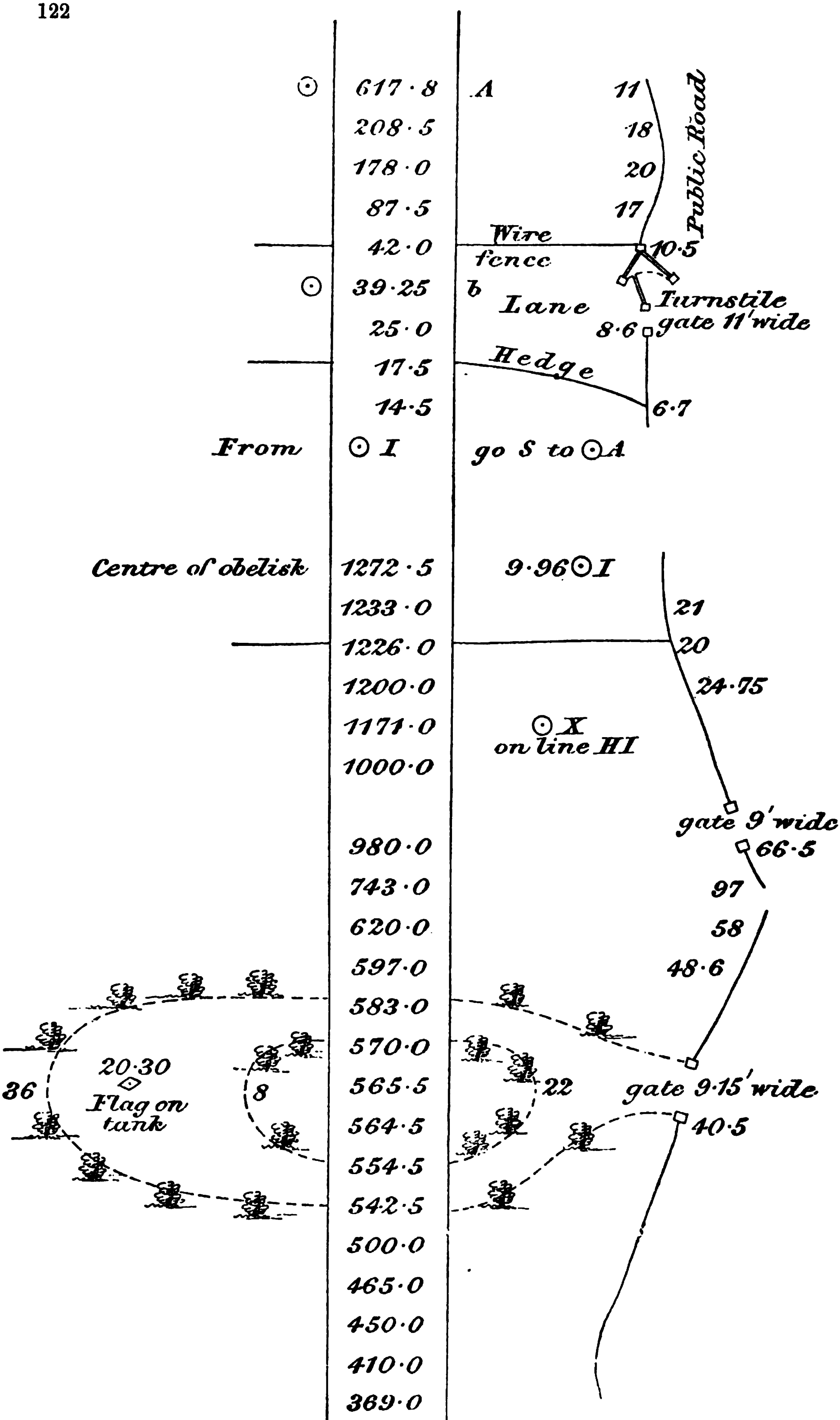
POINT	OBSERVED ANGLE	WHOLE CIRCLE BEARING	DISTANCE
BC			
C	165°·39′		114·0
K	208°·42′		279·7
y	187°· 7′		126·25
F	181°·19		
G			

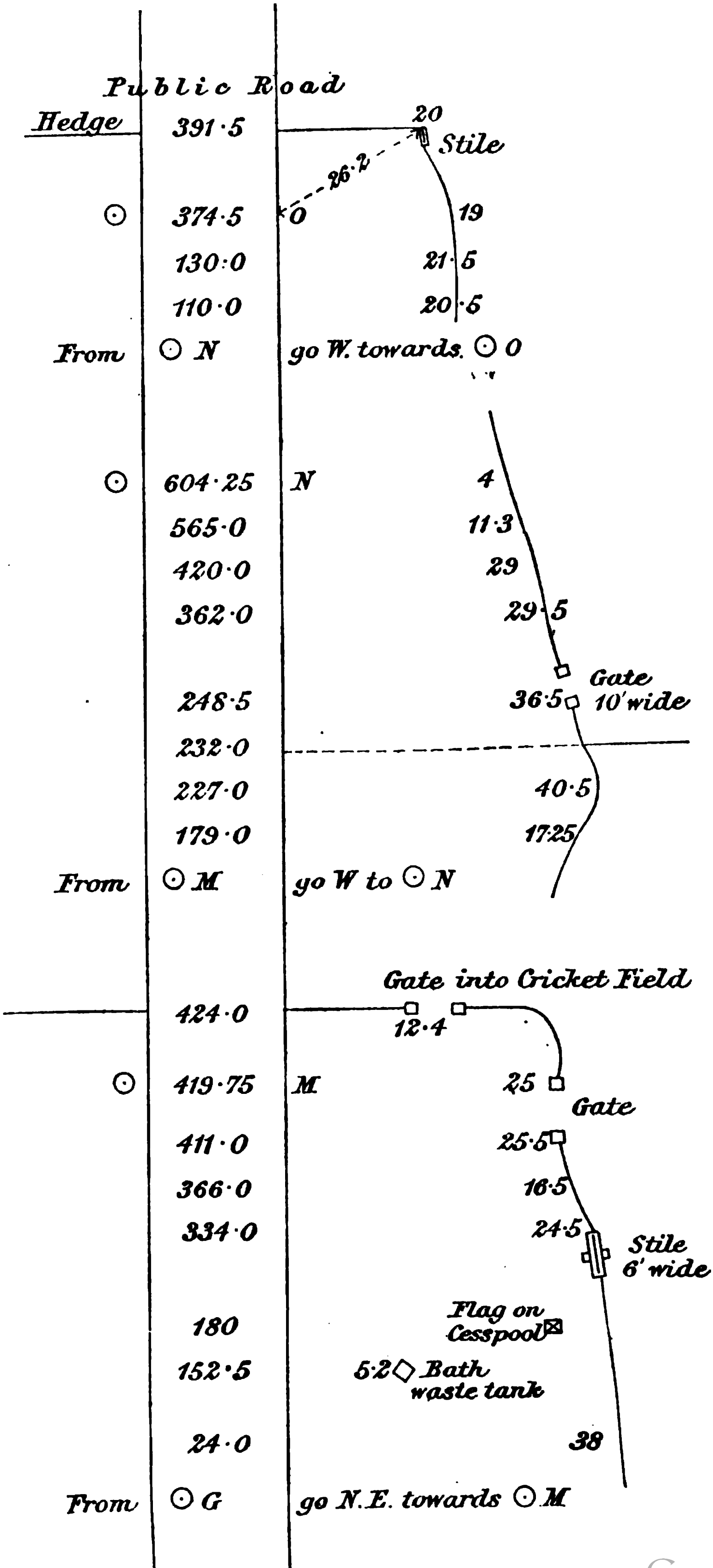

417·10
18·6
Hedge
Laundry
400·0
18·5
14·4
364·0
13·5
339·0
12·2
329·0
13·3
300·0
19·5
294·0
18·0
260·0
2·5
203·5
6·7
191·5
7·0
174·0
19·3
147·0
26·7
120·0
31·0
100·0
27·5
76·3
33·0
50·0
29·5
11·5
26·0
From ·⊙B go N to ⊙ C
End of Line
Hedge
312·5
300·0
13·45
⊙
284·8
B
250·0
12·75
200·0
13·0
150·0
13·2
100·0
13·25
50·0
12·75
11·25
⊙A
13·20
0·00
Hedge
Commence at S.W. corner of Estate, go N.

	448·5	Hedge
⊙	433.0	E
From	⊙ D	go N towards E
⊙	87·7	D
	22·5	3/75
Hedge / Lane	17·0	·75 Old Gate Post
Old Gate Post ·5	9·0	
	4·0	
From	⊙ C	go E to D
⊙	711·75	C
18·3	700·0	18
⊙	684·75	w · Lane · Hedge
15·3	650·0	
	645·0	
18·7	600·0	Gate
	585·5	6·6
Hedge ⊙	565·5	c
	564·0	8·7
	562·5	·9 Wood Fence
Wire Fence 8·8	561·5	
Gate Post	550·0	11·9
	537·5	
To N. side of Eleanor House	500·0	
	453·0	
To S. side of Eleanor Ho.	440·7	
	425·6	
Drain	423·25	
⊙ B to ⊙ C		contd.


Hedge
361·0
2 gate post
349·0
20·1
345·0
Y
325·0
10
300·0
5·5
25·25
275·0
3·5
gate 10' wide
28·6
263·0
Hedge
250·0
3·7
30·25
241·0
230·0
4·5
208·0
6·25
33
200·0
7·0
31
Lane
150·0
2·5
26
120·0
118·0
4·7
20·5
100·0
5·5
82·4
19·5
11·5
gate
16·5
Recreatⁿ Hall
36·75
Wood fence
1·15
gate post
30·75
26·1
30·0
8·8
Wood fence
16·5
28·0
5
0·0
26·2
gate post
From
F
go N to
G
213·5
F
5
189·0
8·9
gate post
Boy's House
172·4
W
101·75
20·4
100·0
6
50·0
11·4
From
E
go W to
F


349·0
300·0
226·0
200·0
93·5
82·45
79·5
68.5
68·25
66·25
50·0
8·0
7·75
14·25
23·25
23·5
20·15
6·25
Gate
4·5
18·3
Entrance
Road
0·5 ⊙ T on line H I
Hedge
From ⊙ H go S to centre of obelisk near ⊙ I
594·75
588·5 H
66·25
Line of N end of Alpha Ho.
From ⊙ G go W towards ⊙ H
593·0 G
515·0
508·5
499·0
460·5
415·0
370·0
367·5
49·25
24·5
23·25
20·25
12
Gate post
Hedge
Lane to farm
To 66·25 on line G H
6·3
ALPHA HO.
To 66·25 on line G H

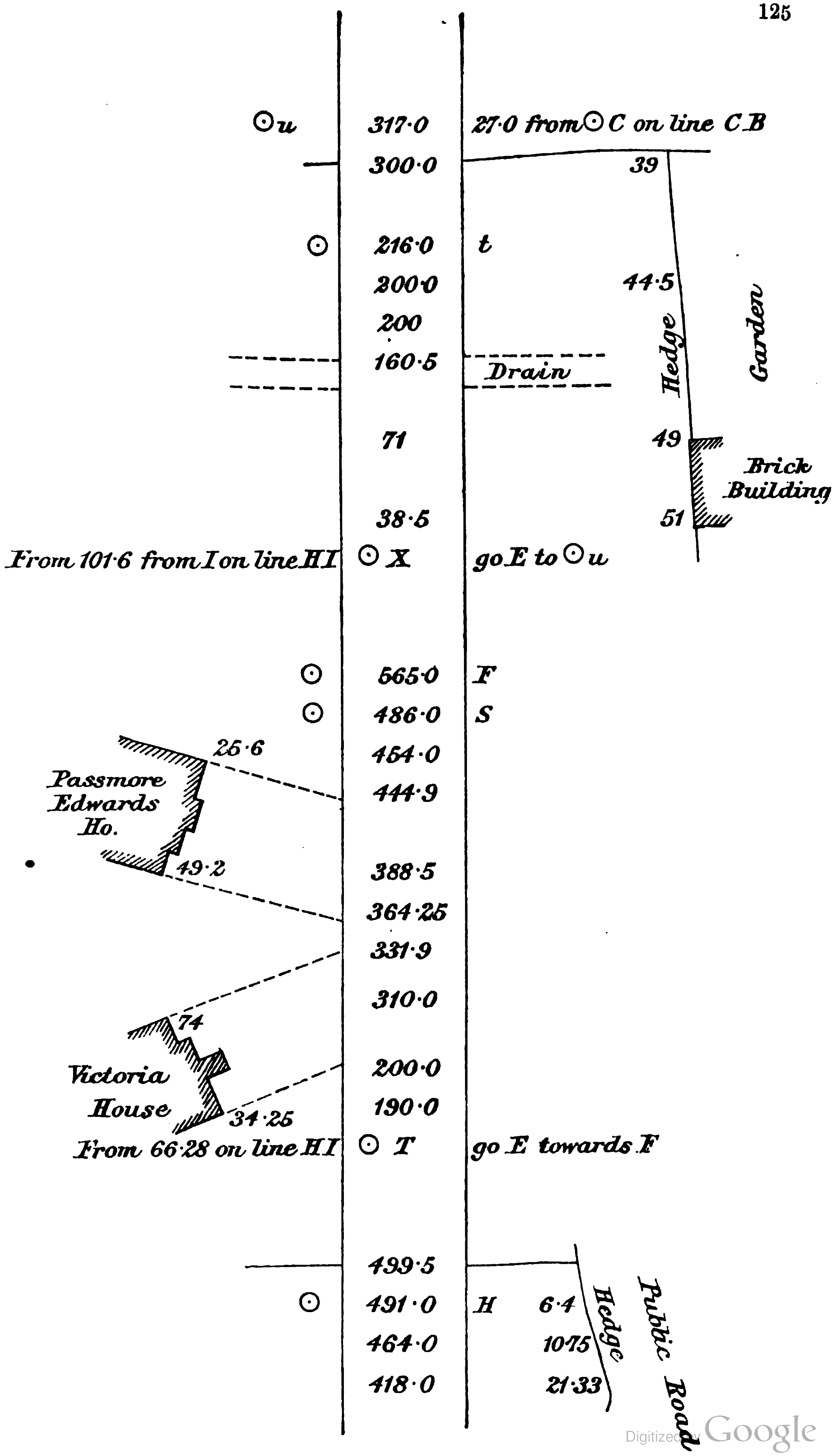

617·8
A
11
18
20
17
Public Road
208·5
178·0
87·5
42·0
Wire fence
10·5
39·25
b
Lane
Turnstile
gate 11' wide
8·6
25·0
Hedge
17·5
14·5
6·7
From ⊙ I go S to ⊙ A
Centre of obelisk
1272·5
9·96 ⊙ I
1233·0
21
1226·0
20
1200·0
24·75
1171·0
⊙ X on line HI
1000·0
gate 9' wide
980·0
66·5
743·0
97
620·0
58
597·0
48·6
583·0
570·0
565·5
22
gate 9·15' wide
20·30
Flag on tank
36
8
564·5
40·5
554·5
542·5
500·0
465·0
450·0
410·0
369·0

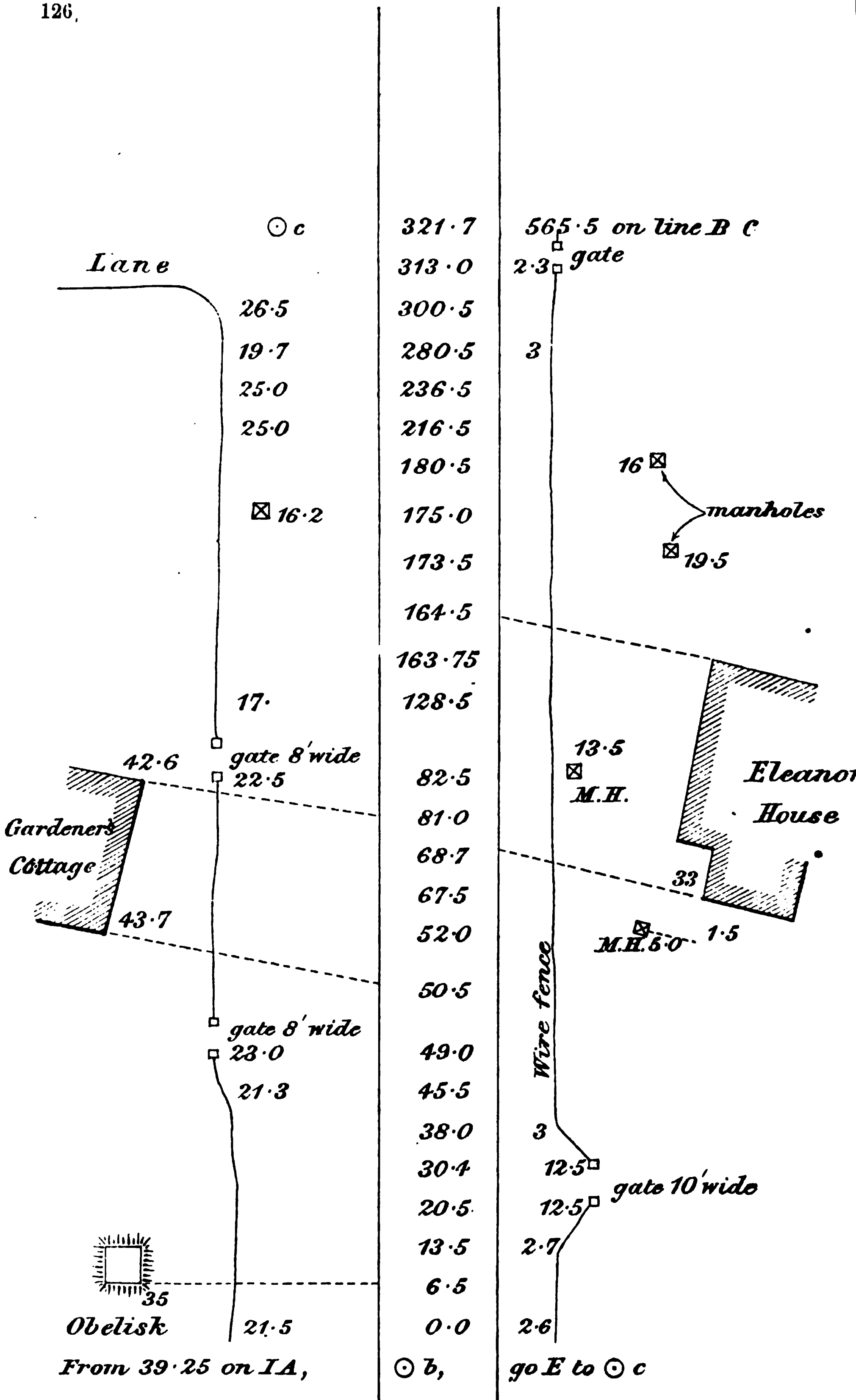

Public Road
Hedge
391·5
20
Stile
26·2
374·5
0
19
130·0
21·5
110·0
20·5
From ⊙ N go W. towards ⊙ O
604·25
N
4
565·0
11·3
420·0
29
362·0
29·5
Gate
248·5
36·5
10′ wide
232·0
227·0
40·5
179·0
17·25
From ⊙ M go W to ⊙ N
Gate into Cricket Field
424·0
12·4
419·75
M
25
Gate
411·0
25·5
366·0
16·5
334·0
24·5
Stile
6′ wide
180
Flag on
Cesspool
152·5
5·2
Bath
waste tank
24·0
38
From ⊙ G go N.E. towards ⊙ M

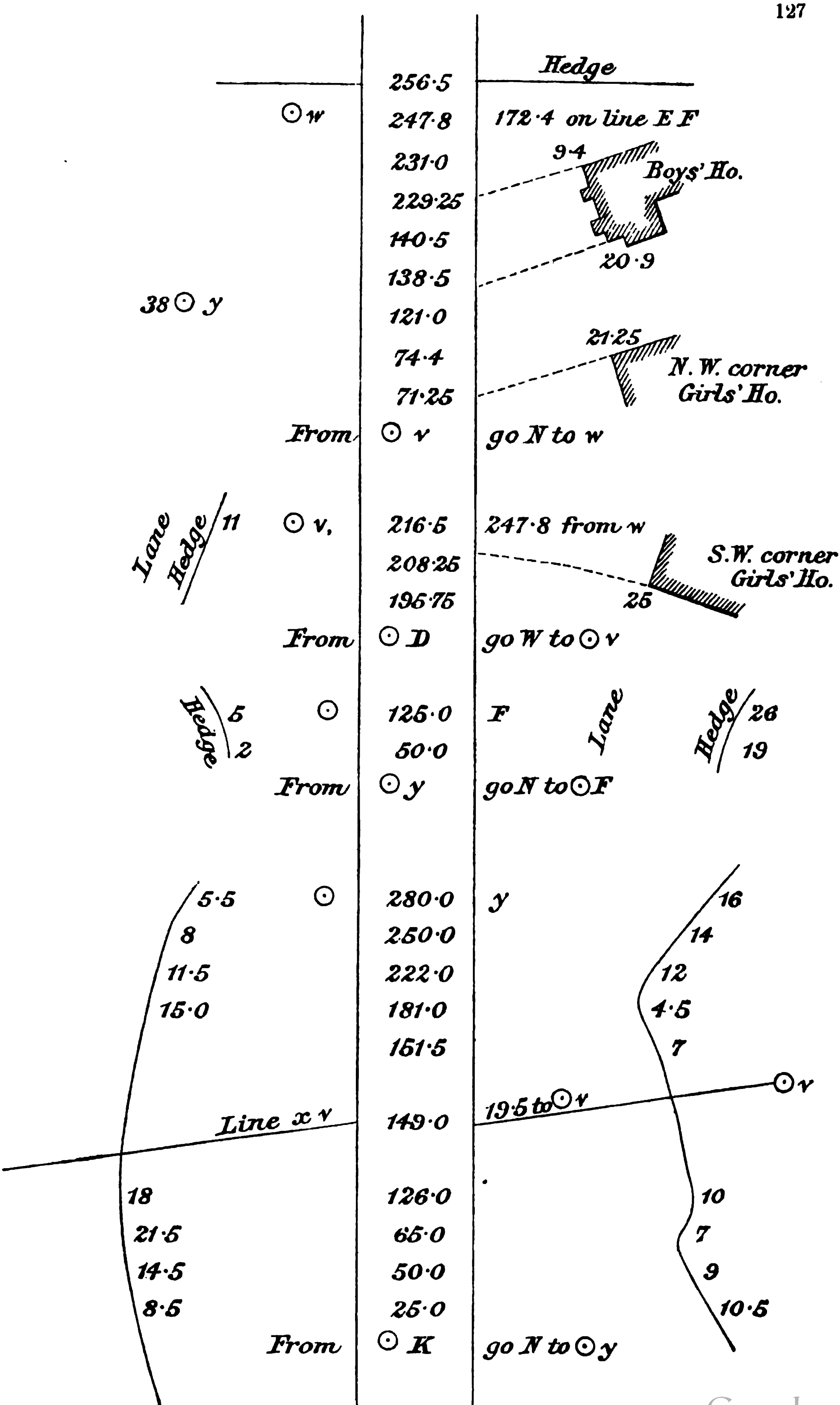
	400·0	17·75
	386·0	17·2
	292·0	29·5
	275·0	26·5
	2?8·0	18·75 Hedge
	205·0	13·15
	192·0	9·4 Public Road
	182·0	8·25 gate
	150·0	16·75
	29·0	15·25
	0·0	5·75
From	⊙ P	go S towards H

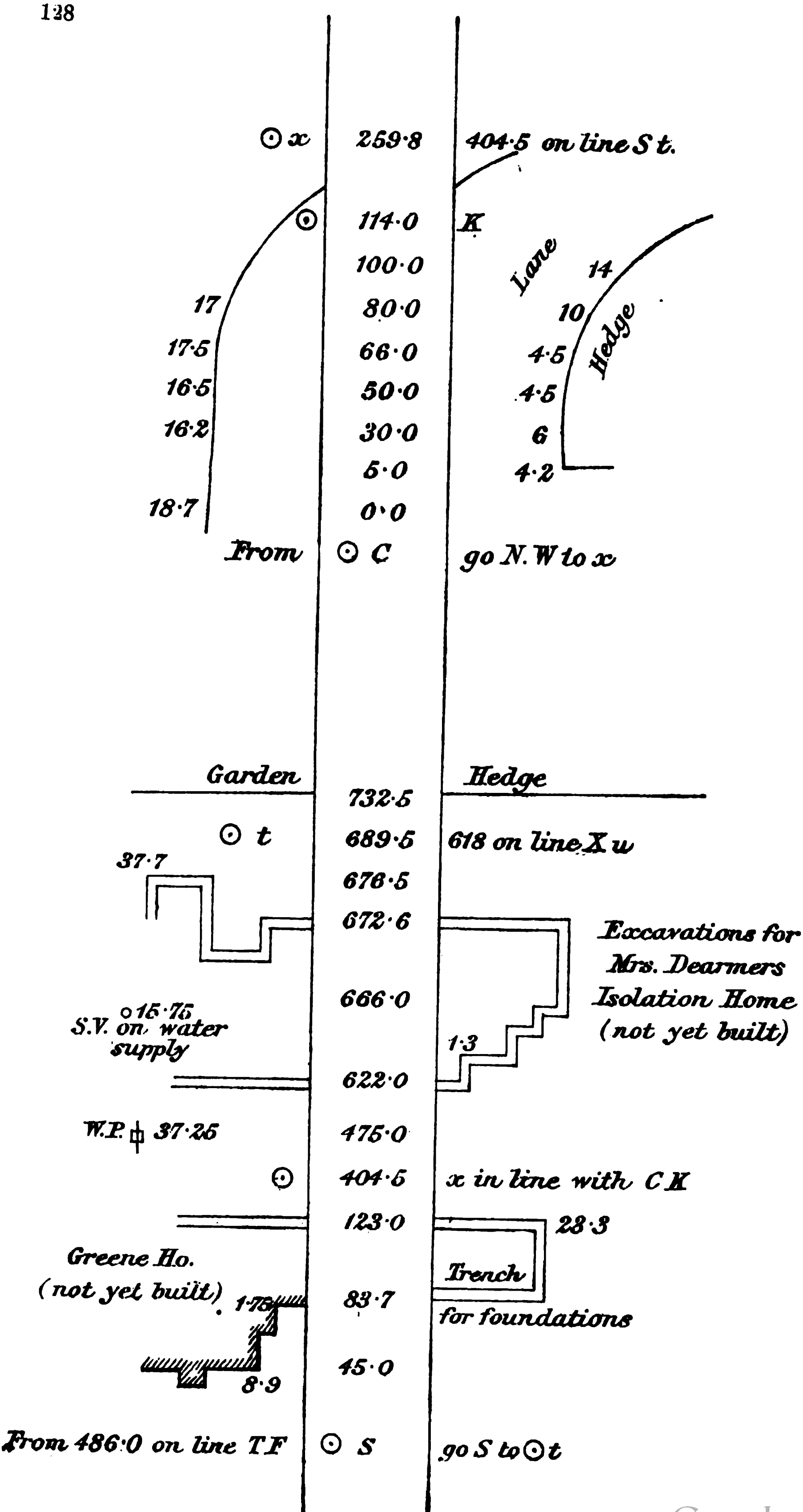
⊙	219·7	P
	216·0	4·75
	203·0	3
	171·0	3·5
	164·0	7·33 Hedge
	135·0	9 Public Road
	108·0	3·1
	50·0	8·5
	7·0	6·4
From	⊙ O_1	go S to ⊙ P

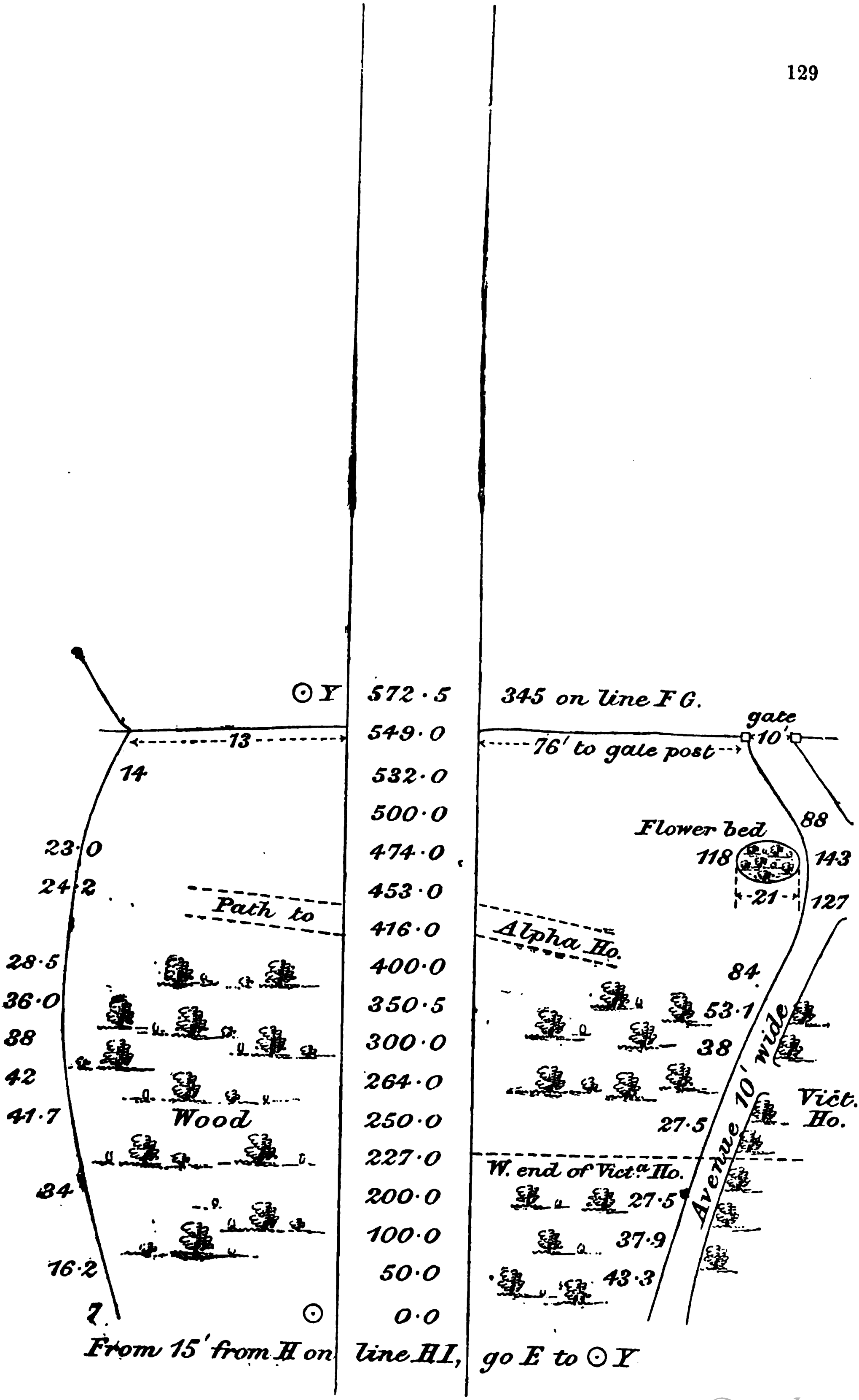
	202·8	Hedge Public Road
⊙	196·0	O_1 8·75
	113·0	20·8
	37·5	9
From	⊙ O	go S towards O_1

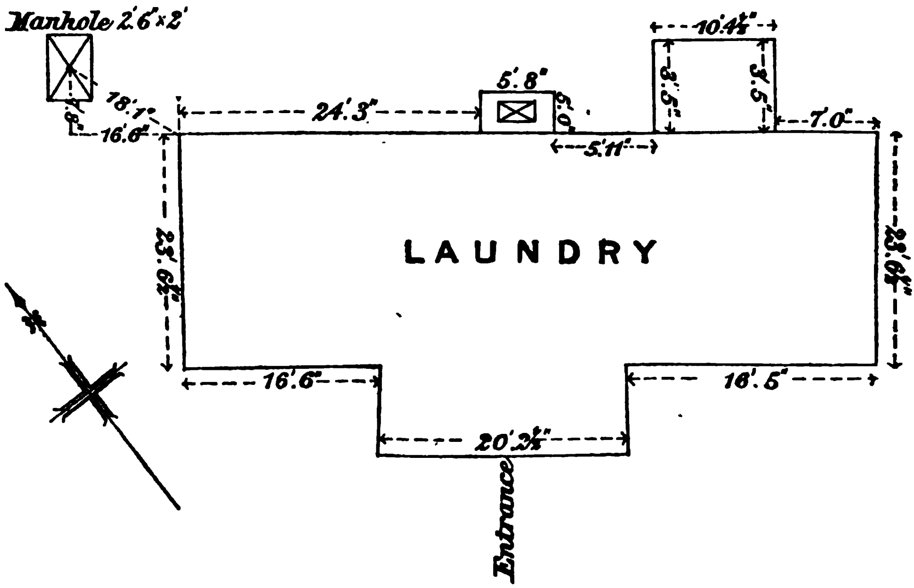

⊙ u
317·0
27·0 from ⊙ C on line CB
300·0
39
⊙
216·0
t
200·0
44·5
200
Hedge
Garden
160·5
Drain
71
49
Brick Building
38·5
51
From 101·6 from I on line HI
⊙ X
go E to ⊙ u
⊙
565·0
F
⊙
486·0
S
25·6
454·0
Passmore Edwards Ho.
444·9
49·2
388·5
364·25
331·9
310·0
74
Victoria House
200·0
190·0
34·25
From 66·28 on line HI
⊙ T
go E towards F
499·5
⊙
491·0
H
6·4
Hedge
Public Road
464·0
10·75
418·0
21·33

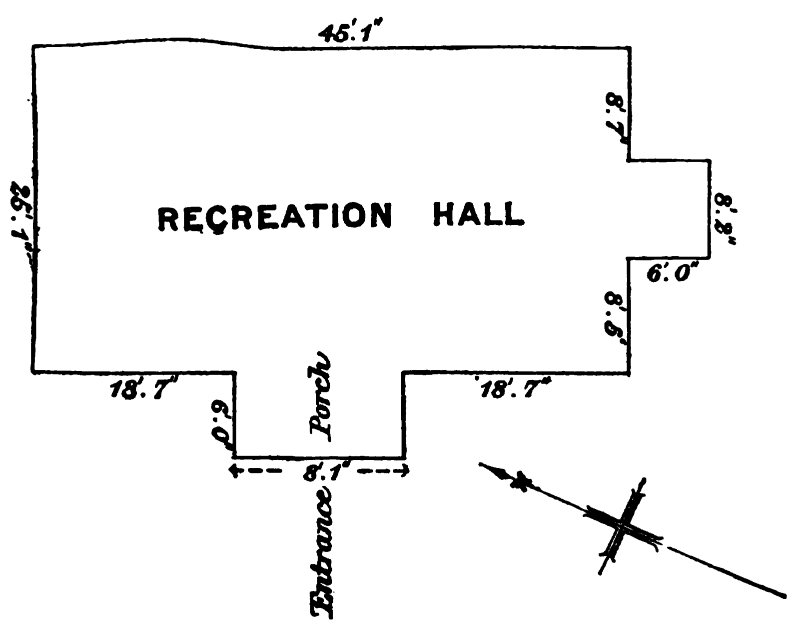

⊙ c
321·7
565·5 on line B C
Lane
313·0
2·3 gate
26·5
300·5
19·7
280·5
3
25·0
236·5
25·0
216·5
180·5
16
manholes
16·2
175·0
173·5
19·5
164·5
163·75
17·
128·5
gate 8' wide
42·6
22·5
82·5
13·5
M.H.
Eleanor House
Gardener's Cottage
81·0
68·7
67·5
33
43·7
52·0
M.H. 5·0
1·5
50·5
Wire fence
gate 8' wide
23·0
49·0
21·3
45·5
38·0
3
30·4
12·5
gate 10' wide
20·5
12·5
13·5
2·7
35
6·5
Obelisk
21·5
0·0
2·6
From 39·25 on IA, ⊙ b, go E to ⊙ c

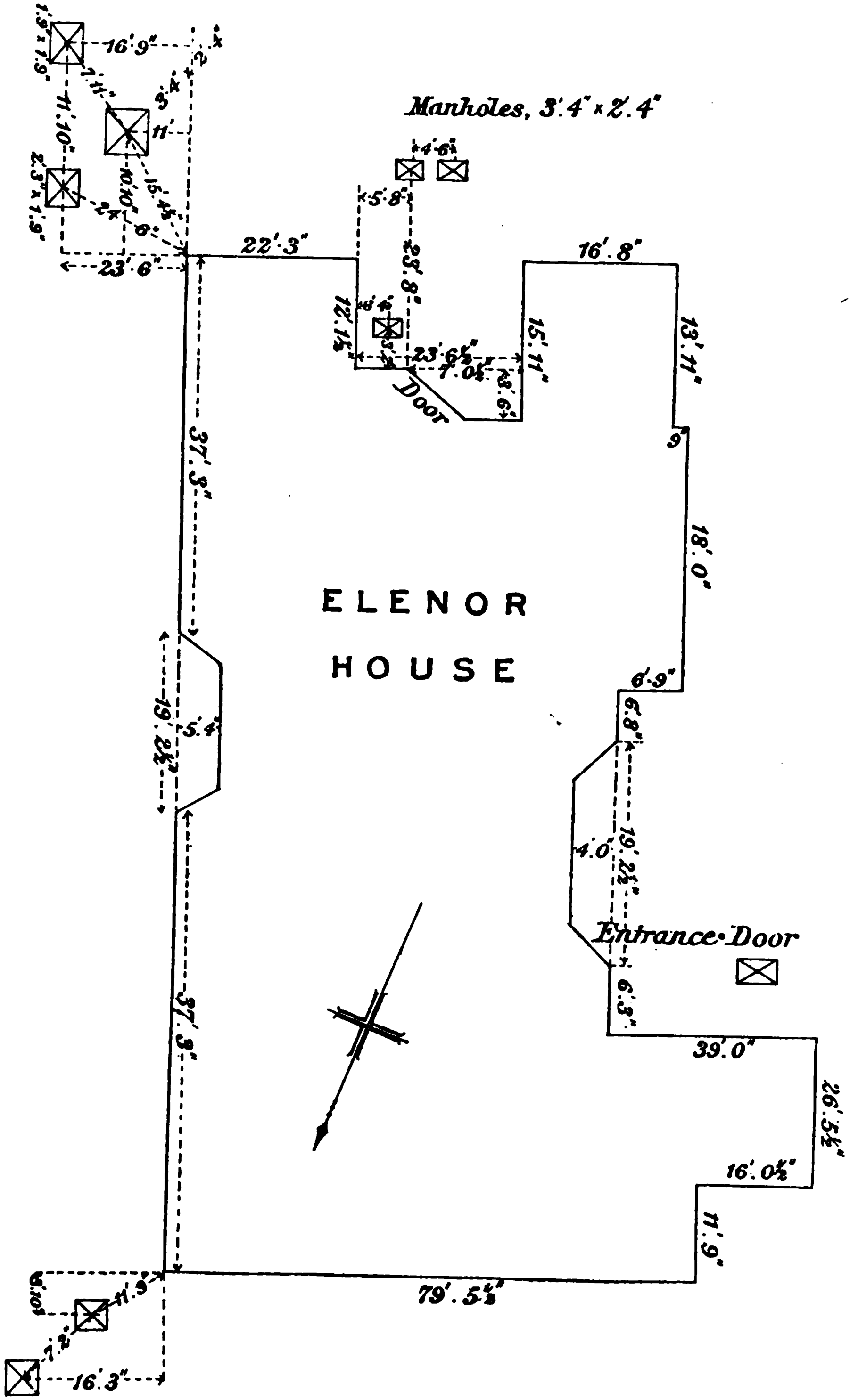

Hedge
256·5
⊙ w
247·8
172·4 on line E F
231·0
9·4
Boys' Ho.
229·25
140·5
138·5
20·9
38 ⊙ y
121·0
21·25
74·4
N. W. corner
Girls' Ho.
71·25
From ⊙ v go N to w
Lane
Hedge
11
⊙ v,
216·5
247·8 from w
208·25
S.W. corner
Girls' Ho.
196·75
25
From ⊙ D go W to ⊙ v
Hedge
5
⊙
125·0
F
Lane
Hedge
26
2
50·0
19
From ⊙ y go N to ⊙ F
5·5
⊙
280·0
y
16
8
250·0
14
11·5
222·0
12
15·0
181·0
4·5
151·5
7
⊙ v
Line x v
149·0
19·5 to ⊙ v
18
126·0
10
21·5
65·0
7
14·5
50·0
9
8·5
25·0
10·5
From ⊙ K go N to ⊙ y

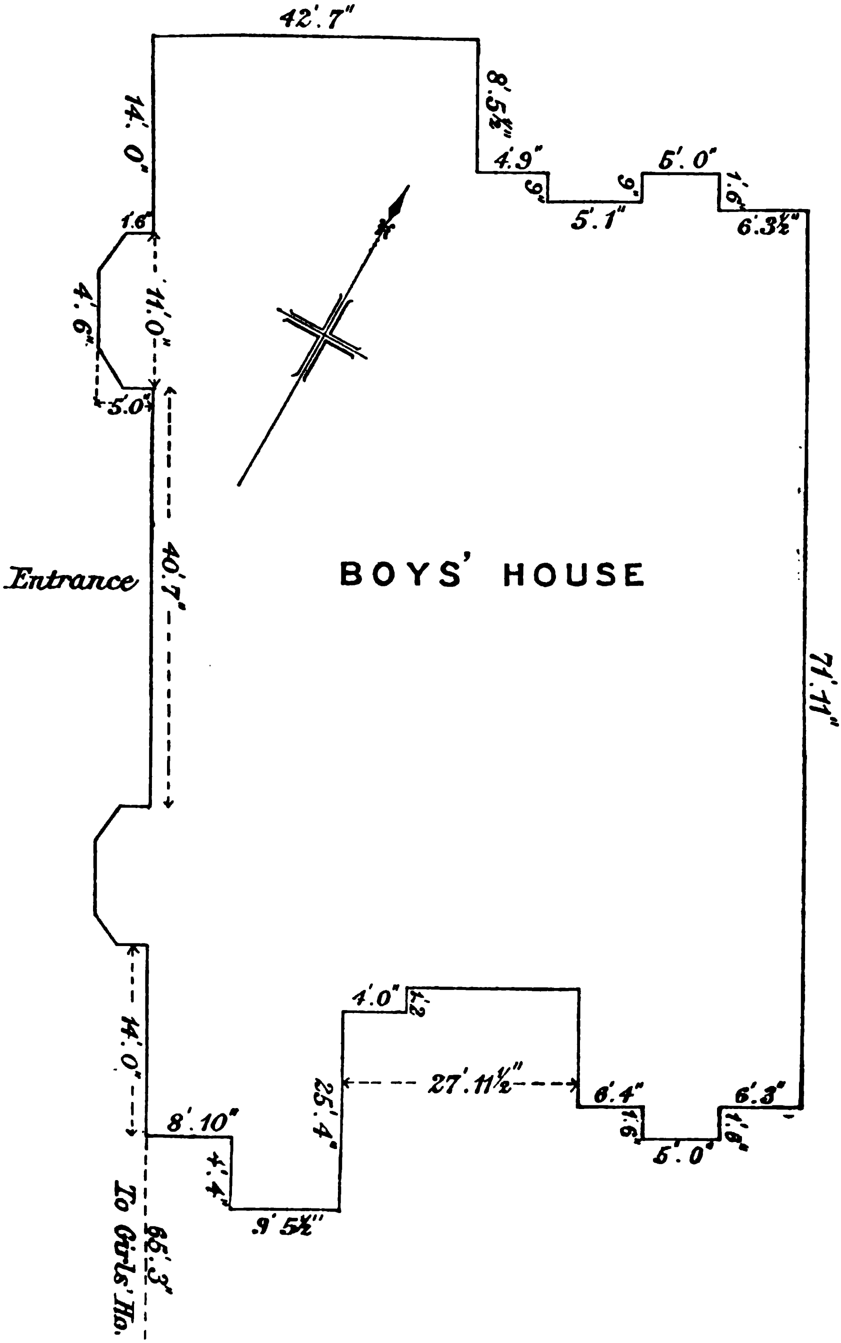

⊙ x
259·8
404·5 on line S t.
114·0
K
Lane
100·0
14
17
80·0
10
Hedge
17·5
66·0
4·5
16·5
50·0
4·5
16·2
30·0
6
5·0
4·2
18·7
0·0
From ⊙ C go N. W to x
Garden
Hedge
732·5
⊙ t
689·5
618 on line X u
37·7
676·5
672·6
Excavations for Mrs. Dearmers Isolation Home (not yet built)
666·0
o 15·75 S.V. on water supply
1·3
622·0
W.P. 37·25
475·0
404·5
x in line with C K
123·0
28·3
Greene Ho. (not yet built)
Trench for foundations
1·75
83·7
45·0
8·9
From 486·0 on line T F
⊙ S
go S to ⊙ t


⊙ Y
572·5
345 on line F G.
549·0
13
76′ to gate post
gate
10′
14
532·0
500·0
88
Flower bed
23·0
474·0
118
143
24·2
453·0
21
127
Path to
Alpha Ho.
416·0
28·5
400·0
84
36·0
350·5
53·1
38
300·0
38
42
264·0
Vict.
Ho.
41·7
Wood
250·0
27·5
227·0
W. end of Victa Ho.
Avenue 10′ wide
34
200·0
27·5
100·0
37·9
16·2
50·0
43·3
7.
⊙
0·0
From 15′ from H on line HI, go E to ⊙ Y

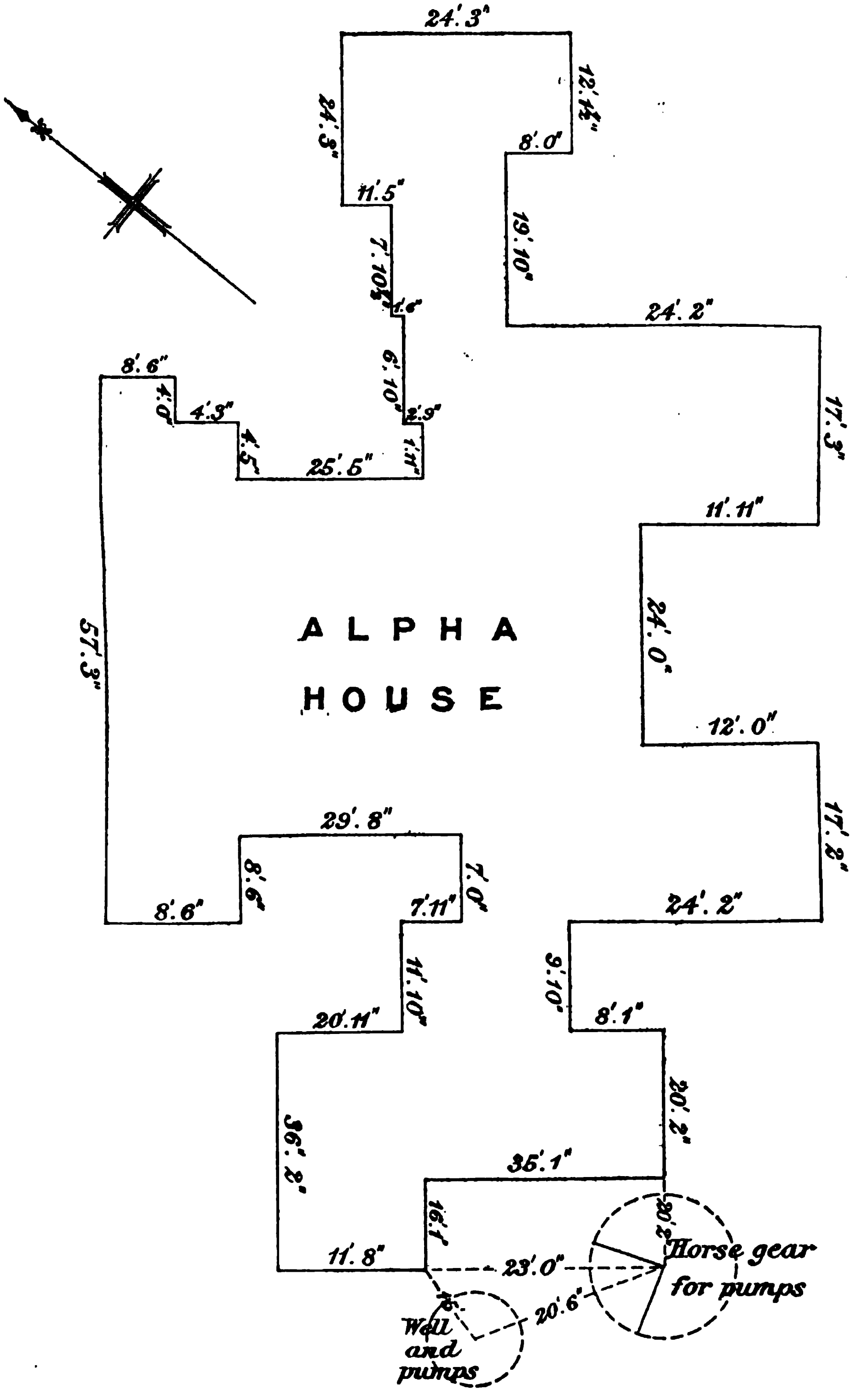

Manhole 2.6 × 2'
18.1"
16.6"
24.3"
5.8"
10.4½"
3.5"
3.5"
7.0"
5.11"
LAUNDRY
23.6½"
23.6½"
16.6"
16.5"
20.2½"
Entrance

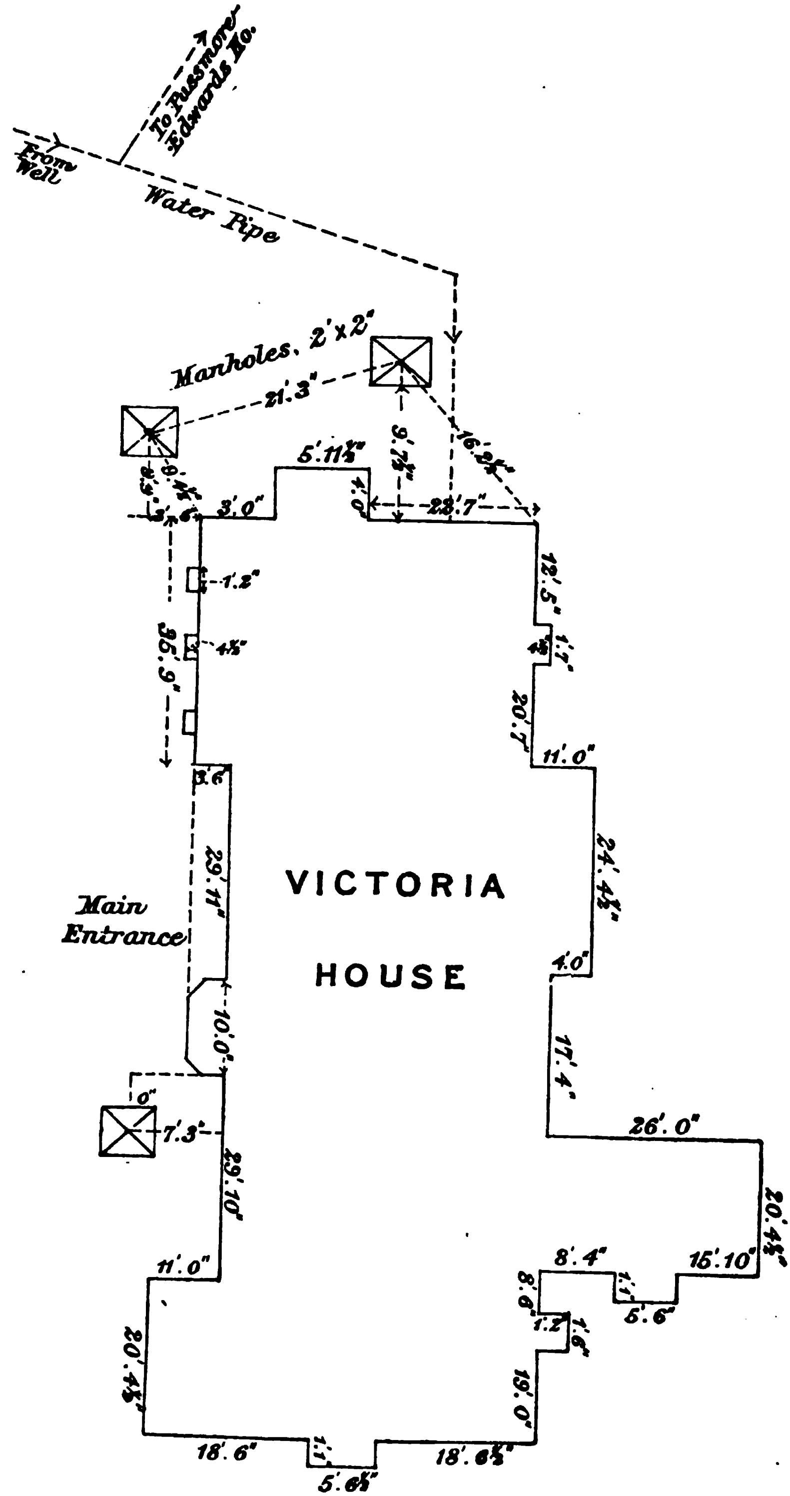

45.1"
8.7
RECREATION HALL
25.1"
8.8"
6.0"
8.9
18.7"
18.7"
6.0
Porch
8.1"
Entrance

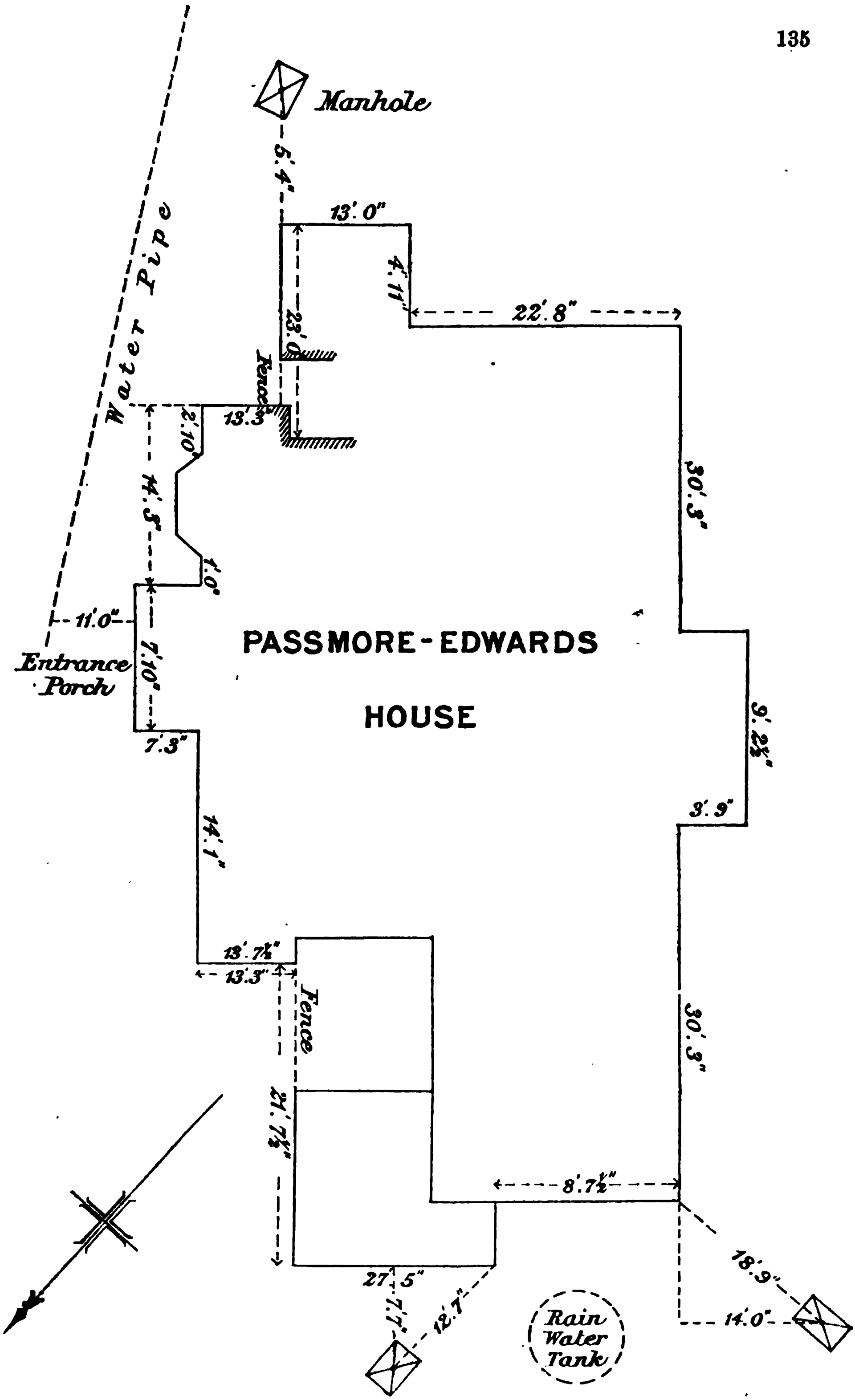

Manholes, 3'.4" x 2'.4"
ELENOR
HOUSE
Entrance·Door
Door
16'.9"
11'.10"
7'.11"
3'.4"
11'
10'.10"
15'.4"
23'.6"
22'.3"
4'.6"
5'.8"
23'.8"
12'.1½"
23'.6½"
7'.0½"
3'.6"
15'.11"
16'.8"
13'.11"
18'.0"
6'.9"
6'.8"
37'.3"
19'.2½"
5'.4"
4'.0"
19'.2½"
6'.3"
37'.3"
39'.0"
26'.5½"
16'.0½"
11'.9"
79'.5½"
11'.9"
7'.2"
16'.3"

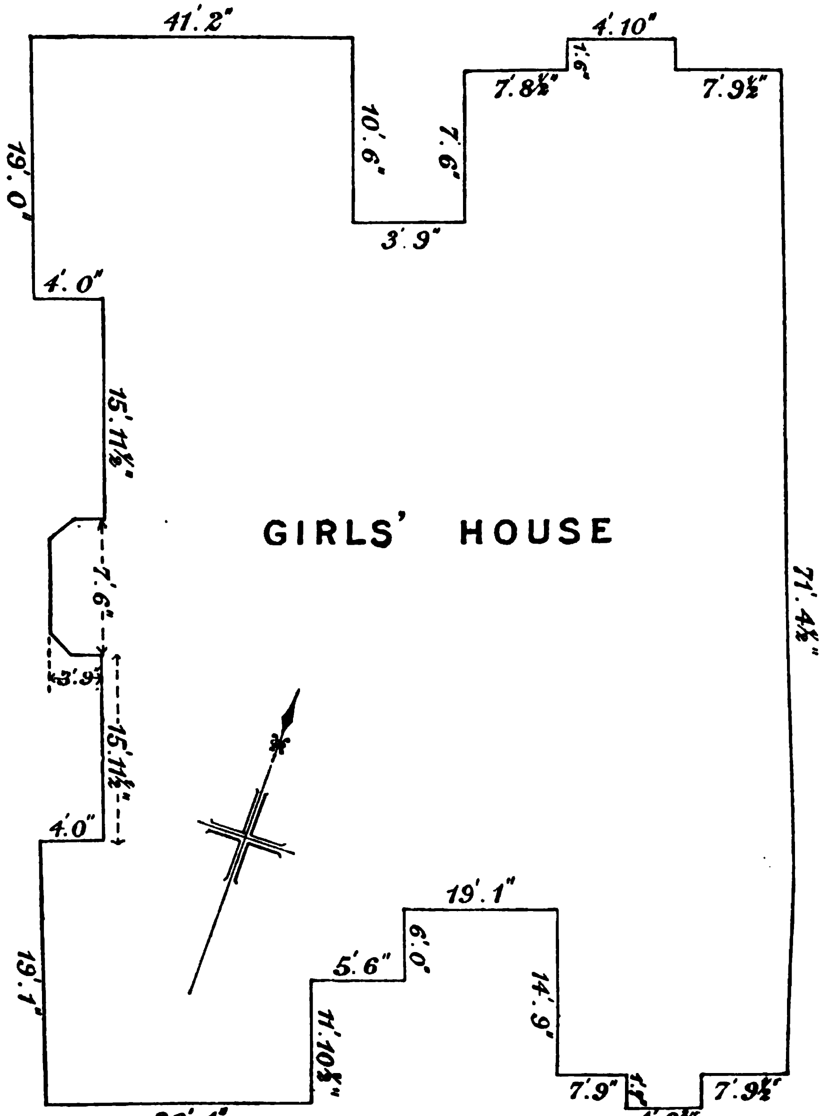


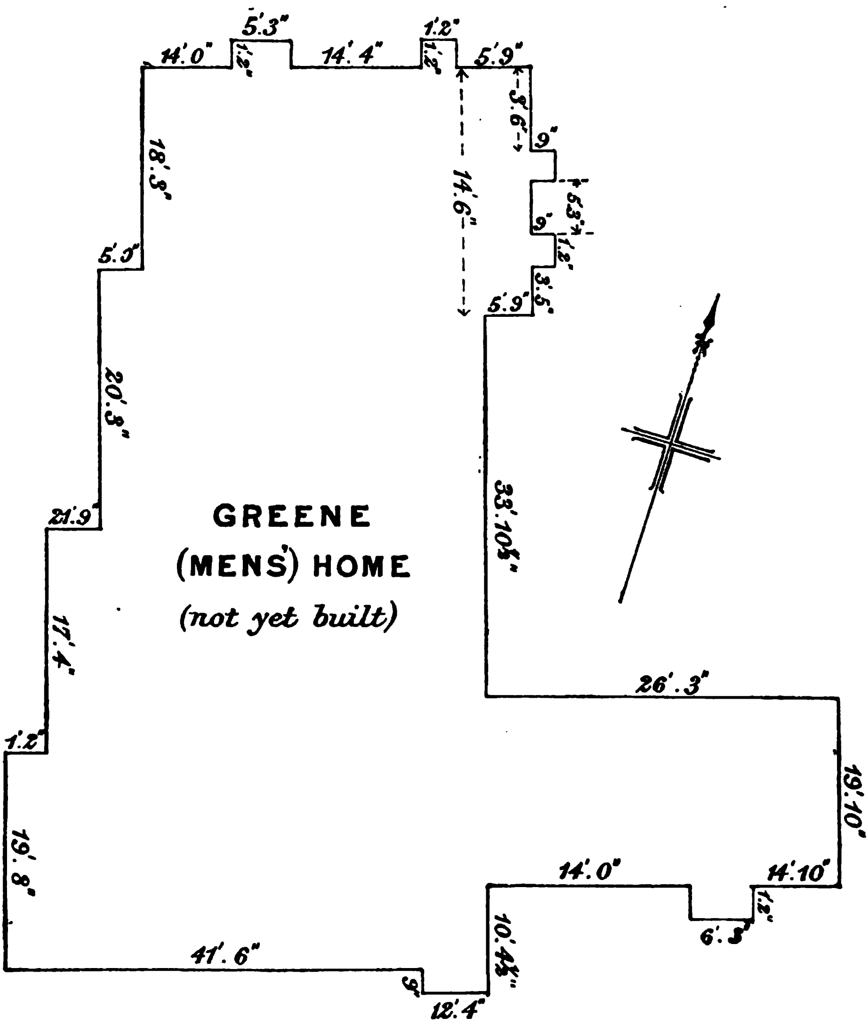
42'.7"
8'.5½"
4'.9"
9"
5'.1"
9"
5'.0"
1'.6"
6'.3½"
14'.0"
1.6"
4'.6"
11'.0"
5.0"
Entrance
40'.7"
BOYS' HOUSE
71'.11"
14'.0"
8'.10"
4'.4"
9'.5½"
25'.4"
4'.0"
1'.2
27'.11½"
6'.4"
1'.6"
5'.0"
1'.6"
6'.3"
55'.3"
To Girls' Ho.

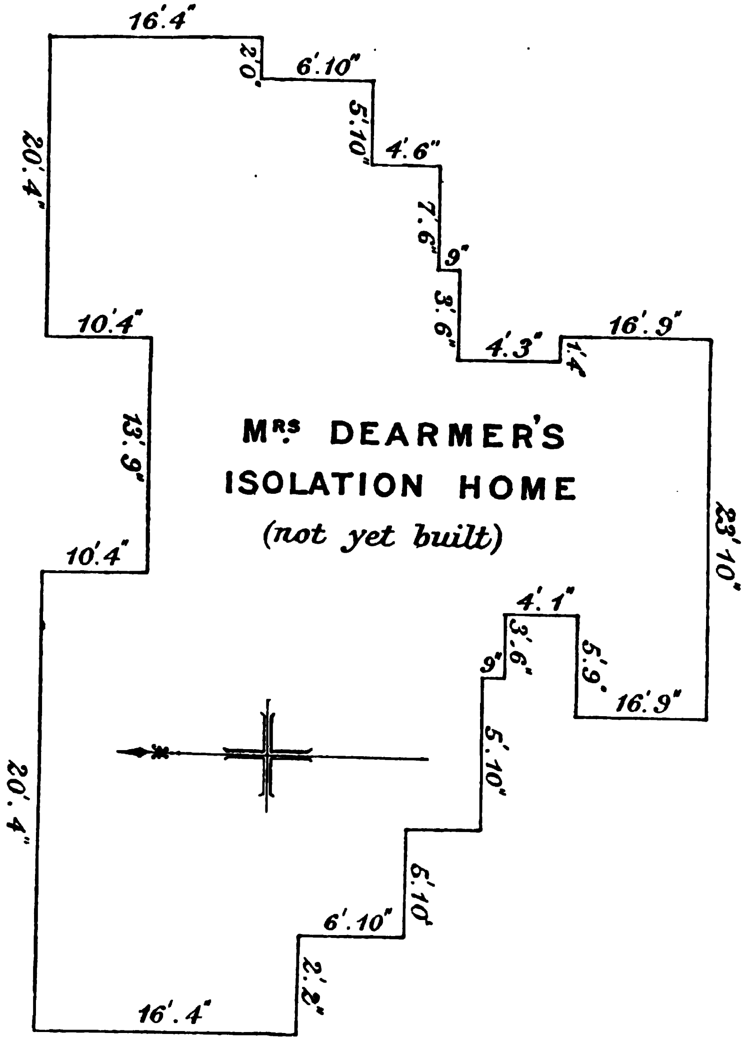

ALPHA
HOUSE
24'.3"
12'.11"
24'.3"
8'.0"
11'.5"
19'.10"
7'.10½"
1'.6"
24'.2"
8'.6"
6'.10"
2'.9"
17'.3"
4'.0"
4'.3"
4'.5"
1'.11"
25'.5"
11'.11"
57'.3"
24'.0"
12'.0"
29'.8"
17'.2"
8'.6"
7'.0"
8'.6"
7'.11"
24'.2"
11'.10"
9'.10"
20'.11"
8'.1"
20'.2"
36'.2"
35'.1"
16'.1"
20'.2"
Horse gear
for pumps
11'.8"
23'.0"
20'.6"
Well
and
pumps


To Passmore Edwards Ho.
From Well
Water Pipe
Manholes, 2'x2"
21.3"
9.7½"
16.2½"
5.11½"
4.0"
3.0"
23.7"
12.5"
1.7"
1.2"
4½"
35.9"
20.7"
11.0"
3.6
29.11"
VICTORIA
HOUSE
24.4½"
Main Entrance
4.0"
10.0"
17.4"
7.3"
26.0"
29.10"
20.4½"
8.4"
15.10"
11.0"
8.6"
1.1"
5.6"
1.2"
1.6"
20.4½"
19.0"
18.6"
1.1"
18.6½"
5.6½"


Manhole
Water Pipe
PASSMORE-EDWARDS
HOUSE
Entrance
Porch
Fence
Fence
Rain
Water
Tank
5'.4"
13'.0"
4'.11"
22'.8"
23'.0
13'.3"
2'.10"
14'.3"
1'.0"
11'.0"
7'.10"
7'.3"
14'.1"
13'.7½"
13'.3"
30'.3"
9'.2½"
3'.9"
30'.3"
21'.7½"
8'.7½"
27'.5"
7'.7"
12'.7"
18'.9"
14'.0"


GIRLS' HOUSE
41.' 2"
4.' 10"
1.' 6"
7.' 8½"
7.' 9½"
10.' 6"
7.' 6"
3.' 9"
19.' 0"
4.' 0"
15.' 11½"
7.' 6"
3.' 9"
15.' 11½"
4.' 0"
71.' 4½"
19.' 1"
19.' 1"
6.' 0"
5.' 6"
14.' 9"
11.' 10½"
20.' 4"
7.' 9"
7.' 9½"
4.' 9½"


GREENE
(MENS') HOME
(not yet built)
14'.0"
5'.3"
1'.2"
14'.4"
1'.2"
1'.2"
5'.9"
3'.6"
9"
5'.3"
9"
1'.2"
3'.5"
5'.9"
14'.6"
18'.3"
5'.5"
20'.3"
21'.9"
17'.4"
1'.2"
19'.8"
41'.6"
9"
12'.4"
10'.4½"
14'.0"
6'.3"
1'.2"
14'.10"
19'.10"
26'.3"
33'.10½"


16'.4"
2'0"
6'.10"
5'.10"
4'.6"
7'.6"
9"
3'.6"
4'.3"
1'.4
16'.9"
20'.4"
10'.4"
13'.9"
Mrs. DEARMER'S
ISOLATION HOME
(not yet built)
23'.10"
10'.4"
4'.1"
3'.6"
5'.9"
9"
16'.9"
5'.10"
20'.4"
6'.10"
6'.10"
2'.2"
16'.4"

The division of the paper into squares, practically renders the surveyor independent of the expansion and contraction, due to alterations of temperature, and of the moisture of the air. Paper absorbs moisture freely, expanding with moisture, and contracting on drying, very considerably. Moreover, the rate of expansion or contraction is not by any means constant in every direction. If a square or rectangle were laid out on a sheet of paper in moderately damp weather, it would not be found to be either the one or the other after a spell of dry weather.

The error, or deformation, in each small square, is insignificant compared with the distortion of the whole sheet. Even if the change in length of an individual square be perceptible, the error can be eliminated by distributing it on applying the scale. When, however, the squares do not exceed about 3 inches, the error in each will not exceed the smallest amount that can be appreciated with an ordinary scale.

If, at any time, serious distortion takes place, perhaps owing to exposure to sun in checking on the ground, or in filling-in detail with the plane-table, the squares afford a ready means of restoring the plan to correctness. It is only necessary to prepare a fresh sheet of paper with correctly drawn squares, and to copy into each, the detail which it contains. Lastly, the squares add materially to the value of the finished plan, for they afford a means of taking out areas and measuring long distances, with greater accuracy than is possible with the short scales usually inscribed on plans. To take out an area, it is merely necessary to count the number of squares which approximately coincide with the figure, and to measure the fractional parts of squares, by which they are in defect or excess of the desired figure. The areas of the squares themselves, are independent of expansion or contraction. The only part liable to error, is the relatively small proportion of the whole area, comprised in the fractional parts of the squares.

The area of the North field of the traverse survey above referred to (marked A) has been calculated by this method, and the result is given in the table on p. 144. The outline of the field and the squares have been reproduced for clearness in fig. 73. The dotted lines show where the boundaries have been 'equalised,' whilst some of the dimensions measured, and recorded in the table, have been marked on the figure for reference.

Plotting by means of a Protractor.

Traverse-surveys may also be plotted by means of a protractor, direct from the field-book.

1. The most efficient protractor for the purpose is that known as Hewlett's, or others on the same principle.

The construction and use of this protractor is fully illustrated by the drawing, (fig. 74).

2. Another method is to fix to the paper (with its zero and 180° points, accurately in the meridian), a circular protractor with two arms (fig. 75), carrying pricking points placed at opposite extremities of a diameter. Then to prick off and number several pairs of points, the lines joining each pair, representing one bearing. These lines all pass through the centre of the protractor. After a sufficient number of pairs of points have been pricked off, the protractor is

removed, and the bearings are then transferred to their proper positions by means of a parallel ruler (as on fig. 76), and each distance is laid off.

These circular protractors are usually provided with verniers, so that 'bearings' may be set off to single minutes or even less.

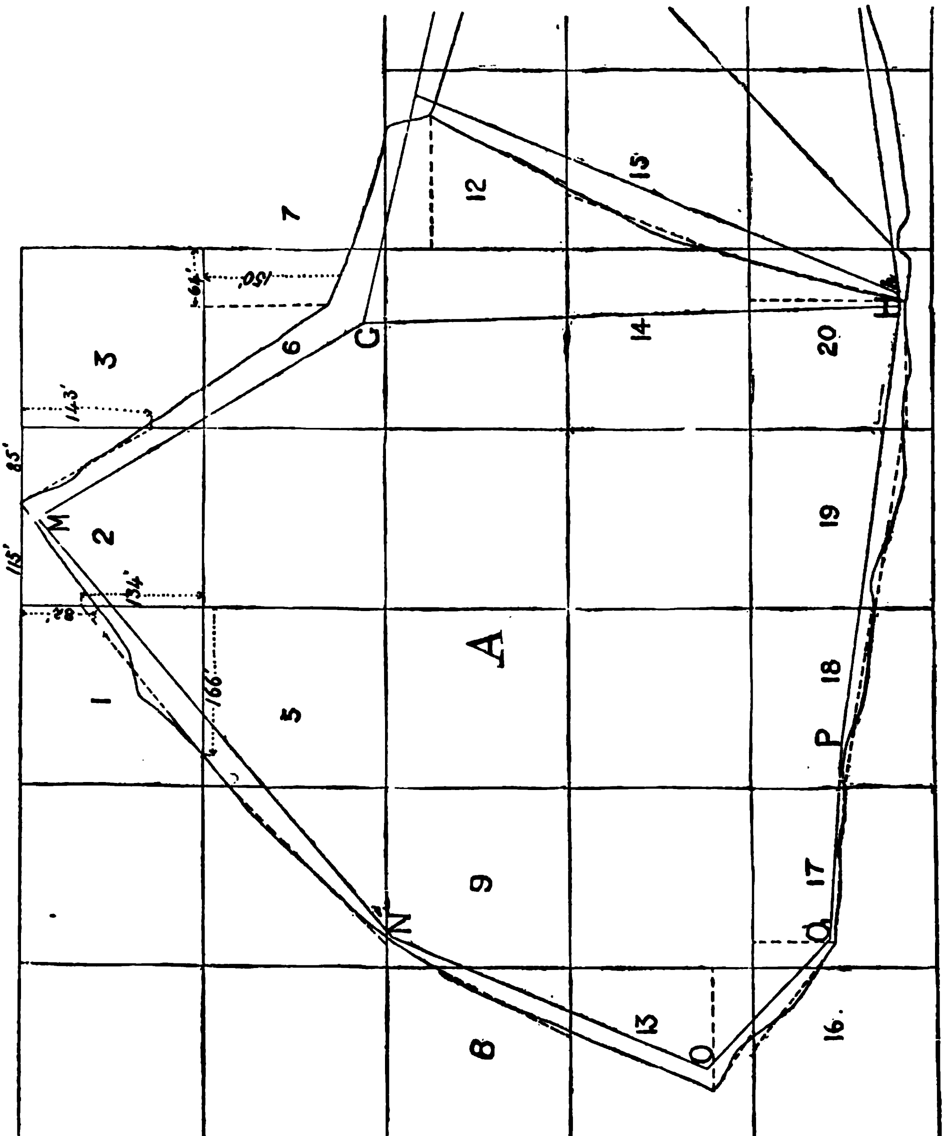


FIG. 73.

If the sides of the polygon are numerous, care is required to ensure that the correct distance is applied to the proper side. The transference of the 'bearing' from the points pricked off, to the position of the line, involves the unavoidable

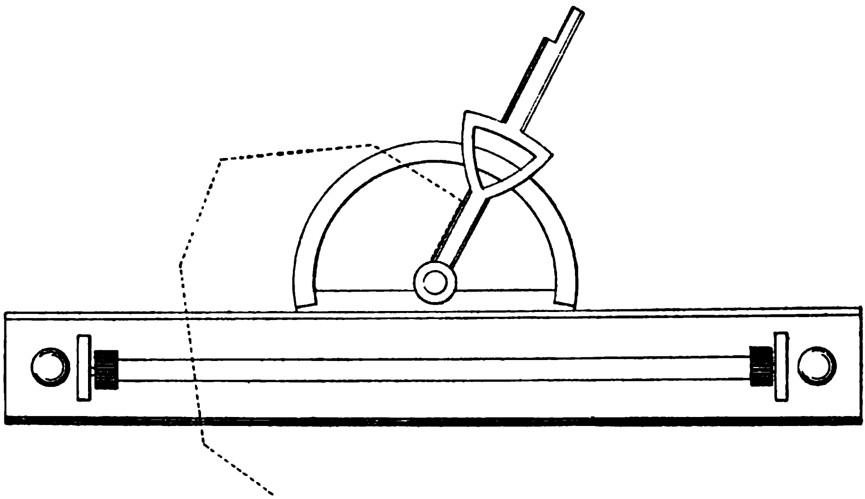


FIG. 74.

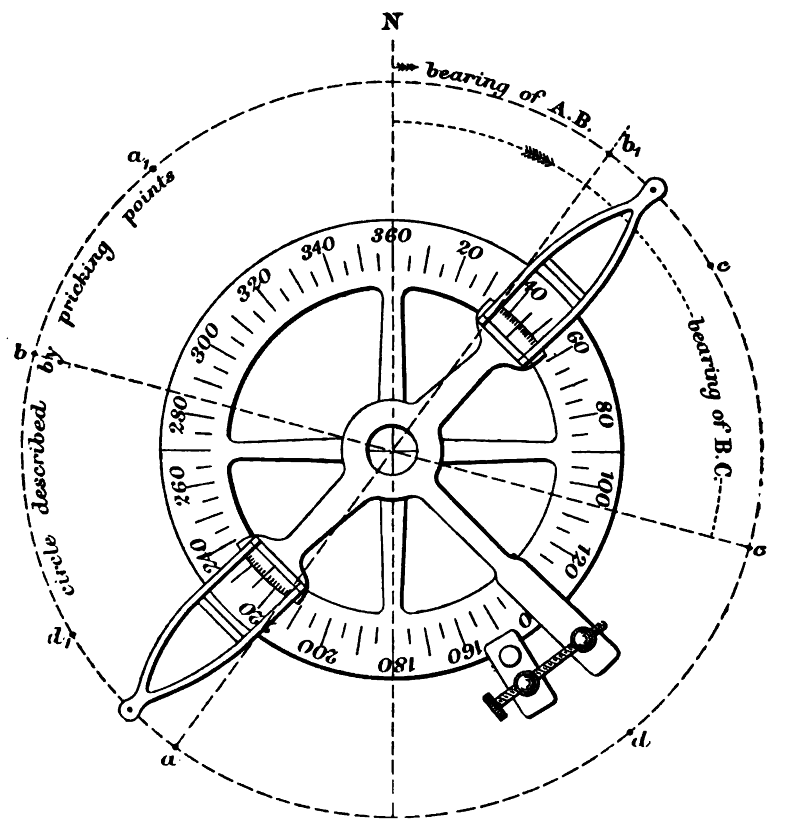

N
bearing of A.B.
b_1
a_1
circle described by pricking points
c
bearing of B.C.
b
c
d_1
a
d
S
360
20
40
60
80
100
120
160
180
200
220
240
260
280
300
320
340

FIG. 75.

error of the parallel ruler, which is probably sufficient to neutralise the accuracy of the protractor. If the degree of accuracy desired, is such as to justify the employment of such a costly and cumbrous instrument as the circular protractor, with vernier and arms, time and labour would certainly be saved by the computation of co-ordinates.

3. A somewhat similar process may be carried out by means of a cardboard protractor (*vide* fig. 77). These can be procured with an internal diameter of about 12 inches. The interior circle is cut out, leaving a ring. This is placed on the

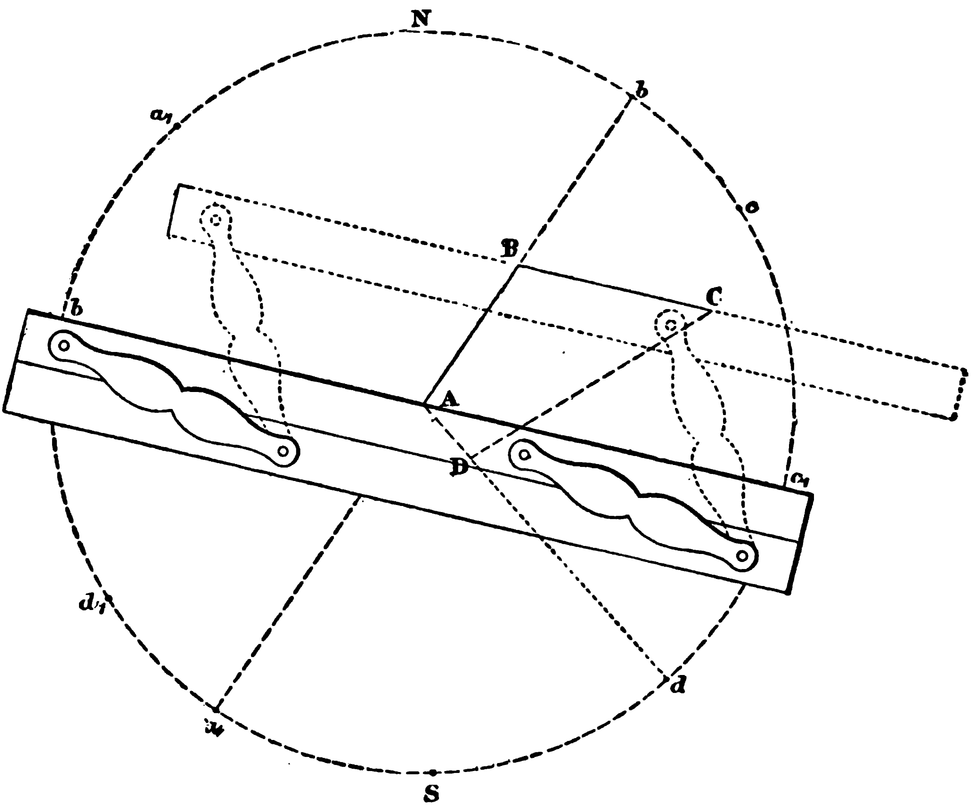


FIG. 76.

paper and secured by weights with zero and 180° north and south, and in such a position as to include within the vacant circle, a considerable part of the work to be plotted. A parallel ruler, preferably one of the sliding sort, is placed with its edge cutting the desired 'bearing' and its 'co-bearing.' Thus, if the bearing to be laid off were 36° the ruler would be laid across from 36° to 216°. The ruler is then extended, and a parallel line is drawn through the point of origin of the line. The distance is then laid off, and a new parallel drawn to the next 'bearing,' without shifting the protractor, as long as the work remains within it.

This method is convenient, when a complicated traverse has to be plotted to a small scale, such as 6 inches to 1 mile, so that a considerable amount of work is contained in the circle of the protractor.

Cardboard protractors are often very accurate, and do not, as a rule, appear to be liable to distortion by variations in temperature and moisture.

The writer has, however, seen exceptions to this rule. A new cardboard protractor, imported into India from a leading instrument maker, was found to be

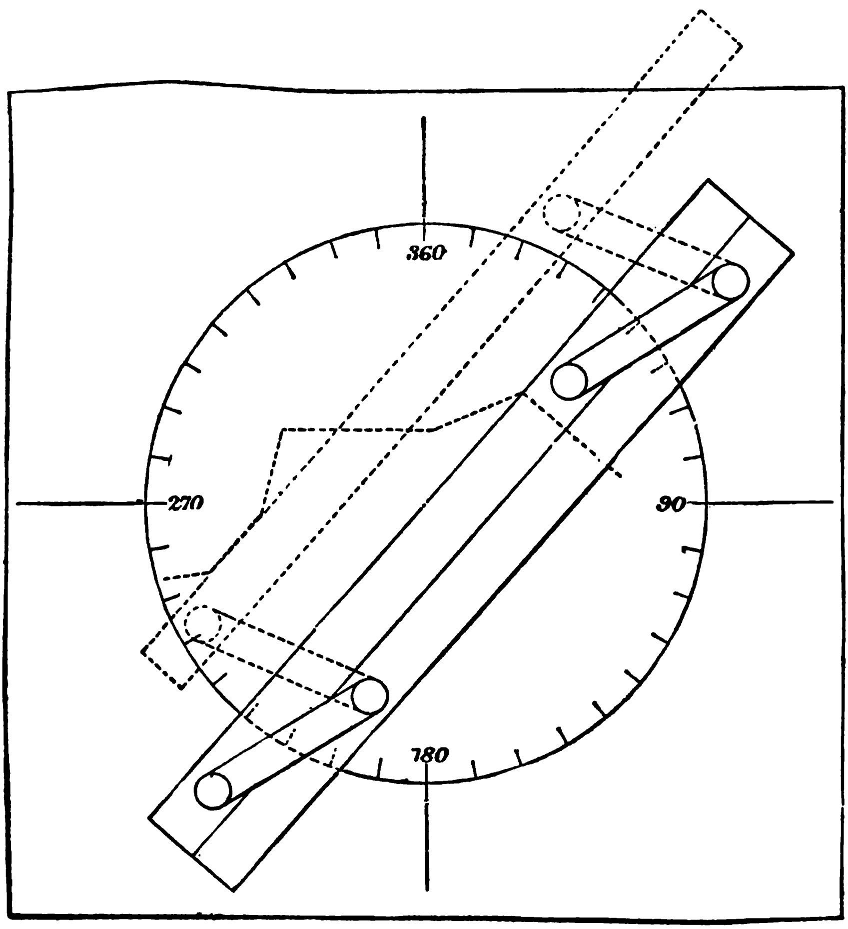


FIG. 77.

so erroneous as to be useless. It is well, therefore, to check a new cardboard protractor (especially in the tropics), by stepping with the dividers, before using it. A parchment paper protractor is often preferred to the above.

When plotting to small scales, and if great accuracy is not aimed at, an ordinary 6-inch protractor may be used, with a parallel ruler to transfer the 'bearings.'

CALCULATION OF THE AREA OF FIELD A, USING SQUARES.

Sign.	Number of Square.	Figure.	Dimensions.	Area. +	Area. −
+	1	triangle	134 × 166	11122	..
+	2	square	200 × 200	40000	..
−		triangle	115 × 82	..	4715
−		”	85 × 143	..	6077·5
+	3	”	44 × 67	1474	..
+	4	”	175 × 172	15050	..
+	5	square	200 × 200	40000	..
−		triangle	34 × 30	..	510
+	6	square	200 × 200	40000	..
−		triangle	91 × 137	..	6233·5
−		trapezium	150 × 64	..	9600
+	7	triangle	47 × 122	2867	..
+	8	”	149 × 77	5736·5	..
+	9	square	200 × 200	40000	..
−		triangle	33 × 51	..	841·5
+	12	trapezium	143 × 50	7150	..
+		”	106 × 150	15900	..
+	13	”	101 × 154	15554	..
+		”	100 × 46	4600	..
+	14	square	200 × 200	40000	..
−		triangle	43 × 12	..	258
+	15	”	61 × 156	4758	..
+	16	”	89 × 69	3070·5	..
+	17	trapezium	78 × 30	2340	..
+		”	94 × 170	15980	..
+	18	”	200 × 115	23000	..
+	19	”	200 × 149	29800	..
+	20	”	161 × 143	23023	..
+		triangle	43 × 171	3676·5	..
	Seven complete squares . .		200 × 200	280000	..
				665101·5	28235·5
	Subtract			28235·5	
	Area in square feet			636866	

Use of loose sheets, and not mounted paper. Surveys, should not be plotted on paper mounted on a board, for the distortion which takes place on cutting off is very great. As many kinds of good paper, such as Whatman's, are, in their original condition, so much buckled as to make accurate

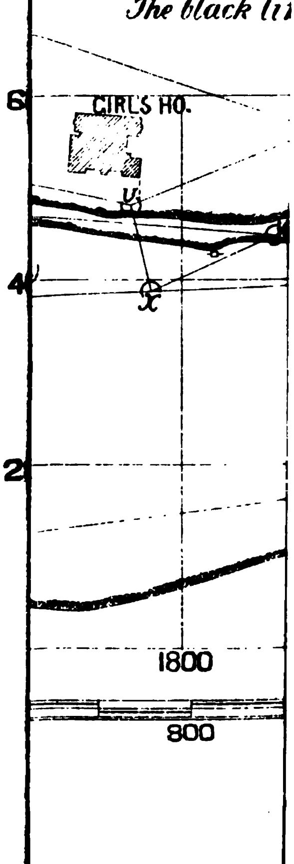
ERSE

ERSE SHEET

	...red ...ing	Cardinal Pts.	Distances (in feet)	Nort...
	57′	N.E.	617·80	160·4
	51′	N.W.	273·50	237·3
	59′	S.W.	711·80	
	43′	N.W.	87·70	66·5
	50′	N.W.	433·00	142·2
	06′	S.W.	213·50	
	15′	N.W.	593·00	130·9
	25′	N.W.	419·70	338·2
10	5′	S.W.	604·30	
	47′	S.W.	374·30	
	50′	S.E.	196·00	
	14′	S.E.	279·70	
	13′	S.E.	491·00	
	33′	N.E.	1274·00	165·2
8				1280·7
				—·4

True bearing of HI...

The black li...



work difficult, the writer has found it convenient to flatten the paper before using it. To this end a number of sheets are mounted on a drawing board, one above the other, by moistening and glueing down the edges in the usual manner. When dry, but not too dry, they are cut off and laid flat in a drawer and allowed to remain there for some weeks, till they have attained the average hygrometric condition of the air of the office. They remain smooth and flat, and are therefore pleasant to work upon, and expand and contract in a fairly regular manner.

Plotting on Millboard, for Plane Table.

If the details of a survey are to be filled in by the plane-table, the traverse or trigonometrical work should be plotted on millboard, which should be clamped to the table, and not attached by glue or gum. Millboard expands and contracts fairly uniformly in all directions, so that if the distances are laid off from a scale of the same material, no inconvenience is experienced, even if the sheet be exposed to varying atmospheric conditions for several weeks. For plane-table work, the millboard should be of a grey or drab colour to avoid the fatigue to the eye which white paper causes.

Comparison of methods of Plotting.

Good work can no doubt be done with the protractor, if very carefully used, with the disadvantage, however, that a closing error which may be due to either incorrect field-work, or to errors in draftsmanship, cannot be located without going over the whole work again.

The protractor can only be used safely by an experienced and thoroughly trustworthy man. It cannot be entrusted to an ordinary draftsman, for should he force the traverse to close (and there is temptation to do so), his action could be only discovered by going over the whole of his work. Probably his delinquency would not be discovered until the plan was finished.

In extensive surveys, for reasons of economy, much of the work must be done by deputy, employing persons at low rates of pay, for operations of routine detail, leaving the head surveyor to exercise general supervision, and to conduct the most important and difficult operations.

In such cases, and whenever really accurate work is desired, the method of co-ordinates alone is applicable, for it admits of positive check, at every stage of the work, even before plotting is commenced.

The labour of computing the 'latitudes' and 'departures' may seem serious, but with a little practice the office boy can do this as well as the most scientific surveyor, and perhaps even quicker and better. The surveyor has only to set up the traverse and hand it to the computer, whose work can be checked in a variety of ways, for instance, the last '*side*' and '*bearing*' might be omitted from the traverse sheet and introduced and computed by the surveyor himself.

Another advantage of the method of co-ordinates is that several draftsmen can be employed simultaneously, in plotting and filling in.

The traverse sheets and field-books can be distributed amongst several persons, who can work each on a separate sheet of paper.

If, on the other hand, the protractor be used, it is convenient, and almost necessary, to have the whole plan laid down at once on a single sheet of paper. The risk of damage (by upsetting ink and the like) to a large plan, which must be worked on for a long time, is most serious. Again, all important plans, such as town plans, should be compared and checked on the ground, before publication. Subdivision into numerous sheets facilitates this operation.

Although the protractor cannot compare with co-ordinates as to accuracy and convenience of plotting, and should never be used on the main lines of important work, still it is a most useful appliance. It is often desirable to make a preliminary plot, as work progresses, to see how the work comes in. It may also be used for laying off bearings, taken to lateral points, at some distance from the main traverse, or angles taken with the pocket sextant, between the main line and objects beyond the proper distance of an offset. It is, moreover, often convenient to make a preliminary rough plot, for the purpose of ascertaining the best method of correcting errors, or of completing or extending the work, and for this purpose any good 6-inch, or 4-inch circular protractor, will suffice.

Degree of accuracy in angular and linear measurements compared.

In traverse-surveying, angular and linear measurements are combined. The degree of accuracy of the result will depend on that of the least accurate of these two classes of measurement, together fixing points in space.

As far as the instrument itself is concerned, angular measurements will, with ordinary care, be more accurate than linear measurement, as the following considerations will show.

With the ordinary chain and arrows, the ground not being peculiarly favourable, an error of *plus* or *minus* one part in a thousand, would not be excessive. That is to say a distance of 1000 feet or links, as measured on the ground, might be anything from 999 feet or links, to 1001 units.

Now, an angle of 1 minute subtends at 1000 feet, a sine or tangent of 0·29 feet. Suppose, to fix our ideas, the line were east and west.

Errors of chainment may displace the terminal point of the line by a foot east or west. An error of one minute in the 'bearing' would only displace the same point by less than one-third of a foot north and south.

If, however, the direction of the line were determined, not by a linear and an angular measurement but by a second linear measurement of the same length as the first, and at right angles to it, the position of the terminal point would be fixed within a square, having a side of 2 feet (± 1). If *two* angular measures accurate to 1 minute were used, then the point would be fixed within a square having a side of 0·58 feet, (± 0·29).

Now a good 4-inch theodolite, in proper adjustment, should by a single reading, determine an angle to within ± 1 minute. Such an instrument would therefore have a degree of accuracy three times greater than ordinary chaining, and equal to some of the more accurate methods of linear measurement, such as the long steel tape.

Manner of using the Theodolite to get accurate Observations.

So far, as to the power of the instrument itself. The manner of using it, must now be considered, and the influence of circumstances extraneous to it, examined. The latter principally consist in accuracy of centering at the point of observation, and correct bisection of the points observed.

The centre of the theodolite must be placed exactly, and perpendicularly, above the centre of the point of observation, by means of the plumb-line. The shorter the distance to the point observed, the more important is accurate centering.

For example, let A, B, and C, be three points, for the sake of simplicity assumed to be in a straight line (*vide* fig. 78). Suppose that A B and B C each measure 100 feet. Now suppose that the surveyor set up his theodolite not exactly over the point B, but at P, one-tenth of a foot from it, and in a direction perpendicular to A B C. He would measure the angle A P C. Then $\tan PAB = \tan PCB = \frac{0 \cdot 10}{100}$ $\therefore$ $\angle PAB = \angle PCB = 0° \; 3' \; 26''$ and the angle A P C would read 180° 6′ 52″ instead of 180° 0′ 0″, thus introducing an error of 6′ 52″. This is perhaps an extreme case, but it is one which might, with want of care, easily occur in a complicated survey, such as that of a town.

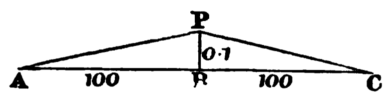


FIG. 78.

Next, as to accuracy of bisection of the points observed to. It may not always be possible to see the actual peg marking the station. Supposing that owing to a bush or an undulation of the ground, the surveyor at P could only see the top of a rod, held at A. A careless staff-man might hold it so that its summit was one-tenth a foot out of plumb, without the surveyor detecting it from the instrument. This would at once introduce an error of 3′ 26″ in the observed angle. Or a similar error might be produced by holding the staff, not on the centre of the peg, but at the side of it.

It is therefore desirable that station-points be so placed that the peg itself is visible in the telescope of the theodolite when observing, so that the bottom, or point of the ranging rod, may be bisected. It is well to arrange the stations beforehand, so as to secure this condition, increasing if necessary their number.

If, for any reason, the surveyor has to observe to the top of a rod of considerable length, then he should make sure that it is *securely* and *accurately* fixed, *perpendicularly* above the station-point, and made fast with guys or a pile of stones. To effect this he may use the plumb-line of the theodolite, holding it at arm's-length and making the rod coincide with it, in two directions, approximately at right angles to each other. A good plan in such cases, is to use a thick plumb-line with a heavy bob, suspended from a light tripod instead of a rod. The plumb-bob may be brought exactly over the point, and any oscillation, on account of wind, will not affect the top of the line to an appreciable extent.

In accurate surveys, where the distances are short (town surveys for example), ordinary rods should not be used. The very thickness of the rod, when a strong sun shines on one side of it, may introduce an appreciable error. There will be a tendency to direct the cross-hairs, not to the true middle of the rod, but rather towards the sunny side of it.

In such cases, the surveyor should observe to the point of a lead pencil, or arrow, held on the peg or bolt, and a pencil mark, or a centre punch mark, should be made at the point at which it is held, to enable the theodolite to be set up exactly over the point observed to.

As regards accuracy in reading angles. Unless the method employed in measuring distances is much more accurate than ordinary chaining, a single reading of one vernier, or at the outside the mean of two or three, will usually suffice. It is not worth while to spend time in repeating angles, or to use heavy and powerful theodolites. The readings should be taken with care, and booked to the smallest fraction of a degree that can be rapidly read. The seconds of arc can usually be eliminated, in correcting the angles or bearings.

Accuracy in angular measurement is therefore to be attained rather by care in centering the instrument, and in bisecting the true point of observation, than by the use of large instruments, or by precision in reading to the last second of arc. Inexperienced surveyors, finding a serious closing error, in their angular measurement, are but too apt to attribute it to a defect in their instrument, rather than to their own want of care in regard to the above points.

Setting-out Traverses.

The surveyor will, as a rule, save time in the end by going round an intended traverse with two assistants carrying rods, pegs, &c., and arranging and marking the station-points before commencing actual measurements. The points should be arranged so that, whilst coinciding (within the limits of moderate offsets) with the boundary of the area to be delineated, the sides should be as long as possible. If, for the purpose of determinating a crooked line, short sides are absolutely necessary, it is well to arrange the points so that the bearing of further sides can be determined independently of the short sides.

Thus, if it be necessary to introduce the short sides B C, C D, D E, and E F (fig. 79), in order to delineate a sharp bend in a road, it will be well to arrange so that the angles A B F and B F G can be observed as well as the angles A B C, B C D, C D E, D E F, and E F G, so that the bearing of F G and of the sides beyond it, can be determined by angles referring to the relatively long side B F, and therefore free from the possible accumulation of error in the angles at C, D, and E, which affect short sides. Still better, if the line B F can be chained, for the polygon B C D E F can then be treated as a subsidiary polygon.

Station-points should be so placed that the theodolite can be easily set up over them, and so that the surveyor can read his instrument without interruption from traffic. For this reason, the middle of a road is not a desirable station-point.

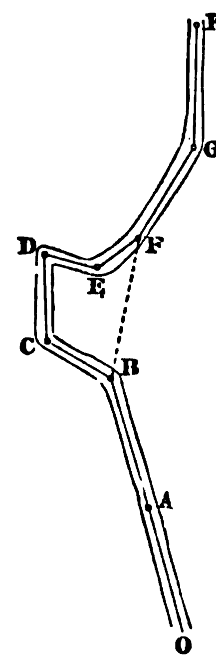


FIG. 79.

Permanent Location of Points.

It is most important to secure the permanency of station-marks, or at least to provide the means of finding them if taken up, or lost. If the surveyor has not the means of marking his points in a permanent manner, for example, with substantial dressed stones, well-bedded, he will do well to establish the position of his station-points by means of measurements to fixed and permanent objects, such as gate-posts, corners of buildings, jambs of doors, corners of steps and the like. Each point should be determined by three measurements. In a country where walls abound it is convenient to mark points of reference on the wall with red paint, inscribing the distances on the wall thus (*vide* fig. 80).

The dots are marked at round distances from the station-point. By holding the ring of the tape to the point marked 16, and the graduation 16 + 12 = 28 to the point marked 12, and stretching the tape with an arrow held at the division 16, the station-point can be accurately recovered. Marks made with iron-oxide paint, such as is used for ships' bottoms, are very permanent. Such measurements

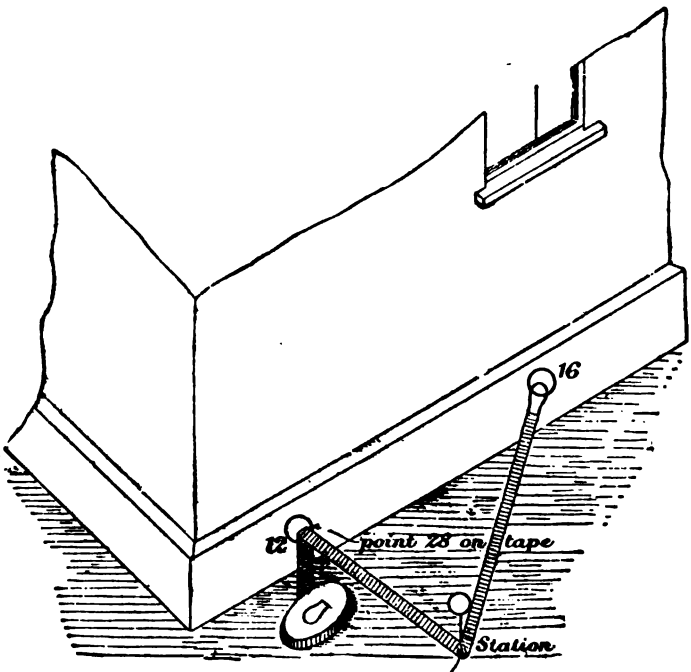


FIG. 80.

should be entered in the field-book with clear sketches, and notes. This information will be found to be of the utmost value, should the surveyor have to verify his work, or if he wishes to extend his survey. Pegs and even nails driven into roads are far from permanent, for boys will devote much energy to their removal.

Limits of 'Permissible Errors' discussed.

Since, unless by accident, there is sure to be some error in the summation of angles, and of 'latitudes' and 'departures,' it will be well to examine the amount of error which is unavoidable, and therefore permissible.

The total error of closing, as shown by the summation of 'latitudes' and 'departures' should not exceed the limit of permissible error, proper to the appliances used for measuring, and to the character of the ground. This should be the case whether the angles have been corrected before com putation of 'latitudes' and 'departures' or not. *Angular error* obviously does not necessarily tend in the same direction as *linear error*.

The limits of 'permissible error' in chaining, have been discussed under the head of chain-surveying.

It is here assumed that the 'permissible error' in linear measure, means the error which is unavoidable in measuring with a given chain, an accurately known distance, such as that between two trigonometrical points, or between the extremities of a base line, previously measured with some appliances of the greatest accuracy. Suppose that a distance of one mile were set out absolutely accurately, the chain measurement thereof might make the length 5285 feet or 5275 feet, without exceeding the prescribed 'limit of error' for ordinary chaining. But two successive measures with the same class of chain should not differ by 10 feet. They should not differ by half that amount, though the mean of the two might be five feet from the truth.

Some chaining errors are constant in direction, such as an error due to the incorrect length of the chain, but this would have no effect upon the closing of a polygon, provided that the same chain were used throughout. What has to be considered is really the difference between two measurements, which should not certainly exceed one-half of the total 'permissible error.' This refers to an independent polygon, all sides being measured with the same appliance. Cases will occur where one side is measured with superior accuracy. For example, a traverse may connect two trigonometrical points. The distance between them, as determined trigonometrically, is probably accurate to 1 in 20,000 or less, and must be taken as correct. The trigonometrical distance, forming one side of the polygon, the error must be wholly in the traverse-work. The full 'permissible error' may therefore be applied to the traverse alone.

The writer has observed that when an extensive closed traverse, including two trigonometrical points, had been 'set up,' the closing error of the polygon, setting aside the trigonometrical points, that is to say, treating them as traverse-points merely, might not be more than 1 in 5000. But when the traverse was divided into two polygons, with the trigonometrical distance as a common side, the error of each (as completed by the trigonometrical side, which must not be altered), was considerably greater, often nearly 1 in 1000.

The difference between absolute 'permissible error,' and the 'permissible difference' between two measurements must not be lost sight of, though the ratio is not easy to determine. The writer has found that with fairly careful work, distances being measured with a 66-foot steel band, total closing errors of less than 1 in 2000 could be obtained with ease.

The error in summation of angles, or angular error, must now be considered. The greater part of this will be due to errors of centering and bisection, which the surveyor has it in his power to reduce to a minimum. With an ordinary theodolite it is possible to read an angle to within $\pm 15''$ of arc. If we take therefore $\pm 25''$ as the total error from all causes, a margin of $\pm 10''$ will be allowed for centering and bisection errors, equivalent to a lateral displacement of object of nearly 1 inch in 500 feet, and therefore a liberal allowance for unavoidable errors, exterior to the instrument. The average error of any one observation may be taken at $\pm$ 25″. But this error will not be always in the same direction, but will, to some extent, tend to compensate each other. Consequently the total error must not be proportional to the number of the sides. Probably, as in other matters of measurement, it will increase in proportion to *the square root of the number of sides.*

Thus, the closing error of a polygon of four sides would be

$$25'' \times \sqrt{4} = 50''.$$

$$\text{Of 9 sides } 25'' \times \sqrt{9} = 1'\ 15''.$$

$$\text{Of 100 sides } 25 \times \sqrt{100} = 4'\ 10'',$$

and so on.

The regulations of the Indian Revenue Survey allow an error of 1′ in every five sides, without reference to the total number of sides. This seems to be an undue degree of accuracy if the sides are few, and somewhat lax if they be numerous.

In polygons of a moderate number of sides, surveyed with reasonable care, the total error will be so small as to require little or no systematic correction. The 'angles' or 'bearings' may be first corrected, and often a judicious elimination of the seconds will suffice to bring about the desired result, taking the next highest or lowest minute, as the case requires.

Adjustment of Final Errors.

The final error in summation may be corrected by a judicious addition to, on deduction from, 'latitudes' and 'departures,' the corrections being in proportion to the lengths to which they are applied.

Cases may however arise in which it is necessary to apply some more systematic method of correction. For example, the closing error of a long and complicated traverse may be quite within permissible limits, but yet so serious as to require careful correction, in such a manner as to reduce relative distortion to a minimum. Or, a long traverse, made by some rough method such as pacing and compass, and therefore having a serious closing error, may require correction in the best manner.

It is capable of mathematical demonstration that, all things being alike subject to error, the most probable correction may be obtained by the following rule, viz.

As the sum of all the distances is to each particular distance, so is the total error in 'departure' *to the correction of the corresponding* 'departure,' *each correction being so applied as to diminish the whole error in* 'departure.' Proceed in the same way for the correction in 'latitude.'

The demonstration of this rule, due to Bowditch, is given in a Treatise on the Adjustment of Observations, by T. W. Wright, B.A., New York, 1884.

The attached diagram (fig. 81) and Table C, give an example of the application of this rule to a rough traverse, made with compass and pacing, in which the closing error is large. The corrections for each side were measured graphically by the simple construction shown on the figure.

In correcting by this method, both angles and sides are altered, the former perhaps unduly, considering that they are more likely to be correct than the sides.

Take the case of a side 1000 feet long, bearing east. Further, suppose that the ratio of correction for latitude were 1 in 2000. The difference of latitude for this line would be zero, but the correction applied would be 0·5 of a foot altering the observed bearing by 0° 1′ 44″ which exceeds the 'probable error' of an angle.

In accurate work, especially if the angular error be small and has been corrected before computation of 'latitudes' and 'departures,' it is better to adopt a method which alters the sides alone.

Let a polygon A B C D E F G H have (fig. 82) a closing error such that when plotted the second position of A becomes A_1. Join A A_1 and produce it to cut the opposite side of the polygon in X. Bisect A_1 A in O. Then we may consider the polygon to be divided into two parts X C B A and X D E F G H A. If commencing from X, we replot the part X C B A enlarging each side in the ratio of X A to X O, without any alteration in the angles, the point O will be reached. In like manner in X D E F G H A_1, by drawing a similar figure, each side being diminished in the ratio X A_1 to X O, the point O will be reached.

Some care must be taken in the selection of the line O X which may be called

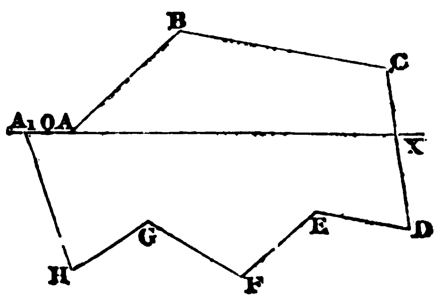


FIG. 82.

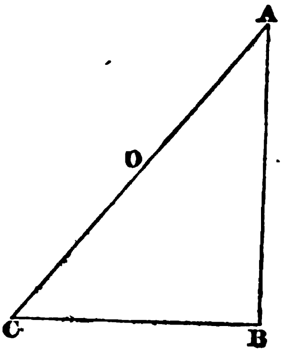


FIG. 83.

the 'axis of adjustment.' For instance the error of closure might place A_1 nearly south of A, in which case the line A A_1 would not cut the polygon at all.

Again, it may be that the axis of adjustment cuts off but a small part of the polygon, and is therefore short, thus necessitating a large percentage of correction.

To apply this principle practically, plot the polygon approximately with protractor and scale to a small scale.

Lay off A B parallel to the meridian (fig. 83) to represent to some convenient but much larger scale, the error in latitude, and B C perpendicular to the same representing error in departure. Join A C and bisect it in O. Then the line A C is parallel to the axis of adjustment, and measured on the scale used to lay off the errors in 'latitude' and 'departure' gives the total closing error, and A O or C O the closing correction.

Next, with the parallel ruler, draw the axis of adjustment X Y (fig. 84) parallel to A C, in such a position as to get the greatest length for X Y.

Inasmuch as the total closing error will be exceedingly small in comparison

[*To pull out between pp.* 152 *and* 153.]

15	75	15	57	15	57	15	N.E.	200	108·2	0·3	..	..	168·2	1·1	..	..	6567·9	10171·8	15
16	63	30	45	30	45	30	N.E.	260	182·3	0·4	..	..	185·4	1·6	..	..	6675·5	10338·9	16
								3068	1853·2	5·0	..	..	1438·8	10·6	889·4	6·1			
17	80	45	62	45	62	45	N.E.	300	137·3	0·5	..	..	266·7	1·6	..	..	6858·2	10522·7	17
18	79	30	61	30	61	30	N.E.	200	95·4	0·3	..	..	175·8	1·1	..	..	6996·1	10787·8	18
19	97	00	79	00	79	00	N.E.	250	47·7	0·4	..	..	245·4	1·2	..	..	7091·8	10062·5	19
51	[illegible]	30	282	30	77	30	N.W.	100	2·2	0·2	..	..	..	..	97·6	0·5	4591·5	10954·0	51
52	309	15	291	15	68	45	N.W.	93	33·7	0·1	..	..	..	..	86·7	0·6	4593·9	10855·9	52
53	360	00	342	00	13	00	N.W.	200	190·2	0·3	..	..	..	..	61·8	1·0	4627·7	10768·6	53
54	350	15	332	15	27	45	N.W.	80	70·8	0·1	..	..	..	..	37·2	0·4	4818·2	10705·8	54
55	284	30	266	30	86	30	S.W.	160	..	..	9·8	0·2	..	..	159·7	0·7	4889·1	10668·2	55
56	325	30	307	30	52	30	N.W.	110	67·0	0·2	..	..	..	..	87·2	0·6	4869·5	10507·8	56
								8897	2571·8	8·5	2639·5	5·9	3328·5	..	2860·7	22·7			
57	313	30	295	30	64	30	N.W.	80	34·4	0·1	..	..	..	..	72·2	0·5	4946·7	10420·0	57
58	338	00	320	00	40	00	N.W.	60	46·0	0·1	..	..	..	..	38·6	0·3	4981·2	10347·3	58
59	333	00	315	00	45	00	N.W.	90	63·6	0·2	..	..	..	..	63·6	0·5	5027·3	10308·4	59
60	261	15	243	15	63	15	S.W	240	..	..	108·0	0·3	..	..	214·3	1·3	5091·1	10244·3	60
61	318	00	300	00	60	00	N.W	33	16·5	0·1	..	..	..	..	28·6	0·1	4983·4	10028·7	61
								9400	2732·3	9·0	2747·5	6·2	3328·5	25·1	3278·0	25·4	5000·0	10000·0	1
									+9·0		−6·2		−25·1		+25·4				
									2741·3		2741·3		3303·4		3303·4				

with X Y it will be amply accurate to augment the sides in one portion, and diminish them in the other, in the proportion of X Y to the half closing error. The distance X Y may be scaled from the plot with sufficient precision.

If the axis of correction falls mid-way between two points, the correction for the side between, must not be applied to the whole side, but to the *difference* of the two parts into which it is divided, and as though belonging to the larger part.

As this method involves no alteration of the angles, the 'latitudes' and 'departures' will be proportionate to the corresponding distances. It is merely necessary to correct each of the 'latitudes' and 'departures' in the ratio that the sides would have been corrected, that is to say in the proportion of the length of the axis of adjustment, to the half closing error. The adjustment of the side, intersected by the axis, must be deduced from the proportion of the length of the side, to the difference of the parts into which it is divided. In most cases the correction will be a negligible quantity.

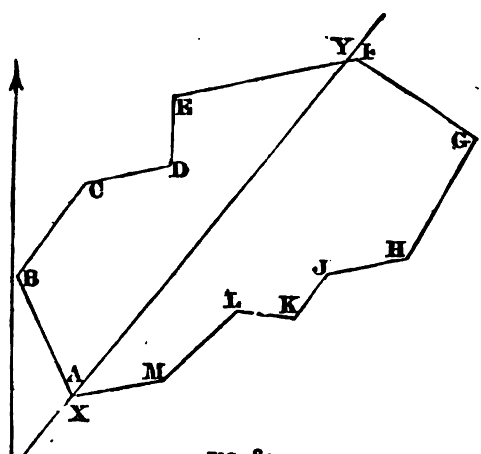


FIG. 84.

Table D, shows the results of this method, as applied to the rough traverse, already treated by Bowditch's method.

Shortrede, in the introduction to his traverse tables, suggests the following method, in which the correction is made by altering the measured data, namely the sides and angles. The traverse is 'set up' and the 'latitudes' and 'departures' computed, direct from the field-book, without any previous correction.

The difference of 'latitude' and of 'departure,' which a difference of 1′ in 'bearing' would give, with a given length of side, is tabulated in a special table, ('difference of latitude' and 'departure' for a variation of 10″ at the middle of each degree, with distances from 1 to 100).

With this, the angles are corrected so as to sum correctly whilst giving a correction in the right direction. Next, the 'difference of latitude' and 'departure' for one foot of distance with each 'bearing' are tabulated. This done, any error remaining after the correction of the angles is eliminated by lengthening or diminishing the sides in due proportion.

This method is, as the writer admits, laborious, and unnecessarily elaborate, for most ordinary cases.

If numerous abutting or interlacing polygons have to be surveyed it is well to defer final adjustment until the whole is complete, and then adjust all simultaneously, in such a manner that the correction of the common part, is the same in each polygon. Thus if the district A B C D E F G H (fig. 85) were divided into four parts by traverses meeting in a common point O, it would be prudent to defer final adjustment till the whole is finished.

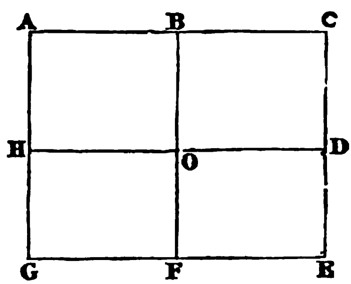


FIG. 85.

Then set up the exterior traverse A C E G, and make temporarily the necessary corrections according to any rule. Next set up A B O E, B C D O, D E F O, F G H O, and carry into each the corrections already proposed for the sides, which are common to them, and the great polygon, correcting any remaining error by adjustment of the common sides of the minor polygons, (making the same adjustment in each).

The question of the strict adjustment of a number of interlacing polygons, so as to arrive at the most probable result, is one of no small difficulty, and comes more properly within the scope of 'higher geodesy' than for consideration in the present part of this treatise. These preceding remarks will, however, serve to indicate the precautions that the surveyor should take to prevent the inconvenient accumulations of errors which may occur when a survey is made by piecing individual portions together. The leading maxim is to work from the whole to the part.

When 'bearings' have been observed directly, or when they have been computed from uncorrected angles, a correction applied to one 'bearing' to make the angles sum correctly must be applied to all subsequent 'bearings.' Thus, supposing that the final 'bearing' error of a polygon of 24 sides were 3′ and that it is desired to correct this by altering the sixth, twelfth, and eighteenth angles, by adding or deducting one minute from each. Then the 'bearings' from station 24 to station 6 will stand, those from 6 to 12 would each be increased by one minute, from 12 to 18 by two minutes, and from 18 to 24 by three minutes. By altering one 'bearing' only, without carrying forward the alteration to all subsequent 'bearings,' no correction of the summation of 'exterior' or 'interior' angles is made, for the same quantity is added to one angle and deducted from the other.

Checks on Traverses, not being 'Closed Polygons.'

Hitherto traverses which form a 'closed polygon' have been considered, and such cases will occur in surveys of estates, towns, or parishes. Whenever practicable, the traverse should be closed, so that there may be a full and complete proof of the accuracy of the work, a condition very properly demanded by the Governments of the principal colonies, for all surveys of grants of land.

Cases will occur, however, in which it is impracticable to close the traverse.

14	78	00	60	00	60	00	N.E.	300	150·0	$\bar{1}$·40	..	..	259·8	$\bar{2}$·43	..	..	6411·5	9909·9	14
15	75	15	57	15	57	15	N.E.	200	168·2	$\bar{1}$·01	..	..	168·2	$\bar{1}$·57	..	..	6560·1	10167·3	15
16	63	30	45	30	45	30	N.E.	260	182·3	$\bar{1}$·70	..	..	185·4	$\bar{1}$·74	..	..	6667·2	10333·9	16
								3068	1853·2	$\bar{5}$·27	..	..	1438·8	$\overline{13}$·42	899·4	..			
17	80	45	62	45	62	45	N.E.	300	137·4	$\bar{1}$·28	..	..	266·7	$\bar{2}$·40	..	..	6847·9	10517·6	17
50	282	15	264	15	84	15	S.W.	90	..	..	9·0	+0·08	..	..	89·5	+ 0·82	4598·7	11046·6	50
51	300	30	282	30	77	30	N.W.	100	2·2	+0·02	..	..	..	..	97·6	+ 0·91	4589·6	10956·3	51
52	309	15	291	15	68	45	N.W.	93	33·7	+0·32	..	..	..	..	86·7	+ 0·81	4591·8	10857·7	52
53	360	00	342	00	18	00	N.W.	200	190·2	+1·80	..	..	..	..	61·8	+ 0·57	4625·8	10770·3	53
54	350	15	332	15	27	45	N.W.	80	70·8	+0·66	..	..	..	..	37·2	+ 0·34	4817·8	10707·9	54
55	284	30	266	30	86	30	S.W.	160	..	..	9·8	+0·09	..	..	159·7	+ 1·49	4889·3	10676·4	55
56	325	30	307	30	52	30	N.W.	110	67·0	+0·62	..	..	..	..	87·2	+ 0·81	4879·5	10509·2	56
								8897	2571·8	$\bar{4}$·51	2639·5	$\overline{19}$·27	3328·5	$\overline{31}$·83	2860·7	+14·86			
57	313	30	295	30	64	30	N.W.	80	34·4	+0·31	..	..	..	..	72·2	+ 0·67	4947·1	10421·2	57
58	338	00	320	00	40	00	N.W.	60	46·0	+0·43	..	..	..	..	38·6	+ 0·35	4981·8	10348·3	58
59	333	00	315	00	45	00	N.W.	90	63·6	+0·60	..	..	..	..	63·6	+ 0·59	5028·2	10309·3	59
60	261	15	243	15	63	15	S.W.	240	..	..	108·0	+1·02	..	..	214·3	+ 2·00	5092·4	10245·2	60
61	318	00	300	00	60	00	N.W.	33	16·5	+0·14	..	..	..	..	28·6	+ 0·26	4983·4	10028·9	61
								9400	2732·3	$\bar{3}$·03	2747·5	$\overline{18}$·25	3328·5	$\overline{31}$·83	3278·0	+18·67	5000·0	10000·0	1
									$\bar{3}$·03		$\overline{18}$·25		31·83		18·67				
									2729·27		2729·25		3296·67		3296·67				

Traverse corrected by the Axis Method.

[*To pull out between pp.* 154 *and* 155.]

For example, the survey of a long line of railroad, or a road through an unexplored country, not previously mapped. Or, in mapping a new country, there may be long traverses which will ultimately close, but only after many months' work. Such would be the case, in the survey of the coast line of an island. The traverse would only close, after one or more seasons' work. In the meantime, some check is desirable, in order that any gross errors may be detected in time.

In the first place it is prudent to have the main lines roughly check-chained. A chain should be used of different length to that employed by the surveyor. If he uses the Gunter's chain, the check-chain might be 100 feet. Or better, the check-chain might be shortened so as to be, say only 99 links. This will prevent any bias. In a level country the perambulator might be used. By this means the surveyor will be sure that no gross error in chaining has been made.

The sun and stars afford the means of checking the angular work. The 'azimuth' of any line, *corrected for 'convergence of the meridian,'* hereafter treated of, should agree with the 'bearing' of the same, as obtained by 'angular measurement.'

The determination of 'latitudes' astronomically, does not afford an efficient check unless indeed the traverse is very extensive, for the surveyor will not, as a rule, have instruments capable of determining 'latitude' to within say 10" of arc or about 1000 feet.

A very efficient check may be obtained by observing the 'bearings' of conspicuous objects, natural or artificial, both in the direction of the line, and on either side of it.

Thus suppose that the coast line (fig. 86) were to be surveyed. Commencing at A, and having erected good signals at the promontories B, and C, the surveyor determines the 'true meridian' astronomically. He sees, in addition to the signals B, and C, a conspicuous silk-cotton tree on the ridge to the left, a cocoanut tree near the shore in the bay, a well-marked rock on the ridge which terminates on the promontory B, and a tree on the island to the eastward. He takes the 'true bearings' of these objects and proceeds to traverse the coast line, till he approaches the cocoanut tree in the bay. He cannot indeed erect his theodolite over the tree, but he can range out a point in line between it and A, and place the theodolite there, within a few feet of the tree, making the position of the theodolite the terminal point of this section of the traverse. Next he observes to the silk-cotton tree, to A, to the island, and to the signal at B. The rock, and point C, are invisible. The back 'bearing' to A, should coincide with the original 'bearing' taken from A. If the direct method of observing 'bearings' has been used, any error in angular measurement will be seen at a glance, and can be corrected. The 'distance' and 'bearing' of the cocoanut tree, from the station of the theodolite are noted. It is therefore evident that, the positions of the silk-cotton tree and the island are fixed, *approximately*. The traverse is proceeded with until B is reached. At this point 'bearings' are obtained to A, the silk-cotton tree, the cocoanut tree, the island, to station C, and to a new object, a palmiste tree. The rock cannot be distinguished. Suppose that the point B forms the termination of the day's work. The 'bearing' of A from B should coincide (± 180°) with the bearing originally obtained from A, thus checking the angular work. Then the difference 'latitude' and 'departure' of the cocoanut tree and the point near it, where the theodolite stood, and of B may be computed,

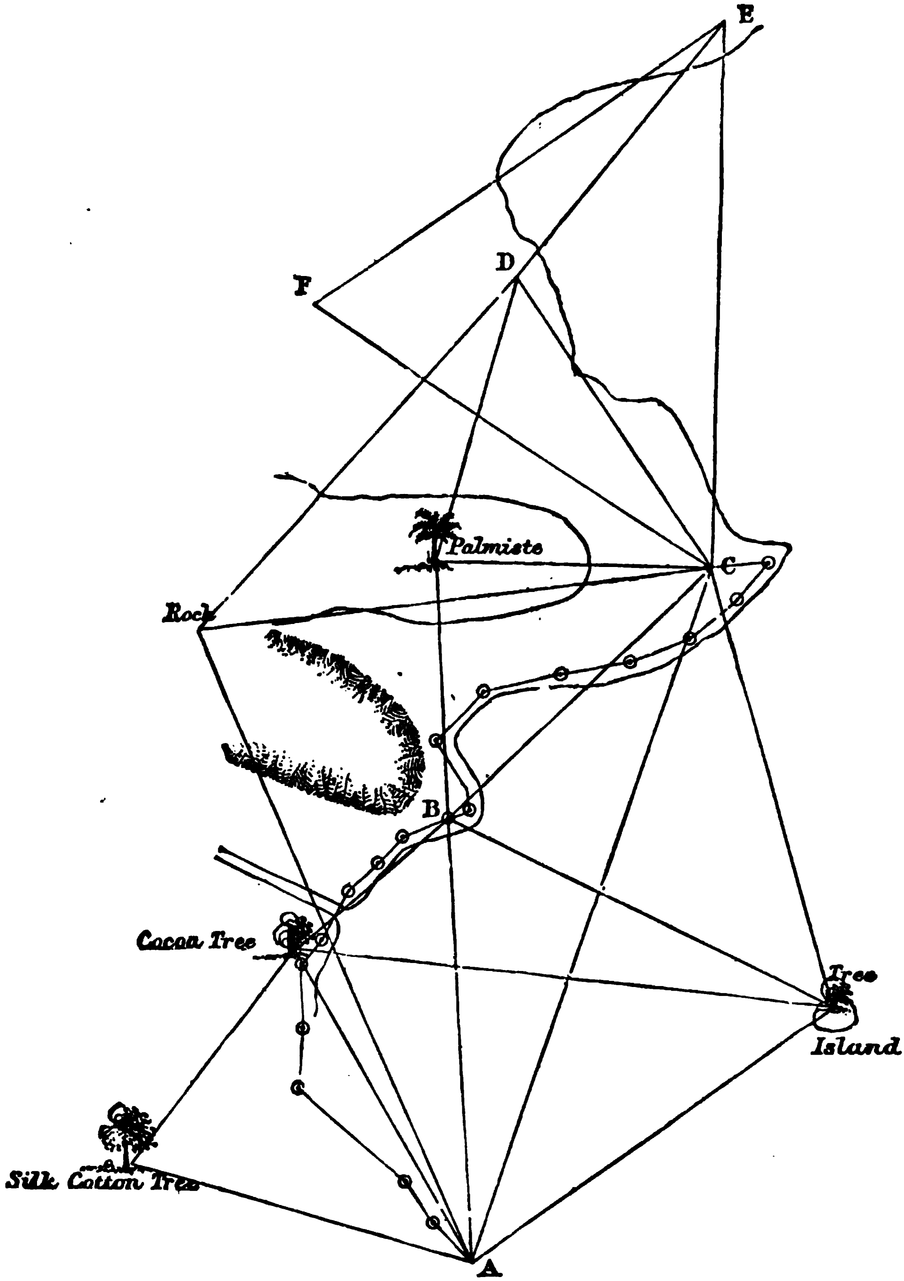


FIG. 86.

and their approximate positions pricked off. Now with a protractor, lay off the 'bearings' at A to the silk-cotton tree, the cocoanut tree, to C, the rock, and the island. Next at the point near the cocoanut tree, lay off the 'bearings' taken there to fix the several points. Then at these points, draw the 'back bearings' observed at B. Their intersections should coincide with each other, and with the determination of the position of B by the traverse. If they do so with fair accuracy, the surveyor may rest assured that his work is consistent. It is indeed affected by unavoidable error in chaining, but no *gross undetected error* can have taken place.

Starting again from B, the correct 'bearing' A B as observed from A may be used. The 'bearings' of the traverse B C will then be independent of any slight errors that may have accumulated on the way thither.

On reaching C, 'bearings' will be obtained to A, B, the rock, the palmiste, to the new stations D, E, and F, and to the island, thus giving the means of checking future work, and a fairly accurate sketch plan of the coast will be produced, as the surveyor proceeds. On this, he should sketch in by eye, the general features of the country, and the ridge and valley lines. The different stations should be numbered and lettered, and distinctly referred to, in the field-book. This sketch will greatly assist the final computation and adjustment of the traverses, an operation which may be deferred until the close of the working season. Then, the distances between the several points and their ordinates may be computed, and not merely laid off by protractor. The mean value of each point, as determined from different sources, may be taken as its approximate position.

The subsidiary points, thus determined, will be of great value, for they assist in delineating topography, such as ridge and valley lines, &c. They will be most useful in laying out grants of land, and lastly they will be of great assistance in laying out a network of triangulation, when the value of property justifies so elaborate a method of surveying.

Permanent marking of principal points.

In such work, it is hardly necessary to point out the importance of marking *permanently*, all the principal points of the traverse, such as A, B, and C. They should be marked with pillars of masonry or concrete, or with large cairns of stones, having a stake or bar of iron in the middle.

If there be rock, a hole could be drilled with a jumper and an iron or copper plug inserted in it.

In the stoneless steppes of Russia, the angles of boundaries of property are marked by digging two deep trenches intersecting each other. Into these, a foot or two of charcoal is placed, and the earth is then filled in. The boundary post or pillar is put in its proper place. The charcoal is absolutely permanent, and could hardly be removed so completely as to leave no trace. Thus even if the post or boundary-stone be removed, the position of the point could be recovered, very appoximately, by uncovering the charcoal.

By leaving permanent marks, the whole of the traverse-work becomes available for filling in a subsequent trigonometrical survey. The positions of the main points would be fixed by triangulation, and the several traverses adjusted to them. The detail would then be plotted from the original field-books.

In short, if the leading points be not marked in a permanent manner, the survey loses half its value.

Traversing with Compass and Chain.

Hitherto it has been assumed that the 'angles,' or 'bearings' of the traverse, have been measured with the theodolite. This instrument should always be used when accuracy is desired. For many purposes, however, the compass will give sufficiently accurate results more rapidly and more conveniently than the theodolite, and it requires less skill in its use than the more elaborate instrument. The compass gives the angle which a line makes with the line of magnetic force, passing through the place of observation, or what is called a 'magnetic meridian.'

The angle which the 'magnetic meridian' of any place makes with the 'true meridian' is called the 'variation' or 'declination' of the compass. Within areas of moderate size, the 'variation of the compass' may be taken as constant. In other words, the 'magnetic meridians' may be assumed to be parallel, like the meridians used in the traverse computation. The limits within which this assumption may be taken as correct, will be discussed later on.

Consequently, the 'magnetic bearings' may be reduced to 'true bearings' by adding or deducting the 'variation of the needle.'

The 'variation' may be determined astronomically, by observing the 'magnetic bearing' of some line, the 'true bearing' of which is known.

Each 'bearing,' taken by the compass, is independent of those which precede or follow it. Any error, whether due to centering, bisection of the object, or to reading, affects only the side to which it refers, and is not carried forward.

The compass, as usually made, is not susceptible of great accuracy, but with a common prismatic compass, 'bearings' may be read to about a quarter of a degree. Even when using a theodolite set up by its compass the accuracy of the 'bearings' depends on that to which the compass can be read, say ¼ of a degree.

So far as to the instrument itself, next as to the parallelism of the successive 'magnetic meridians.'

Changes in 'Magnetic Variation,' local, annual and diurnal.

Excepting over relatively small areas, these are by no means parallel. A chart of the world is now published by the Admiralty, giving the 'variation' of the needle (from the true north), at different points of the earth's surface. Lines are drawn through places where the 'variation' is found to be the same.

An inspection of the chart for 1895 shows that the 'variation' at Paris is almost exactly 15° west. At Dover it is 16° W., and at Land's End, 19° W., whilst on the west coast of Ireland in Connemara it is 23° W. From Dover to Connemara is about 360 nautical miles, as the crow flies. Hence the direction of the needle varies 7° in 360 miles, or at the rate of 1′ 10″ per nautical mile. Going eastward again, the 'variation' decreases, till at St. Petersburg the needle points due north.

It will thus be seen that the 'magnetic meridians' are by no means parallel. They are moreover subject to very considerable local distortion, probably owing to the presence of magnetic rocks either above or below the surface of the ground. These local 'variations' cannot be shown on a chart of the whole world.

The 'variation of the compass' at the same spot changes from year to year, as the following record shows.

Station.	Date.	Variation.
Paris	1580	11° 30′ east.
"	1663	0° 00′ "
"	1780	19° 55′ "
"	1805	22° 05′ "
"	1835	22° 04′ "

(For 'variation' at Greenwich, *see* Appx. C, p. 282.)

Now the 'variation' in 1895 at Paris was 15° W., decreasing about 7′ annually. Similar changes have taken place all over the world. In 1600, the compass pointed due North at the Cape of Good Hope, now it points 29½ West of North.

The accuracy of compass bearings is further reduced by the fact that there is a diurnal oscillation of the needle about its mean position.

Dr. Carl Bauerfeind, in his treatise on surveying, gives the following daily variations :—

Places.	Daily variation. Summer.	Winter.
St. Helena . .	0° 4′·06	0° 3′·03
Greenwich . .	0° 8′·16	0° 7′·02
Munich . .	0° 10′·77	0° 5′·88
Toronto . .	0° 10′·70	0° 6′·64

The same authority states that the maximum 'deviation' to the east occurs at 8 A.M., and the maximum westerly at 2 P.M. (near Munich).

The daily oscillation is greater in high latitudes then in low. It also varies in extent from year to year.

Lastly, the needle is subject to occasional oscillations of an irregular character, which are known as 'magnetic storms.' When these take place, the needle vibrates rapidly, moving as much as one degree from its normal position. Magnetic storms occur when there is an aurora borealis. Convulsions of nature, such as earthquakes, appear also to produce them.

It is clear, therefore, that the compass cannot be accepted as an instrument of precision, not so much an account of any inherent defect in the instrument itself, but owing to the fact that the starting point of the angular measurement, the 'magnetic meridian,' is constantly varying.

It is also evident, that for ordinary surveying purposes, it is not advisable to sacrifice the simplicity of the compass, in order to obtain what is after all an illusive degree of accuracy in taking readings.

For exploratory work, military reconnaissances, and for filling in details of surveys, especially when they are to be plotted to a small scale, the compass is invaluable.

Many cases will arise in the practice of the surveyor, in which, by division of labour, and by judiciously proportioning the degree of accuracy to the requirements of the case, always working from the whole to the part, and making the measurement of the whole, more accurate than that of the several parts, compass surveys can be made at a cost that is admissible, which would not be the case if all details were surveyed with the skill, care, and accuracy, that is necessary for the main polygon.

It may be well to point out the objections to a practice which obtains in many of the minor colonies. In several cases a true north and south line has been set out in some public place, in order that surveyors may determine the variation of their compasses. The variation so obtained is exact, *only* at the *station* and *time* of observation. It by no means follows that the same variation of the compass will obtain at some other point a few miles away, and at some other hour or date.

It is therefore absolutely necessary that a surveyor using a compass should be capable of determining its variation on the spot by astronomical observation. It will be seen in a chapter devoted to this subject that this operation really presents no great difficulty (Part II.).

Areas of closed Polygons.

The area of any closed polygon may be determined direct from the traverse sheet, without plotting, and therefore free from all errors in scaling off distances, and from those due to shrinkage of paper.

It is evident that the area of the polygon A B C D E F G (fig. 87) is equal to that of the figure c C D E F G g less the figures c C B A G g.

The figure c C D E F G g is composed of the figures c C D d, d D E e and so on. Similarly for the figure c C B A G g. Again the areas of the small figures are $\frac{c\mathrm{C} + d\mathrm{D}}{2} \times cd$, $\frac{d\mathrm{D} + e\mathrm{E}}{2} \times de$, $\frac{e\mathrm{E} + f\mathrm{F}}{2} \times ef$, and $\frac{f\mathrm{F} + g\mathrm{F}}{2} \times fg$.

The area of the polygon may therefore be obtained by the following precept.

Add together the 'perpendiculars' of the successive pairs of points. Multiply each 'sum' by the 'difference of latitude' between the two points, making the 'products' as '*north-products*' and '*south-products*' respectively, according as the multiplier is a '*northing*' or a '*southing*,' then,

Sum the 'north products' and the 'south products,' and half the 'difference' of these 'sums' will be the 'area of the figure.'

Three cases may occur:

1st. The '*meridian*' and '*origin*' may be entirely outside the polygon, so that all '*meridional distances*,' '*differences of latitude*,' and '*perpendiculars*' are respectively either all '*northings*,' or all '*southings*,' all '*eastings*,' or all '*westings*.'

2nd. The '*origin*' may be outside, but the '*meridian*' may traverse the polygon. The '*meridional distances*' will either be all '*northings*' or all '*southings*,' but the '*perpendiculars*' will be some '*eastings*' and some '*westings*.'

3rd. The '*origin*' may be within the polygon. Some of the '*meridional distances*' and the '*differences of latitude*' will be '*northings*,' some '*southings*,' and some of the '*perpendiculars*' will be '*eastings*,' some '*westings*.'

The first case presents no difficulty. The sum of the '*north products*' will be greater or less than the sum of the '*south products*,' according to the quadrant in which the polygon is, and according to the order in which the successive points are taken. The lesser sum must in any case be deducted from the greater.

The second case may be treated independently, as follows.

Assume two auxiliary points X and Y (fig. 88), in which the meridian cuts the lines A B and D E respectively. The 'departure' of X and Y will evidently be zero. The 'latitudes' may be computed as follows.

As the 'departure' of A is to the sum of the departures of A and B, so is the difference of 'latitude' between A and X, or a X, to the 'difference of latitude' between A and B.

The 'latitude' of Y is determined in the same manner

The polygon is then divided into two, X B C D Y and Y E A X, each of which may be treated by the above rule.

In the case shown in the sketch, the first sum is zero + B *b*, which is to be multiplied by X *b*, the 'difference of latitude' between X and B found as described.

The last sum is *d* D + zero, and it is multiplied by the 'difference of latitude,' Y *d*. The last side of the right-hand half of the polygon is the 'difference of latitude' Y to X, which need not be taken into account because the 'departures' of X and Y are zero.

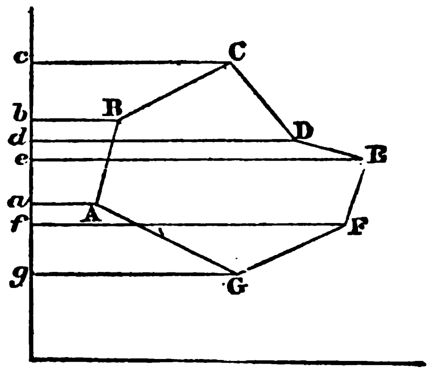


FIG. 87.

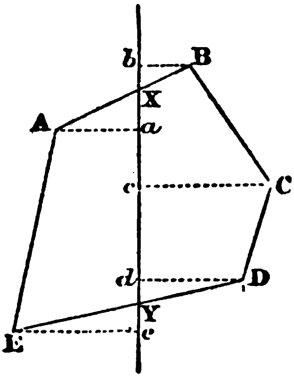


FIG. 88.

The left-hand half Y E A X may be treated in the same manner, and the sum of the two areas will be the area of the whole figure.

A little consideration will show that the area may be obtained without separate *additions* and *subtractions*, but merely by reversing the order in which the points are taken in one half of the polygon. Thus, in the right-hand half, if we proceed in the direction X B C D Y, 'south products,' such as (*d* D + *c* C) × *d c*, &c., are *greater* than the 'north products,' the only one of which is (zero + *b* B) X *b*.

If, on the other hand, we proceed round the left-hand half in the direction X A E Y, the 'south products' are *greater* than the 'north products.'

Consequently, the successive 'products' may be taken out in the original direction and the area will be obtained merely by reversing the 'products,' by putting the 'north product' in one half, under the 'south products' in the other half, and *vice versâ*.

The third case may be treated in a precisely similar manner.

Probably the easiest way of treating the second and third cases is to refer the points to a line parallel to the '*meridian*,' and to an '*origin*' outside the polygon, by adding to, or deducting from the '*meridional distances*' and '*perpendiculars*,' a constant quantity, and thus transferring them from the '*prime meridian*' running through the polygon, to an '*auxiliary meridian*' and '*origin*' outside.

The computation of areas from 'latitudes' and 'departures' is most accurate. The results are obtained directly by computation. The area may be taken out without even plotting the survey, and certainly without any measurement on the plan. It is, therefore, free from errors due to incorrect plotting, incorrect measurement on the plan, or to shrinkage of paper.

If the polygon be a survey of an estate or district, the actual boundary may not coincide with the principal survey lines. It will more probably be an irregular figure, sometimes *inside*, sometimes *outside*, the sides of the polygon, and determined by offsets from the main lines, as in chain surveying.

Even in this case the area of the whole survey may be determined without plotting on paper. The area of the strips between the main survey lines, may be taken out direct from the field-book, by adding together successive pairs of offsets and multiplying them by the distance between the successive offsets.

Half the sum of the products will be the area of the strip, which will be additive or subtractive according as it is outside or inside the main polygon.

A complete traverse with a table of offsets, therefore, forms a reliable record of both the form and area of a property without being plotted on paper at all. It may be plotted at any time from these data, and to any desired scale without the slightest risk of confusion or error.

Problems connected with Traverse Surveying.

1. Given the 'co-ordinates' of two points, that is to say their 'meridional distances' and 'perpendiculars' respectively, *to find the bearing of one point from the other.*

Find the 'difference of latitude' of each point. If their 'meridional distances' are of the same name as in cases 1 and 2 (fig. 89), this will be effected by deducting the smaller from the greater. If of opposite name as in case 3, then they must be added together.

Then find the 'departure' in like manner, *subtracting* when of the same name and *adding* if of opposite names.

Lastly, divide the 'departure' by the 'latitude,' and the quotient is the tangent of an angle which can be taken from the table. If we denote this angle by θ, the bearing can then be deduced in the four cases which may arise, as follows:—

1.	Departure 'Easting,'	Diff. of Latitude 'Northing,'	$\theta =$	bearing.
2.	„ „	„ „ „ 'Southing,'	$180^\circ - \theta =$	„
3.	„ 'Westing,'	„ „ „ „	$180^\circ + \theta =$	„
4.	„ „	„ „ „ 'Northing,'	$360^\circ - \theta =$	„

If the 'Departure' be much greater than the 'Difference of Latitude,' it will facilitate reference to the tables to divide the 'Difference of Latitude' by the

'Departure' instead. This will give the tangent of an angle (ϕ), from which the bearing can be deduced as follows :—

1.	Departure 'Easting,'	Diff. of Latitude 'Northing,'	$90^\circ - \phi =$	bearing.
2.	" "	" " " 'Southing,'	$90^\circ + \phi =$	"
3.	" 'Westing,'	" " " "	$270^\circ - \phi =$	"
4.	" "	" " " 'Northing,'	$270^\circ + \phi =$	"

2. *To find the 'distance between two points,'* the 'co-ordinates' of which are known.

Find the 'differences of latitude' and 'departure' as in case 1.

Square each, add the squares together, and take the square root of the sum, this will be the '*distance*' required.

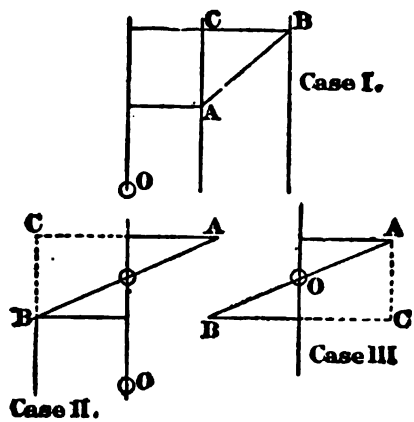


FIG. 89.

This procedure is laborious, unless an extended table of squares is at hand. Even the use of logarithms does not greatly facilitate matters, unless one has a table of '*sum and difference logarithms*,' which but few logarithmic canons contain.

Generally, when the 'distance between two points' has to be found, the 'bearing' also is required. Consequently the following procedure is more practical.

Compute the 'bearing' as in (1). Then the 'distance' is obtained by dividing the 'departure' by the *sine* of the *bearing*, or the 'difference latitude' by the *cosine of the bearing*. Or in other words multiply the 'departure' by the *cosecant* of the *bearing*, or the 'difference latitude' by the *secant of the bearing*. In practice, the choice between these rules is decided by the principle, that it is desirable to work from the larger of two known quantities. Therefore, if the 'departure' be larger than the 'difference latitude' work from it, and *vice versâ*.

3. *To find the 'meridional distance' of a point on a certain line* (the 'co-ordinates' of whose terminal points are given), *that shall have a given 'perpendicular,' and the converse.*

This is a case which occurs very frequently in plotting extensive traverses, when the line to be plotted extends beyond the fixed margin of the sheet. It is therefore desired to find either the 'meridional distance' or the 'perpendicular' of the point in the line in which it cuts the margin line of the sheet.

Find the 'differences of latitude, and departures,' of the terminal points as in case (1). Suppose that the line cuts the 'eastern' or 'western' margin of the sheet, then the 'meridional distance' of the intermediate point is sought, together with its 'perpendicular' (namely the perpendicular of the margin line).

Find the 'departure' of the desired point from one or other of the given points.

Then, *as the 'departure' of the two given points, is to the 'departure' of the point sought, from one of the given points, so is the 'difference latitude' of the given points, to the difference latitude of the point sought, from that same point.* By adding the 'difference latitude' to, or subtracting it from, the 'meridional distance' of the starting point, the 'meridional distance' of the intermediate point to be laid off on the margin line, is found.

If the top or bottom margin be cut then the 'perpendicular' of the intermediate point may be found by the same rule, merely substituting '*difference latitude*' for '*departure*.'

The 'differences latitude and departures' will be found from the traverse sheet by applying any correction that has been necessary, to the calculated value of these quantities.

The 'corrected bearing of the line' will also be found in the traverse sheet. The 'meridional distance' of an intermediate point can be found by multiplying the 'departure' of the intermediate point, from the known point, by the *tangent of the 'bearing*,' and the 'perpendicular' by multiplying the 'difference latitude' by the *co-tangent of the 'bearing*.'

An example is here inserted, showing how to calculate an area direct from the Traverse Sheet and Field-Book, being the calculation of the area of the field A (fig. 73), (*vide* 'Example of Traverse Survey,' plate II.).

In this case, as the points *h* and Y are not given on the traverse sheet, we must first calculate their co-ordinates.

From the field-book, H*h* = 15′, H I = 1274′, hence the 'Difference Latitude' and the 'Departure' of *h* from H may be found by multiplying those of I from H (which are got from the traverse table) by the fraction $\frac{15}{1274}$ or ·01177.

The 'Difference Latitude' of *h* from H will therefore be,

$$165 \cdot 1 \times \cdot 01177 \text{ or } 1 \cdot 9 \text{ N},$$

and the 'Departure'

$$1263 \cdot 0 \times \cdot 0177 \text{ or } 14 \cdot 9 \text{ E}.$$

Similarly, we find that for G from Y,

the 'Difference Latitude' = 54·7 N
and 'Departure' = 242·0 W.

We can now construct the following subsidiary table by reference to the main traverse sheet:—

Point.	Latitudes.		Departures.		Co-ordinates.	
	North.	South.	East.	West.	North.	East.
H	1·9		14·9		34·9	937·0
h	532·4		206·8		36·8	951·9
Y	54·7			242·0	569·2	1158·7
G					623·9	916·7

The other points are taken from the main traverse sheet, and the area of the polygon O O_1 P H *h* Y G M N may then be calculated as follows:—

CALCULATION OF THE AREA OF THE POLYGON O O_1 P H *h* Y G M N.

Points.	Perpendiculars.	Sum of Perpendiculars.	Latitudes.		Products.	
			North.	South.	North.	South.
H *h*	937·0 951·9	1888·9	1·9	..	3588·9	..
h Y	951·9 1158·7	2110·6	532·9	..	1124738·7	..
Y G	1158·7 916·7	2075·4	54·9	..	113939·5	..
G M	916·7 697·9	1614·6	358·2	..	578349·7	..
M N	697·9 233·3	931·2	..	386·4	..	359815·7
N O	233·3 88·3	321·6	..	345·2	..	111016·3
O O_1	88·3 231·2	319·5	..	134·1	..	42844·9
O_1 P	231·2 450·5	681·7	..	14·4	..	9816·5
P H	450·5 937·0	1387·5	..	66·5	..	92273·8
					1820616·8	615767·2
					615767·2	
					2)1204849·6	
Area in square feet	. . .	. . .	. .	. .	602425	

The area of the ground between the survey lines and the fence can now be calculated either direct from the field-book, or by 'equalising boundaries' where the fence is nearly straight, and using only the extreme ordinates or offsets for each straight bit, remembering that the line *h* Y is outside of the fence, and hence the area computed from the field-book with that line must be subtracted. In the following computation the boundaries in some places, particularly from O_1 to H, have been equalised (*vide* fig. 73).

CALCULATION OF AREAS BETWEEN THE SURVEY LINES AND FENCES.

Number.	Ordinates or Offsets.	½ Sum of Offsets.	Length.	Area.	Remarks.
1	0 / 20·25	10·125	9	91	Line Y G.
2	20·25 / 49·25	34·75	223	6749	
3	38 / 24·5	31·25	310	9687·5	G M.
4	24·5 / 16·5	20·5	32	656	
5	16·5 / 25·5	21·0	45	945	
6	25·5 / 25	25·25	13	328	
7	5·75 / 17·25	11·5	179	2058·5	M N.
8	17·25 / 40·5	28·875	52	1501·5	
9	40·5 / 36·5	38·5	21·5	828	
10	36·5 / 29·5	33	13·5	445·5	
11	29·5 / 29	29·25	58	1696·5	
12	29 / 4	16·5	184·5	3044	
13	4 / 21·5	12·75	130	1647·5	N O.
14	21·5 / 19	20·25	244·5	4951	
15	19 / 20	19·5	17	331·5	
16	20 / 9	14·5	30	435	O O₁.
17	9 / 20·8	14·9	75·5	1125	
18	20·8 / 8·75	14·775	83	1236	
19	7 / 8·5	7·75	220	1705	O₁ P.
20	10 / 27	18·5	360	6660	P H.
21	27 / 7	17·0	142	2414	
				48535·5	

CALCULATION OF AREAS BETWEEN THE SURVEY LINES AND FENCES—*continued.*

Number.	Ordinates or Offsets.	½ Sum of Offsets.	Length.	Area.	Remarks.
1	7 / 42	24·5	264	6468	A Y (to be subtracted).
2	42 / 28·5	35·25	136	4796	
3	28·5 / 13	20·75	149	3092	
4	13 / 16	14·5	15·5	225	
	Deduct			14581	
	Area in square feet			33954·5	
	Area of polygon			602425·0	
	Total area in square feet, with polygon			636379·5	

The effect of the spherical form of the Earth on Survey problems.

At the commencement of this chapter it has been stated that in all points but one, namely, the convergency of the meridians, even a large area may be treated as a plane, without introducing any serious distortion. Further it has been stated that no other course is practicable than to divide up a large survey into a number of small areas, each of which is so small, that within it, the spherical form is negligible, and such is obviously a necessary procedure since a spherical surface cannot be unwrapped, and flattened, without either stretching or tearing. The only plan therefore, is to treat the globe as though it were a many faced solid, each face exceedingly small in proportion to the whole surface, so small that the difference between the spherical and the plane surface, in each little element, is negligible.

It is now desirable to see how large these small areas may be, without producing any appreciable distortion. This can be done by means of a simple numerical calculation.

Data, treating the Earth as a Sphere.

For this purpose the earth may be assumed to be a true sphere, having the following mean dimensions.

	feet.	
Earth's mean radius	20,889,000	log 7·3,199,176.
Arc subtending one degree	364,582	„ 5·5,617,950.
Arc subtending one minute	6076·36	„ 3·7,836,437.
Arc subtending one second	101·273	„ 2·0,054,925.

(Rankin's 'Civil Engineering.')

These arcs are the distances measured on the surface of the earth, between points which subtend an angle of one degree, one minute, or one second respectively, at the centre of the sphere. For example, two lines, drawn from the centre of the earth to two points 6076·36 feet apart, measured on the surface of the earth, would (on the above assumption) be inclined to each other at an angle of one minute, whatever be the relative direction of one point from the other. The nautical or sea mile, is the length of one minute of a degree of latitude at the mean level of the sea, its mean value being nearly 6076 feet, though a value commonly taken for the nautical mile is that of one minute of longitude at the equator, or 6086 feet.

'Great Circles,' 'Meridians,' and 'Parallels of Latitude' described.

A 'great circle' is any circle on the surface of the sphere whose centre is in the centre thereof, or in other words it is the section of a sphere by a plane passing through the centre. 'Parallels of latitude' are not great circles, their centres are at various points in the polar axis, and they have all shorter radii than great circles.

The property of the 'great circle,' most important to the surveyor, is that any 'straight line,' as ranged in the field, is a portion of one. If the line joining any two points on the ground were prolonged indefinitely, a great circle would be described, and the surveyor would go round the globe, and return to the starting point. Again, if a theodolite were set up at any point (excepting on the equator) and so adjusted that when the index was at zero the telescope pointed *due north*, then, if the index were set at 90°, the telescope would now point *due east*. Suppose now that, with the telescope last named, two pegs were driven in its exact line of collimation, and the line so marked out were prolonged, by ranging continually over land and sea, it would be found that after some considerable distance had been set out, the direction was no longer *due east*, but appreciably to the *south* thereof.

Any number of pegs, marking such a line, would appear to be in an exactly straight line, but the azimuth of each peg from the next would, if determined astronomically, vary continually. This can easily be seen by applying a string to a terrestrial globe.

The only 'great circle' which always makes the same angle with a given meridian is either the equator, or is itself a meridian.

On the other hand, if a line were set out, in short lengths, from any point—not on the equator, and each successive short length were laid out exactly east or west, (as with a perfectly correct compass for example), then the points on that line would not form a great circle, but a parallel of latitude. On the ground, the successive pegs would not be in a straight line, but in a curve, the radius of this curve being less, as the latitude increases. It would run through all places having the same latitude as that of the starting point, ultimately returning to it. If the starting point were on the equator, then, as the line cuts all the meridians at right angles, the equator (itself a great circle) would be set out. An example of the curvature of a 'parallel of latitude' will be given later on.

If a line were set out and continued in some direction other than *due* north, south, east, or west, so as to cut each successive meridian at the same angle, then, (as a glance at a globe will show) this line, if prolonged indefinitely, would form a spiral, circling round the globe, with diminishing radius or curvature, always approaching, but never reaching either pole. The turns of the spiral become more and more rapid as the pole is approached.

A portion of such a spiral is traced by a ship, when she steers any course but N, S, E, or W. If steered either *due east* or *due west* she would trace out a 'parallel of latitude' and return to the starting point. It is a property of Mercator's projection of the earth, that these spirals or 'courses' are projected as straight lines. Hence the use of this, otherwise misleading form of projection, in navigation. The difference of the spiral 'course' from the 'great circle,' may be seen by comparing a map on Mercator's projection with a globe.

Measured distances, parts of 'Great Circles.'

Distances which the surveyor measures on the ground are therefore arcs of 'great circles.' In projecting them on the plan, as straight lines, no error is committed as regards the true distance from one point to another. If, however, numerous points were taken and the several distances from one to the other measured with absolute accuracy, so as to build up a network of triangles or polygons, the various triangles or other figures, which the several measurements form, could not be fitted together, on account of the spherical form of the earth. It remains to be demonstrated how such measurements can be adjusted so that a series of triangles can be fitted together on paper, &c. *Vide* Part II. of this treatise.

CHAPTER V.

MINOR TRIANGULATION.

Definition of term, 'Minor Triangulation.'

THE present article is entitled 'Minor Triangulation,' because the methods to be indicated are not so complicated, or minutely accurate, as those employed in dividing up a 'grand' or primary triangulation. When a survey of a great country is made, it is usual to commence by determining, by triangulation, the position of a number of points at distances from each other of from 20 to 100 miles. This work is executed with the highest degree of accuracy possible. The most powerful instruments are employed, and the spherical form of the earth is taken into account. A description of the methods necessary to attain this high degree of accuracy does not come within the scope of this present treatise.

The grand triangulation being finished, the topography of the country is delineated by fixing a number of secondary points. The great triangles are broken up, as it were, into a number of smaller triangles, having an average side of one mile or less. In this 'minor triangulation' instruments of a less powerful character are employed, and the spherical form of the earth is neglected. The methods which will now be described, are those suitable for such a 'minor triangulation.' They will, however, suffice for a survey of an area of 100 square miles or more, and will afford the means of preparing a map of a considerable district, to any desired scale, possessing sufficient accuracy for all practical purposes.

Explanatory Remarks.

In '*chain surveying*,' the positions of points are determined by linear measurement only, and in '*traversing*' by one lineal and one angular measurement. In a 'triangulation,' there is but one lineal measurement, that of *the base line*, from which the relative distances of numerous points from each other are determined, and their co-ordinates calculated.

Triangulation depends upon the well-known trigonometrical rule, that if one *side* of a triangle, and *two of its angles* are measured, the remaining *angle* and *sides* can be calculated, and that, treating the triangles as 'plane,' and not 'spherical,' the three angles sum to 180°. Consequently, if the two angles A and B of the triangle A B C (fig. 90) are measured, the third angle C is obtained by deducting the sum of A and B from 180°. Calling the sides which subtend the angles A, B, C :—*a*, *b*, and *c* respectively. Then by trigonometry,

Method of obtaining and collecting Angles, and computing Sides of Δ's indicated.

$$a : b : c :: \sin A : \sin B : \sin C.$$

Now, suppose that the base *c* is known,

$$\text{Then } a = \frac{c \sin A}{\sin C} \qquad b = \frac{c \sin B}{\sin C}, \tag{1}$$

$$\text{or } a = c \sin A \operatorname{cosec} C \qquad b = c \sin B \operatorname{cosec} C. \tag{2}$$

Though not absolutely necessary, it is desirable to measure the third angle of the triangle. This not only gives a check on the accuracy of the angular measurements, but materially increases the probable accuracy of the work. The three angles will not, in all probability, sum to 180°, but to a few minutes or seconds more or less. The *excess* or *defect*, *over* or *under* two right angles, is called the '*summation error*' of the triangle. This should not exceed some prescribed amount, dependent on the capacity of the instrument used, and the care and skill of the observer.

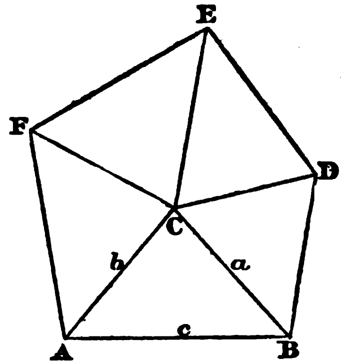


FIG. 90.

Before calculating the sides a and b, the three observed angles A, B, and C, must be corrected by adding to or deducting from each, a small angle, so that the corrected angles sum exactly to 180°. In any given triangle there is no particular reason why one angle should be more in error than another, nor is the probable error in any way dependent on the magnitude of the angle, as it is on the length of linear measurements. So, as far as a single triangle by itself is concerned, the summation error may be divided by three, and the correction so obtained, applied to each angle, in such manner as to make the corrected angles sum to 180° 0′ 0″. Although this simple method of correction suffices for a single triangle, it will be shown hereafter, that when a considerable number of triangles have to be corrected, a more elaborate method of determining the proper correction to be applied to each angle, becomes necessary.

With a series of Δ's (*vide* fig. 90), proceed as follows.

Having corrected the three angles, A, B, and C, the sides a and b, are computed. Then with side 'a' or C B known, the sides C D and B D in the triangle C B D are computed. Proceeding in the same way, the positions of the remaining points in the area to be surveyed, are determined from the first measurement of A B.

Limit of Accuracy attainable under different conditions, discussed.

Before describing the process of triangulation, it will be well to examine superficially, the limit of accuracy attainable under different conditions, and the effect of the form of the triangle thereon.

Let A and B be two points on the meridian, C a third point at a distance from it (*vide* fig. 91). Let A B be 10,000 units in length.

At A and B angles are measured to C. The angle at C is not supposed to be measured. For simplicity in calculation, it will be assumed that the angles at A and B are equal, and that the angle at C is obtained by deducting the sum of A and B from 180°.

Now every angle, however measured, is liable to some error, smaller or greater, according to the power of the instrument, and the care employed in using it. In the present case let the limit of error be ± 30″. That is to say if an angle measured, say 49° 32′ 30″, it might in reality be anything between 49° 33′ 0″ and 49° 32′ 0″.

The rays from A and B to C, may therefore be regarded as two rods, pivoted at A and B, and capable of moving through an angle of 30″ on each side of the true angles to C, at which point they intersect.

It is evident, however, that as each angle is liable to any error up to 30″, the true position of the vertex may be anywhere within the area C_1 C_2 C_3 C_4, according as *both* angles have been observed *too large* or *too small*, or *one too large* and

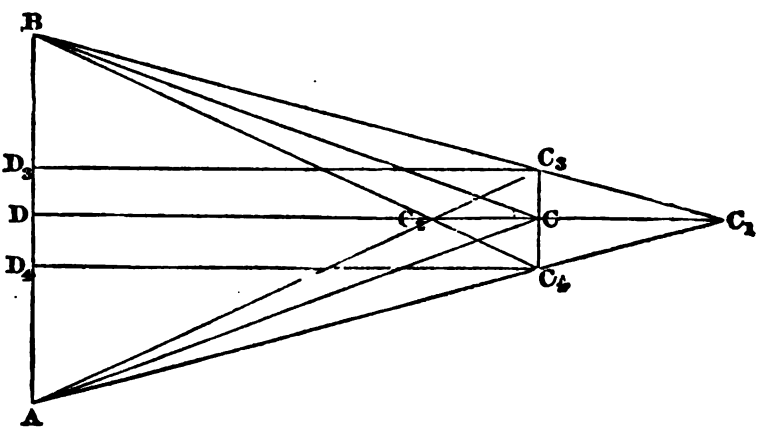


FIG. 91.

the *other too small*, and all that can be asserted is that C is somewhere within that area.

Draw C_2 D through the points C_1 C and C_2 perpendicular to A B, also C_3 D_3 and C_4 D_4. Then D C is the *apparent* 'perpendicular distance' of C, and A D the apparent 'meridional distance.' But the real 'meridional distance' may be anything between A D_4 and A D_3, and the 'perpendicular' anything between D C_2 and D C_1. The line C C_3 = C C_4 may be called the *range of uncertainty* in the 'meridian,' and C C_1 = C C_2 (approximately) the *range of uncertainty* in the 'perpendicular.' The figure C_1 C_3 C_2 C_4 may be called the area of uncertainty. We may also call the ratio $\frac{C\,C_1}{D\,C} = \frac{D\,D_3}{A\,D}$ the *relative uncertainty* or *rate of probable error* in the 'perpendicular' and 'meridian' respectively.

Effect of errors of ± 30″ on various Isosceles Δ's.

The following Table, gives the actual and relative errors for various values of the angles observed, at the extremities of the base of an isosceles triangle. The base is supposed to be ten thousand units in length.

TABLE SHOWING DIFFERENCES IN THE POSITION OF THE VERTEX OF ISOSCELES TRIANGLES OF DIFFERENT PROPORTIONS, DUE TO A VARIATION OF ± 30″ IN EACH OF THE ANGLES ADJACENT TO THE BASE OF 10,000 FEET.

1 A and B Angles at Base.	2 C Angle at Vertex.	3 A D Meridional Distance.	4 CC_3 & CC_4 Difference in Meridional Distance due to a Variation of ± 30″ in Lateral Angles.	5 Difference per 1000 in Length of A B.	6 C D Perpendicular Distance C from A.	7 CC_1 & CC_2 Difference in Perpendicular Distance due to a Difference of ± 30″ in Lateral Angles.	8 Difference per 1000 in Length of C D.
° ′	°						
87·30	5	5000·00	8·344	1·668	114,518·00	{ 383·49 380·95	3·348 3·326
85·00	10	”	4·188	0·837	57,150·00	{ 95·89 95·57	1·672 1·679
82·30	15	”	2·809	0·562	37,978·77	42·70	1·124
75·00	30	”	0·581	0·290	18,660·25	10·85	0·581
67·30	45	”	0·411	0·205	12,071·06	5·81	0·412
60·00	60	”	0·335	0·168	8,660·22	2·91	0·336
52·30	75	”	0·301	0·150	6,516·13	1·96	0·301
45·00	90	”	0·291	0·145	5,000·00	1·45	0·291
37·30	105	”	0·301	0·150	3,836·63	1·15	0·301
30·00	120	”	0·335	0·168	2,866·75	0·97	0·336
22·30	135	”	0·411	0·205	2,071·07	0·85	0·412
15·00	150	”	0·581	0·290	1,339·74	0·77	0·581
7·30	165	”	2·809	0·562	658·26	0·74	1·124

An inspection of this table shows that the *actual*, as well as the *rate of error*, is least when the angle at the vertex is a right angle. As the angle varies from a right angle, the *actual* and *the rate of error* increases slowly at first, then more and more rapidly, tending to become infinite as it approaches zero or 180°. To make this more apparent the values of $\frac{C\,C_3 \times 1000}{A\,B}$ and $\frac{C\,C_1 \times 1000}{D\,C}$ have been plotted as ordinates of a curve, the several values of the angle at the vertex being the abscissæ (*vide* fig. 92).

From this we may see that between

$$C = 25^\circ \quad \text{and} \quad C = 155^\circ$$

the rate of error due to an error of ± 30″, in the angles A and B, is always less than 1 per 1000, which has been taken as the error of good ordinary chaining.

We likewise see the advantage of measuring the third angle C. When this is deduced from A and B its value may vary between the limits $C_2 = C + 1'\ 0''$ to $C_1 = C - 1'\ 0''$. But if C be measured we shall obtain its value to within ± 30″, so that the true position can be neither at C_1 nor C_2, but at some point nearer to C.

It is obvious that similar conditions obtain when the triangle is scalene.

So far, we have assumed that A and B are fixed points, and that A B is fixed both as to direction and length. But when a number of triangles are built up as a network of triangulation, such as A B C D E (fig. 93), every angle is equally

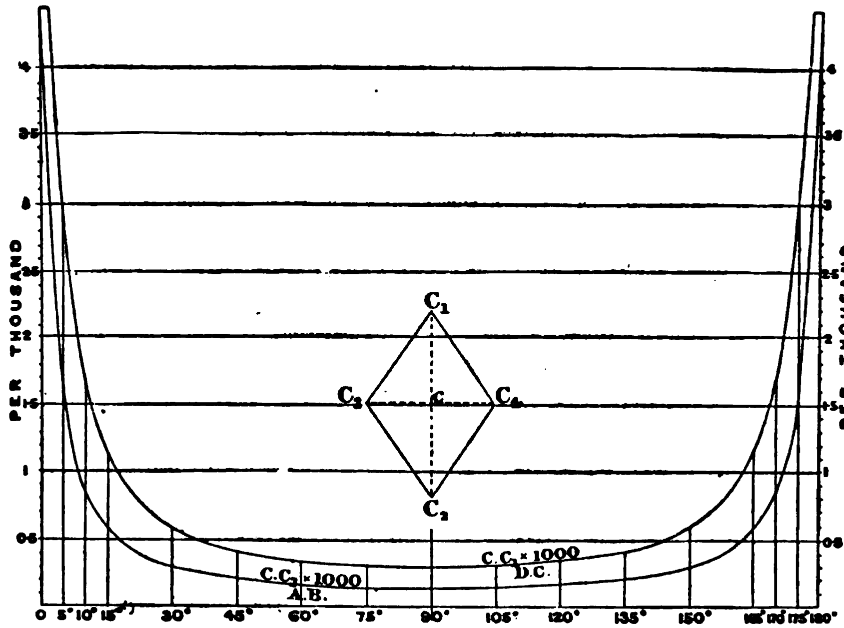


FIG. 92.

liable to error, and also every side, except some one primitive side, such as A B, whose direction and length is assumed to be fixed.

Each triangle, therefore, may be considered as being composed of three slender rods, pivoted at their middle points (fig. 94), and which may be moved

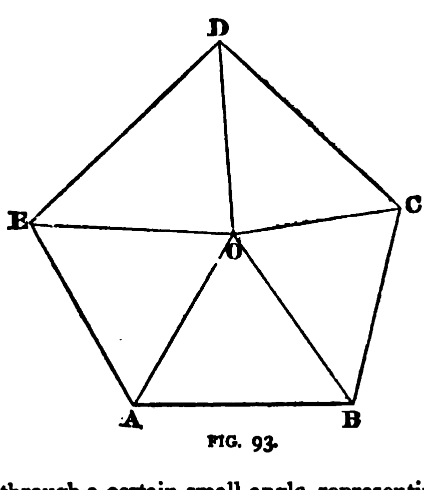


FIG. 93.

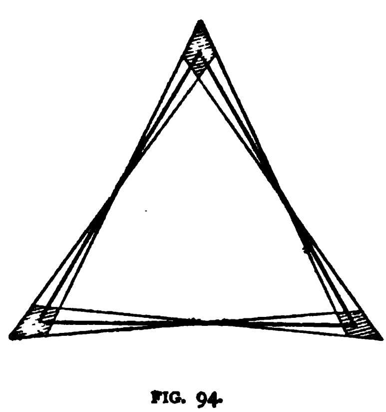


FIG. 94.

through a certain small angle, representing the probable error in the measurement of any angle. The intersections of these rods in their limiting positions, mark out the three areas of uncertainty, within each of which the true position of the point must lie.

Equilateral Δ's the best. When the triangle is equilateral the three areas of uncertainty are equal. It is true that an angle of 60° does not give the smallest area of uncertainty. But if one of the angles of a triangle exceeds 60°, one or both of the others must be less than 60°. An inspection of the 'table, and curve,' shows that the rate of error increases much more rapidly as the angle becomes less than 60°, than it decreases as the angle becomes greater than 60°. Consequently, an equilateral triangle is that in which a given error in each of the three angles, produces on the average of all three sides, the least error, and therefore, the surveyor should endeavour to make the triangles of a network, as nearly equilateral as possible. He will not as a rule, be able to do so exactly, but the nearer to the equilateral, the greater the accuracy to be attained with any given power of instrument.

Sketch of Δ'n of Malta. The sketch here inserted (fig. 95) shows the triangulation of Malta, and is a fair sample of what can be attained in practice, without going to the expense of building artificial stations.

Another System of carrying out a Δ'n. Triangulation may also be carried out as follows. Let A B be the known base, and C, and D, two points whose positions have to be determined (*vide* fig. 96). The several angles at A, B, C, and D, are observed. The three angles of each of the triangles A D B, A C B, A C D, B C D, are obtained by deduction, and each corrected for summation error. Then C A and C B may be obtained from the triangle A C B, and A D and B D from A D B.

C B being known, C D may be computed from the triangle C D B. It may also be obtained from A D in A D B. Similarly, two values of each of the five lines A C, C D, D B, C B, and A B, may be obtained, which would not in all probability be identical. The mean of the two values of each might be taken as the true value, and laid off with the 'beam compass.' Or, both values of each line might be laid off, by striking arcs. These arcs would not in all probability cut in a point, but would form small triangles or rectangles, the centre of magnitude of each of which may be taken as the true position of the point. A very good check on the accuracy of the work would thus be obtained. The process would, however, be laborious, as four triangles have to be computed to determine two points. Plotting by 'beam compass' is also troublesome, owing to the expansion and contraction of the paper.

It would be better, perhaps, to treat such a figure as a traverse, the sides of which are obtained by computation, and the angles by observation, but even then the 'observed angles' would require adjustment.

If a series of triangles be regarded as building up a closed polygon, and the angles not merely corrected to sum 180°, then all requisite checks may be obtained, without the necessity of making two computations for each side.

Fig. 97 shows a network of thirty-four triangles. A B is the known base. If the measurement of the angles were absolutely perfect, the final result obtained by computing a series of triangles successively in any order, must be the same.

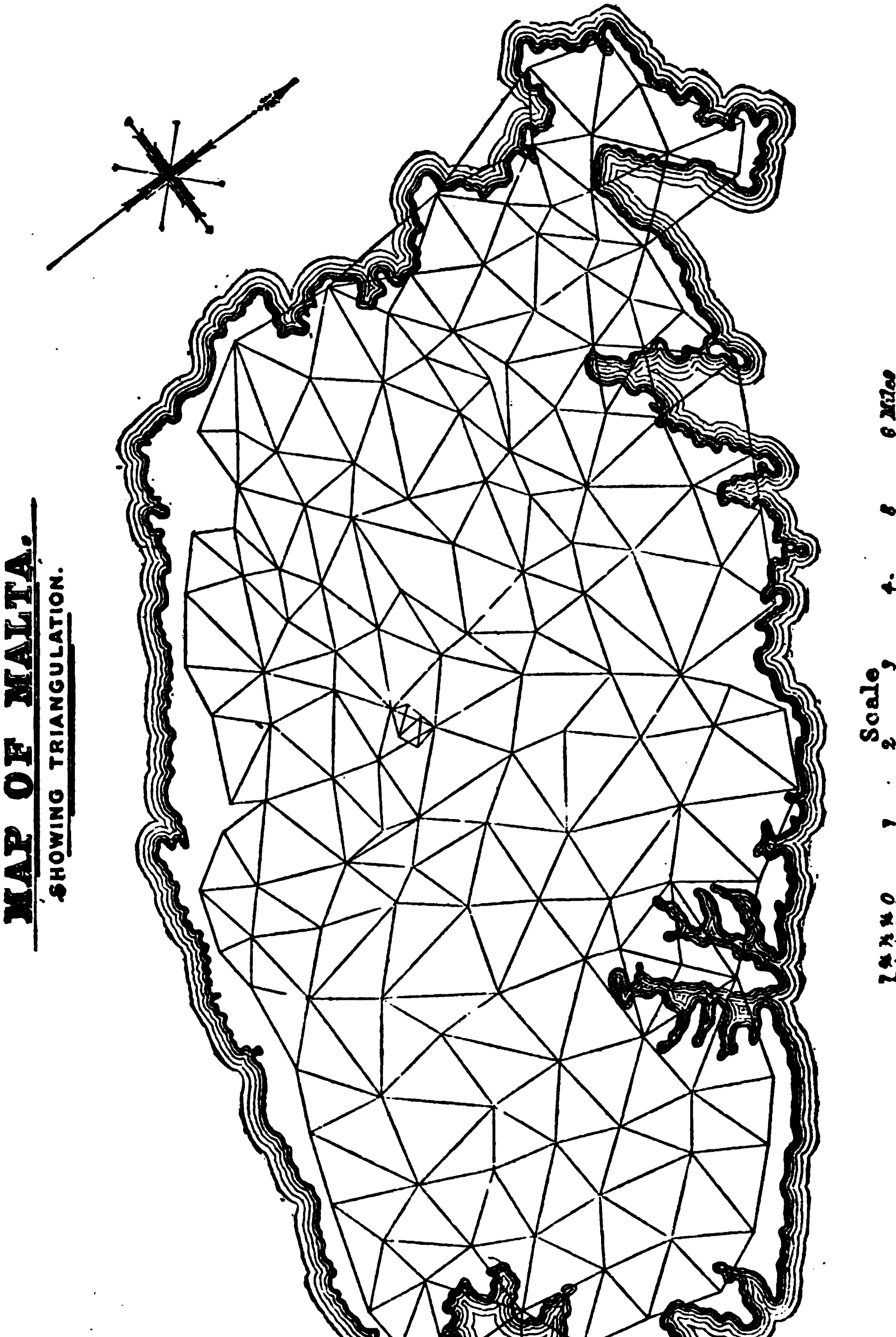


FIG. 95.

Thus, starting at triangle 1, of which the side A B is known, one might proceed to compute in succession triangles 1, 2, 3, 4, &c., to 22, 23. Then the length of the common side of the triangles 23 and 1, as obtained from computation of the series, ought to coincide exactly, with that obtained from the original triangle 1. Again, starting from the common side of the triangles 26 and 2, the triangles 26, 27, 28, &c., to 24 may be computed in series, and the remaining triangle 25, does not require computation, excepting as a check.

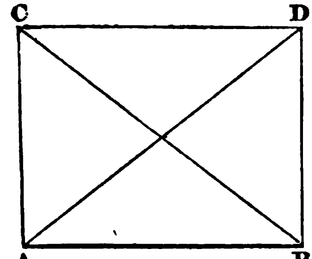


FIG. 96.

'Equations of Condition' of a closed polygon.

In order to ascertain how the necessary corrections may be made, so as to obtain the desired result, it will be necessary to examine the 'equations of condition' of a closed polygon, composed of several triangles fitted together. It is clear that a network of triangles may be divided into a series of interlacing polygons. Thus, referring to fig. 97, the triangles 1, 2, 26, 25, 24 and 23, form a polygon, which interlaces with another composed of triangles 2, 3, 4, 5, 27, and 26, and so on, for the whole network of triangles. It is clear that if the triangles of the first series are so corrected as to satisfy the conditions of a closed polygon, as also those of the second, (using of course, the same corrections for the angles of the triangles common to both

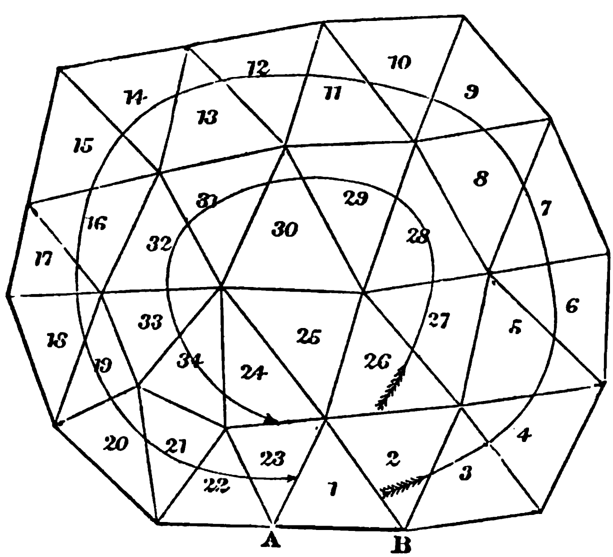


FIG. 97.

polygons), and so on for the whole network, the final value of any side will be the same, whatever order or series of triangles be adopted for its computation. By correcting in this manner, a network of triangles will be obtained which will fulfil the conditions of a series of plane closed polygons, and the corrected angles of the triangles whilst differing but little from the observed values, will satisfy these conditions. The results will therefore be very close to the truth.

To continue, let A B C D E F (fig. 98) be any polygon. Let rays be drawn

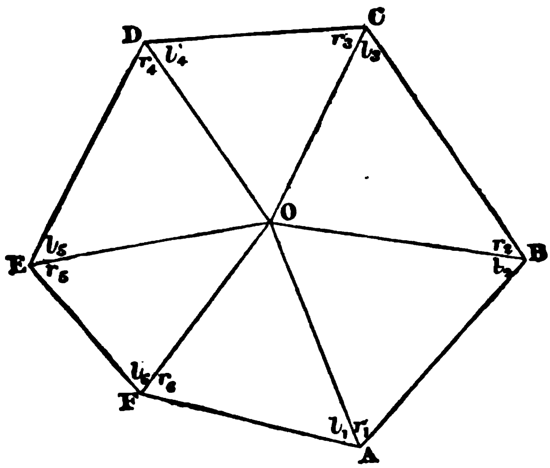


FIG. 98.

to some point O inside the polygon. Then the polygon may be considered as made up of the triangles, A O B, B O C, C O D, D O E, E O F, and F O A.

In the first place, it is evident that the three angles of each triangle must sum to 180.°

Next, the angles at O must sum to 360 or four right angles. This condition will be fulfilled if the angles are observed in the usual manner by keeping the limb clamped fast, whilst the telescope is directed to each point in succession, for

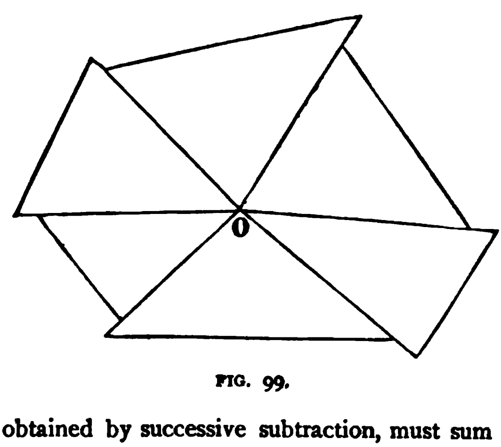


FIG. 99.

the angles being obtained by successive subtraction, must sum correctly. Each angle is liable nevertheless to error, like all other angles. If they were observed independently, (as with a sextant for example), they would not in all probability even sum correctly.

Now suppose that the several triangles, (their angles being corrected to satisfy these two conditions), are cut out in paper. They would then fit accurately at O,(fig. 99) and the three angles of each would sum to 180. Yet there is nothing to secure the coincidence of the extremities of their peripheral sides. They might fit together as in fig. 99, for the third equation of condition of a closed polygon has not yet been fulfilled.

Now suppose that the observer is at O (fig. 98). Then the angles of the several triangles, at the circumference of the polygon, may be called *right hand* or *left hand* as he regards them in succession. Thus $r_1\, r_2\, r_3\, r_4\, r_5\, r_6$ are *right-hand* angles, and $l_1\, l_2\, l_3\, l_4\, l_5\, l_6$ are *left-hand* angles.

Now by plane trigonometry

$$\frac{OB}{OA} = \frac{\sin r_1}{\sin l_2} \qquad \frac{OE}{OD} = \frac{\sin r_4}{\sin l_5}$$

$$\frac{OC}{OB} = \frac{\sin r_2}{\sin l_3} \qquad \frac{OF}{OE} = \frac{\sin r_5}{\sin l_6}$$

$$\frac{OD}{OC} = \frac{\sin r_3}{\sin l_4} \qquad \frac{OA}{OF} = \frac{\sin r_6}{\sin l_1}.$$

Multiplying these equations together,

$$\frac{OB \times OC \times OD \times OE \times OF \times OA}{OA \times OB \times OC \times OD \times OE \times OF} =$$

$$\frac{\sin r_1 \times \sin r_2 \times \sin r_3 \times \sin r_4 \times \sin r_5 \times \sin r_6}{\sin l_1 \times \sin l_2 \times \sin l_3 \times \sin l_4 \times \sin l_5 \times \sin l_6} = 1.$$

The products of the sides on the left of the equation cancelling.

Then we have the precept, that, *In any closed polygon, composed of triangles, the continued product of the sines of the right-hand angles is equal to the continued product of the sines of the left-hand angles.*

The log sines of the several angles may be conveniently taken, in lieu of the natural sines.

We thus have the three equations of condition of a perfect polygon.

(1) *The three angles of each triangle must sum to two right angles.*

(2) *The angles at the central point must sum to four right angles.*

(3) *The sum of the log sines of the right-hand angles must be equal to the sum of the log sines of the left-hand angles.*

So far, the central point has been assumed to be inside the polygon, the case which usually occurs in practice. The same principle applies, with a little modification, to the rarer case, where O is exterior to the polygon (*vide* fig. 100).

Thus, the polygon A B C D E F is obtained by taking away the three triangles A O F, A O B, and B O C, from the figure O C D E F, made up of the triangles C O D, D O E, and E O F. The first-named triangles may therefore be called *negative* triangles, the last three, *positive* triangles. It is unnecessary to repeat the reasoning, which is precisely like that of the first case, except that, (as a little consideration will show), when O is exterior to the polygon the left-hand and right hand angles must be inverted in the *negative* triangles.

For this case the precept then becomes, *The log sine of the right-hand angles, in the positive triangles, added to the log sines of the left-hand angles in the negative triangles, are equal to the log sines of the left-hand angles of the positive triangles, added to the log sines of the right-hand angles of the negative triangles.*

This case might be of use in surveying a coast line. A D O (fig. 101) would then be the *negative* triangle.

Application of the three 'Equations of Condition.'

The three equations of condition are applied as follows. The various angles at the different trigonometrical points of a survey having been measured, the three angles of each triangle are computed from the observed angles entered in the 'field-book.'

To avoid confusion in so doing, it is well to prepare a sketch of the network of triangulation, to approximate scale, laying off the angles by means of a pro-

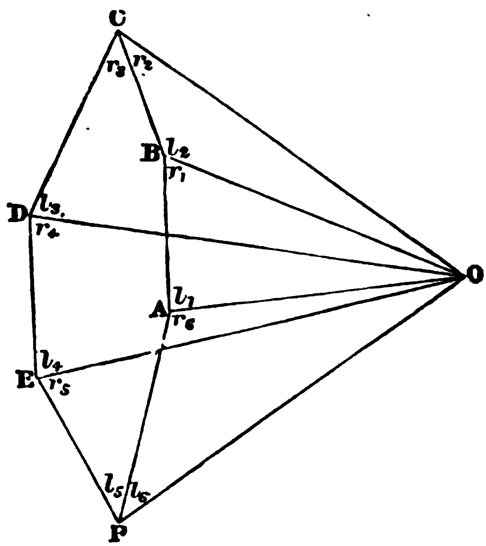


FIG. 100.

tractor. The angles are then inserted each in its proper place (*vide* fig. 102, Diagram of Triangulation).

Correction-Sheet.

A correction-sheet is then prepared (*vide* table E). In this the angles of the various triangles, which, when grouped together form a polygon, are brought together. Thus the first polygon might be composed of the triangles 2, 1, 7, 8, 9, 3, the next, of triangles 3, 4, 12, 11, 10, 9, the third, of 4, 5, 6, 14, 13, 12 and so on.

The central angles of each polygon are inscribed in the left-hand rubric in succession. In the second and third rubrics the corresponding right and left-hand angles of the triangles are inscribed, so that the three angles of the same triangle come in one horizontal line. The log sines of the right and left-hand angles are inscribed against them, in these rubrics. Against each log the log difference for 1″ is inscribed, noting the sign, and remembering that sines of angles greater than 90° *decrease*, and that therefore their *differences* are *negative*. Next the three angles of each triangle are summed, and the sum is carried out in the fourth column. This determines the summation errors.

The correction is now proceeded with. No complete rule can be laid down for determining the proper correction. In a single polygon, the number of possible solutions is infinite, but when several polygons interlace, as the same correction must be made in the same triangle in every polygon, of which it forms a part, the number of possible solutions becomes much reduced.

It must be remembered that all angles are equally liable to error, consequently if the central angles sum to four right angles, as they will do, if observed in the usual manner, they are none the less open to correction. To apply an error in summation of log sines, correct small angles rather than large ones. The sines of small angles increase more rapidly than large angles, in other words the

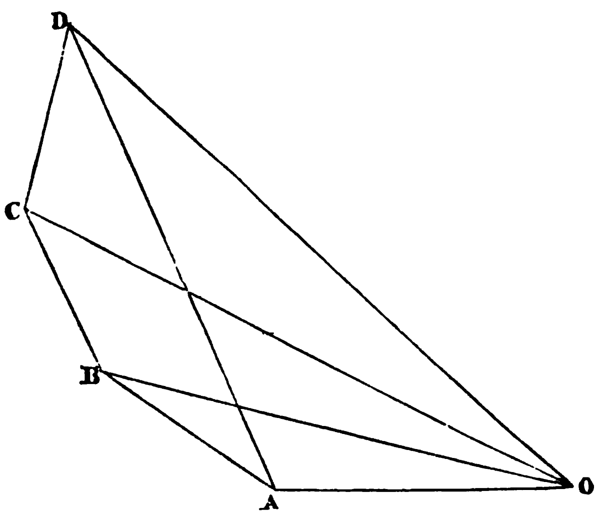


FIG. 101

differences of log sines of small angles are greater than those of large. Therefore, a greater correction in the summation of log sines is produced with a lesser alteration in the observed angle, by altering small angles than by altering large angles.

Conversely, in correcting an error of summation, either at the centre or in each individual triangle, correct large angles rather than small, for the difference of log sine of large angles is small.

First, the summation error should be divided by three, and the quotient applied to each angle, noting the sign + or − as the case may be. These preliminary corrections are written in pencil. The difference for 1 second is multiplied by the correction and the result put in proper position in the columns prepared for the purpose in the second and third rubrics. Then the differences due to the trial corrections are summed, also those of the central angles of the triangles, and the result is noted.

Probably the trial corrections will improve matters, but not sufficiently so. Some corrections must be increased, others diminished, always remembering that the same correction must be applied to the same angle, in every polygon in which the said angle occurs. Often, it will be necessary to make a correction which apparently makes matters worse in one polygon in order to adjust another.

Proceeding in this way, by trial and error, it is quite practicable to select corrections for a network of triangles, so that whilst the angles of the individual triangles, and the angles at every point, sum correctly to the last second of arc, the corrected summation of the log sines of the right and left-hand angles, agree to the first five figures, from the left of the mantissa of the logarithm, and this degree of accuracy will suffice for the purpose of a 'minor triangulation.'

It will at first sight appear that this method of adjustment is laborious. It is not so much so as might be expected. When all is arranged beforehand, and proper forms prepared, the work of adjustment proceeds rapidly. But laborious or not, the precision of the results obtained, fully justifies the labour. A network of compatible triangles has been prepared, whose angles differ from the observed angles by a few seconds only. All errors have been eliminated, including spherical excess, and the result is a network of true, plane, triangles.

Table E, opposite, gives an actual correction sheet taken at random from the survey of Malta. It refers to the series of triangles shown in fig. 102. This portion of the triangulation is not put forward as a model of accuracy, for indeed some of the corrections are rather large. Perhaps this is an advantage, as illustrating the principle more clearly, and as showing what can be done with a somewhat crude triangulation.

If the correction has been carried out to the degree of precision indicated, the final determination of the last side of a circuit of many triangles, will agree with the first to five places of significant figures from the left. It may happen that some one or more polygons, cannot be made to satisfy the three equations of condition, without applying corrections greater than the assumed unavoidable error of angular measure, appropriate to the means of measurement employed.

If this happens, there is nothing for it but to re-observe some of the angles. Generally the position of erroneous observation will be indicated by the correction sheet. It will be found that an angle or pair of angles throughout the whole of the group of polygons which they affect, will always require an undue amount of correction. A little consideration will then show the point at which the observations should be revised.

It is true that with a number of interlacing polygons, and some fixed limit of error of each angle, the number of solutions of the 'equations of condition' is innumerable. The solution of the problem which has the greatest probability of truth, could be arrived at by the method of least-squares. This is however, a method quite beyond the scope of 'minor triangulation,' in which an error of several seconds of arc is unimportant, as regards the accuracy of the final result. The writer has found, that if three different computers correct independently, a network of say 30 triangles, composing about 15 polygons, they will arrive at practically identical results. They will not indeed apply exactly the

„ 9, [illegible]	[illegible]	23	57	−00	51	18	13	9·8923561	16·9	−02	− 34	60	18	18	9·9388572	12·00	−20	− 240	180	00	28	−28	„ „
	360	00	00					Ī·5776276			+170				Ī·5776534			− 92					
								+170							−92								
								Ī·5776446							Ī·5776442								
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15·00	+22	+ 330	179	59	35	+25	„ „ „
„ 22, QFE	85	53	03	+10	41	13	58	9·8189644	24·00	−03	− 72	52	52	45	9·9016569	15·93	+07	+ 111	179	59	46	+14	„ „ „
„ 21, QGF	29	48	00	+02	73	31	43	9·9818011	6·20	−01	− 06	76	40	12	9·9881389	4·98	+04	+ 20	179	59	55	+05	„ „ „
„ 20, QPG	72	33	17	+09	75	00	15	9·9849522	5·63	+05	+ 28	32	26	10	9·7294554	33·13	+04	+ 132	179	59	42	+18	„ „ „
„ 11, QRP	75	09	45	+02	41	55	25	9·8248669	23·45	00	00	62	54	52	9·9495499	10·80	−04	− 43	180	00	02	−02	„ „ „
	360	00	00					Ī·4740798			+125				Ī·4740381			+ 540					
								+125							+540								
								Ī·4740923							Ī·4740921								

Polygon. Point P Centre.
Q, G, H, J, O, R.

No. 11, PQR	62	54	52	−04	75	09	45	9·9852719	5·60	+02	+ 11	41	55	25	9·8248669	23·45	00	00	180	00	02	−02	180 00 00
„ 20, PGQ	75	00	15	+05	32	26	10	9·7294554	33·13	+04	+132	72	33	17	9·9795500	6·62	+09	+ 60	179	59	42	+18	„ „ „
„ 19, PHG	34	02	22	+16	68	19	12	9·9681379	8·37	+04	+ 33	77	38	02	9·9898052	4·62	+04	+ 18	179	59	36	+24	„ „ „
„ 18, PJH	53	52	38	−07	55	51	48	9·9178737	14·28	−02	− 28	70	15	53	9·9737109	7·55	−10	− 75	180	00	19	−19	„ „ „
„ 17, POJ	61	14	40	−12	91	56	25	9·9997509	0·72	−10	− 08	26	48	52	9·6542753	41·67	+25	+1042	179	59	57	+03	„ „ „
„ 10, PRO	72	55	13	+02	38	13	38	9·7915375	26·73	−03	− 80	68	51	12	9·9697234	8·13	−02	− 16	180	00	03	−03	„ „ „
	360	00	00					Ī·3920273			+ 60				Ī·3919317			+1029					
								+60							+1029								
								Ī·3920333							Ī·3920346								

same corrections in every instance, but their results will be the same in quality or direction, if not in quantity.

All three will agree in applying larger corrections to certain angles, lesser to others. In a triangulation of the extent indicated, differences of 10" and more are unimportant.

Method of conducting a Triangulation.

Turning now to the method of conducting a triangulation. The first step is to select the position of a base-line. For this purpose a flat and unbroken piece of country is required. The length of the base-line should not be less than about one-third of the average length of a triangle side. A relatively short base, measured with great care, under favourable circumstances, is preferable to a longer base, measured with less precision, under less favourable circumstances. It is obviously easier to find a place where 1000 feet can be measured correctly than one where 3000 feet can be set out, free from obstructions.

Moreover, it is easy to obtain a secondary and longer base by triangulation. Let A B (fig. 103), be the measured base. From its extremities let angles be taken to C and D. Then without involving any ill-conditioned triangles, the line D C may be made about double A B. Then in like manner from D and C observations can be made to E and F, so that E F the extended base, may be about four times the length of the original base.

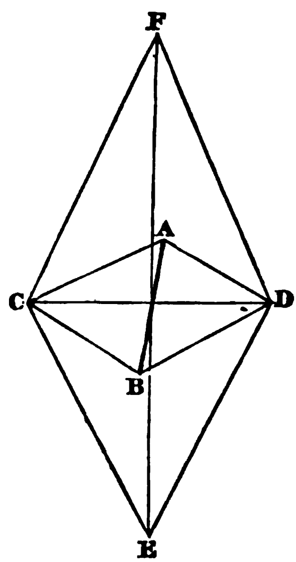


FIG. 103.

It is better to extend the base, by means of four triangles, than by one, for if D C be about twice A B, and F E twice D C, then the angles at C, D, E, and F, would be about 53°, whereas in the single operation the vertical angle would be only about 28°, thus giving a far lower degree of accuracy for the single triangle, than for any one of the four. The errors, moreover, of the four triangles will not always be in the same direction. They will tend more or less to compensate each other, and thus the probable error of a distance, determined by a series of four well-conditioned triangles, will be less than that determined by one ill-conditioned triangle. With care, the length of the 'extended' or 'working base,' may be determined with an accuracy *nearly* if not *quite* equal, to that of the original base.

Even if the surveyor does not possess the means of measuring a base, with the highest accuracy, triangulation is still a valuable method. When a network of triangles is corrected in the manner above described, the *relative* positions of the sundry points will be determined with great precision. The actual distances from point to point, will be determined in terms of the measured base, for if the base be measured too long, then all distances will be relatively too long, and *vice versâ*. The effect of a slight error in the measurement of the base, is not a distortion of the plan, but amounts to its being drawn to a slightly different scale to that intended. If an error of one part in 2500 were made in the base measurement, the result would be that a plan, whose scale was intended to be $\frac{1}{2500}$ might be, in reality, $\frac{1}{2499}$ or $\frac{1}{2501}$, an unimportant difference, and inappreciable on the paper.

Suitable size of Triangles discussed.

The question of the proper distance between trigonometrical points depends largely on local conditions, such as whether the formation of the ground be favourable or not. The best average length of triangle sides, may be discussed on the following lines.

The positions of the trigonometrical points, as determined by the triangulation, must be considered to be accurate. They must not be re-adjusted, but all subsequent detail field-work must be corrected, so as to agree with the trigonometrical points.

The adjustment of chained lines or traverses, should never exceed a quantity perceptible on the plan. Hence the length of the triangle side, depends upon the scale of the plan to be produced, and on the probable accuracy of the chain or traverse-surveying. Suppose that the plan has to be $\frac{1}{2500}$, and that the probable error of chaining or traversing is estimated at 1 per 1000. Now ·001 of a foot (·012 inch) is about the least dimension that the draughtsman can work to. With the scale $\frac{1}{2500}$ adopted, this would represent 2½ feet on the ground. Then the length of the triangle side should be such, that when measured with a chain or estimated from a traverse, an error exceeding 2½ feet cannot accumulate between its extremities. As the error of chaining has been assumed to be 1 per 1000, the average distance between the trigonometrical points should not exceed 2500 feet, say half a mile. If the plan were to be on the scale $\frac{1}{10,000}$ (6 inches to 1 mile nearly) then the error on the paper of ·001 foot would represent 10 feet on the ground. With the same error in chaining, the length of side might be 10,000 feet, say two miles.

Again, in a broken and rugged country where chaining would be inaccurate, the trigonometrical points should be more numerous than in an open, smooth country, where the best work can be expected.

Generally, a side of from three-quarters of a mile to a mile in length is convenient. If much shorter, a very slight displacement of the signal observed to, causes inconveniently large errors. If plans on a large scale are to be prepared, for which according to the principles laid down, more numerous points would be desirable, it would be preferable to resort to 'intersected' subsidiary points, determined in a manner hereinafter described.

Number of Trig. Points.

Even were the plan proposed in the first instance to be on a scale of $\frac{1}{60,000}$, it would be desirable to work with a degree of accuracy that would allow of the survey being plotted to a larger scale at some future date. The number of trigonometrical points within a given area must be considered on its merits in each case. The surveyor will not be able to arrange his points, so that all his sides are equal, and of the desired length. Much depends upon the nature of the country and even on the funds available. The principles above stated will, however, serve as a general guide.

It is economical in the end, and always satisfactory, to spend a considerable amount of time in locating the trigonometrical points so that ill-conditioned triangles may, as far as possible, be avoided, and so that the triangles may be symmetrically grouped, forming polygons without redundant or deficient rays.

Selecting, and Marking, Trig. Points. A considerable number of points should be located and marked before commencing the observation of angles, for if this is not done, it may be found that though well placed as regards each other, some are so situated as to make further extension difficult. It is not essential that the signals should be set up. The points should be fixed and marked in a permanent manner as for traverse-surveying, and it must be remembered, that in the case of triangulation, permanence is of the greatest importance.

Whilst locating points, it is desirable to prepare a skeleton plan to an approximate scale, showing their positions. Such a plan will prove most useful as the work proceeds. For this purpose, a compass, pocket-sextant, plane-table, or even a small theodolite, (using it as a compass), may be employed. For measuring distances approximately, the telescope of the theodolite may be provided with subtense wires, visually subtending 1 foot at 200 feet. Then by using a staff 16 feet long boldly graduated to feet, a rough approximation to distances up to 3000 feet may be obtained, and information obtained that will often be valuable (especially if a staff-holder has to be sent to put up a mark). A Watkins or Weldon range-finder, may be useful, and a powerful binocular is almost essential.

It is not possible to lay down any fixed rule of procedure, for the location of points. Much depends on the nature of the country, and the facilities for getting from place to place. If locomotion were free, the following routine might be followed (*vide* fig. 104). Starting from 1 the point 2 might be selected. The distance 1–2 being measured approximately would serve as a base for the skeleton plan. The compass bearing of 1–2 should be taken. Then 3, 4, 5, 6, and 7 might be located, and their positions determined by intersected bearings, the same being plotted on the skeleton plan, then and there, with the protractor. Then some point such as 7 might be selected as the centre of an interlacing polygon, which might be completed by fixing 8, 9, and 10, and so on. When standing at each point, the neighbourhood should be examined, with a view to ascertaining the possibility of seeing other points, and bearings to natural objects may be taken to serve as guides in fixing further points.

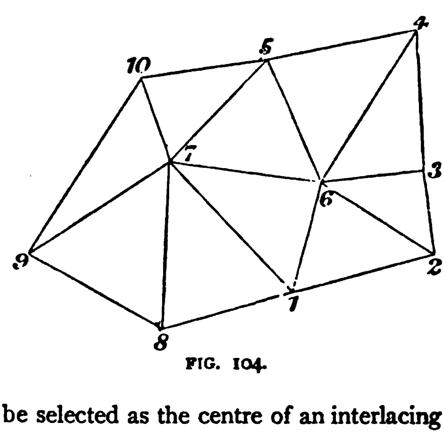


FIG. 104.

To locate points properly is always laborious, especially when new to the work, and the eye is not habituated to judge distances. The assistance of an intelligent foreman chainman is most valuable, a person who can be sent to select a point, with some amount of judgment, and not taking up a position whence but one or two other points can be seen.

Trig. Points should be selected by the Chief Surveyor.

The selection, and location of points, is so important that it should be performed by the chief surveyor himself. The observation of the angles may be reduced to a matter of routine, and may be effected by subordinate surveyors. Thus in an extended survey, the chief surveyor locating points, may keep several assistants fully employed in observing angles.

The value of the trigonometrical points, does not cease with the completion of the survey. They are of the utmost use when the plans, on account of new constructions, have to be corrected up to date. Surveys for new roads, railways, and the like, should be connected to trigonometrical points, as well as those of land granted or sold. If the survey be one conducted by Government, a law should be provided giving power to mark survey points, making it a punishable offence to remove, destroy, or obliterate them. If, on account of the construction of a building, a survey point must be removed, then notice should be given to the officer in charge of surveys, who will, if necessary, fix a new mark, near to the old one. Moreover all Licensed Surveyors should be bound by law, to connect their work to established survey points. This being done, and copies of their surveys and traverse-sheets being deposited with the officer in charge of the Government Surveys, a complete map may be compiled, *pari passu* with the occupation of the land.

The survey points should therefore be marked in the most permanent manner possible. In addition to this, measurements should be made to fixed and permanent objects. These should be recorded on sketches, showing the measurements on plan, and the elevation and appearance of the objects to which the measurements were made. These sketches should be copied into a book kept for the purpose, and accompanied by a full verbal description. Not only should the points be marked in a permanent manner, but means should be provided for their recovery, if they be lost or obliterated.

Masonry pillars are generally useful as survey marks. They should be founded deep enough to be below the range of surface disturbances due to tillage. If made high enough to serve as a stand for a three-screw theodolite, it would be convenient. The funds at the surveyor's disposal will, however, rarely suffice to erect such high pillars of sufficiently substantial construction, to resist the rubbing of cattle and other destructive tendencies.

Signals for Observing to.

The surveyor will therefore have to rest satisfied with marks but little, if at all, above the level of the ground, erecting temporary signals when observing. If there be rock on the surface, a permanent mark may be made by drilling a hole and driving in an iron plug. Or the hole might be filled with Portland cement, a nail being inserted on the top to mark the exact centre. To protect the mark and indicate its position, a cairn of stones may be erected over it, and this will serve to hold up the observing signal.

If no objects in the immediate neighbourhood of the trigonometrical point be available for fixing measurements, it will in some cases be possible to fix by cross bearings, each being determined by two objects in line. Thus, for example, the line joining the left-hand angles of the buildings A and B (fig. 105) might intersect that joining the right-hand angle of C and the left of D in some point X not far from the trigonometrical point O. Then by four measurements *a b c d* the point O would be fixed. Sketches should be made, showing the appearance of the objects when in line.

The trigonometrical points, having to be so low as to permit of the theodolite being placed over them, will not be visible from a distance, hence some temporary mark, or signal, must be erected to observe to.

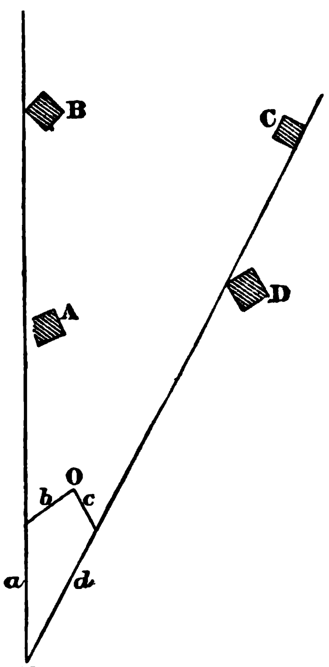


FIG. 105.

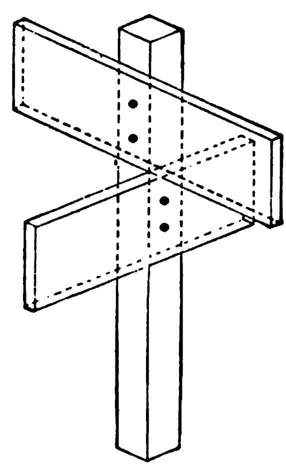


FIG. 106.

If the distances be not great, an ordinary ranging rod answers as well as anything else. If labour be cheap, nothing can be better than rods of this kind, each held on the mark by a man. If the staff-holder stands heels together behind the mark, and facing the observer, with the point of the rod on the centre of the mark, and holding it between the palms of his hands in front of his nose, perpendicularity will be secured.

Flags are of little use as signals. If there be wind enough to display the flag, the staff is apt to be pulled sideways. If there be no wind (the best condition for observing), the flag hangs down and is useless.

It will generally happen that semi-permanent signals, fixed so as to dispense with the attendance of a man, have to be used. A good form consists of a piece of 1½-inch to 2-inch scantling (fig. 106), as straight as possible, to the top of which two pieces of board are nailed at right angles to each other, as shown in the sketch. A coat of whitewash over the boards and rod, makes the signal very distinct with most backgrounds. Sometimes however it may be well to paint the signals red for a green background, black for a pale grey, such as ploughed land. The signal may be fixed over the mark, either with a small pile of stones or by means of three struts, of similar scantling to the signal staff.

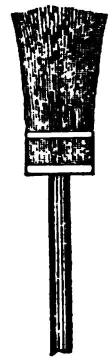


FIG. 107.

A very good signal is made by lashing a number of twigs to a pole or scantling, after the manner of an ordinary besom (*vide* fig. 107). In lieu of ordinary twigs, the writer has used a bunch of bright coloured flowers, often found in tropical forests, which made a most conspicuous signal against a green background.

Admiral Belcher in his treatise on Marine Surveying has suggested a form of signal which would appear to be useful, (*vide* fig. 108). The signal consists of a rope, stout enough to be visible, and stretched with a heavy weight, suspended from a tripod consisting of three poles lashed together. The weight may be a basket or bucket, with a stout spike fixed to the bottom, and filled with stones, earth, or water, obtained on the spot. The rope will in any case be perpendicular. If blown aside by the wind, the displacement at the the top will be slight, probably less than the displacement of an ordinary signal. Any oscillation may be bisected by the wire of the theodolite. If the point of the spike be not exactly below the centre of gravity of the bucket or basket, correct centering may be effected by spinning it, and moving the legs till the point describes a circle having the mark for a centre. The rope might be painted with different colours to make it conspicuous, or a double cone of painted canvas might be attached to it.

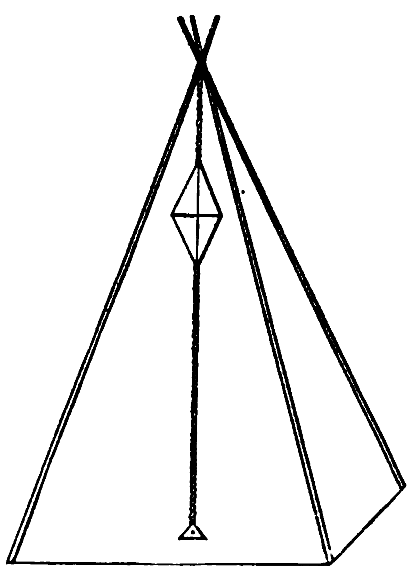


FIG. 108.

The following description of the heliotrope, and luminous signals, is taken from Thuillier and Smyth's Manual of Surveying for India :—

"Vase lights were invented by Colonel Everest nearly fifty years ago, and completely altered the operations of the Great Trigonometrical Survey in India, which had previously to be carried on in the unhealthy season of the rains, in order that the opaque signals, such as flags, might be clearly seen. By enabling observations to be rapidly taken at night, the progress of the work was also much accelerated.

"The vase light consists of a common earthen dish about ten inches in diameter (more or less, according to distance), and filled with cotton seeds and common oil. This is placed upon the station, and to prevent the flame being blown aside, a large earthen pot, in the side of which an aperture has been cut, is inverted over the dish, as shown in the diagram (fig. 109); an aperture is also cut in the top to allow the smoke to escape. Further protection is necessary from high wind by means of grass screens and blankets, leaving merely the requisite opening in the direction of the observer. The materials for this light are procurable in nearly every village.

"Trigonometrical operations in Southern India were entirely conducted by means of the foregoing signals, more especially the vase light for principal stations, and the pole and brush for secondary points. During the last seventeen years, however, signals of modern invention have been employed on account of their superior economy, convenience, and power. These consist of heliotropes, Argand reverberatory lamps, and Drummond lights. The latter surpass all previous contrivances. A ball of lime, about a quarter of an inch in diameter, placed in the focus of a parabolic reflector, and raised to an intense

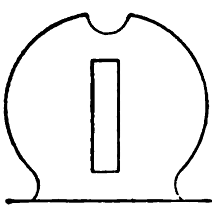


FIG. 109.

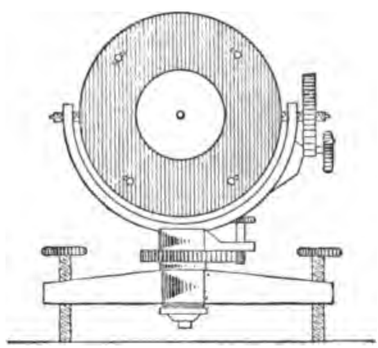


FIG 110.

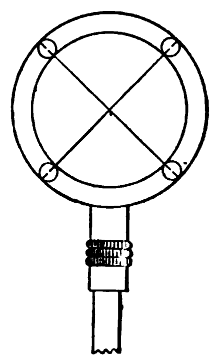


FIG. 111.

heat by a stream of oxygen gas directed through a flame of alcohol, produces a light eighty times as intense as that given by an Argand burner, and is visible even in hazy weather at a distance of 60 to 80 miles.

"The heliotrope consists of a circular piece of flat plate-glass mirror, about 9 inches in diameter, with a small unsilvered aperture in the centre about 0·1 of an inch in diameter, as represented by fig. 110. This mirror is mounted on a frame, which stands on a tripod for the sake of steadiness. The frame admits of the looking-glass being turned on a horizontal as well as a vertical axis. These two motions in altitude and azimuth are regulated by means of rackwork, and they permit the reflection of the sun's rays to be turned in any required direction. In order that it may be directed truly to the observer, a ring with cross wires (fig. 111) is placed at a distance of about three feet; the signalman then looks through the unsilvered aperture in the centre of the heliotrope, and moves the cross wires until they intersect the distant station. Thus the centre of the heliotrope, the centre of the wires, and the observer's station form one straight line. Now, if by means of the rackwork, the mirror is moved in altitude and in azimuth, until the sun's rays fall on the wires, it is evident that the light will proceed straight to the observer's station, but the pencil of rays must be duly bisected by the wires, which intersection can be managed with ease and delicacy by means of a little circle of white paper placed at the crossing of the wires, upon which paper the reflection of the little aperture in the centre of the mirror may be seen like a small dark speck. When the weather is hazy, the signalman will, of course, be unable to see the observer's station, in which case, unless a nearer mark has been given to guide him, or a directing line drawn for him, he will be so far helpless. Under such circumstances, the observer ought to direct one or more heliotropes towards the man, and keep them playing until he has adjusted his apparatus. Similarly, if the man is careless, and neglects to keep the sun's rays constantly shining in the true direction, the observer has only to flash a heliotrope at him to keep him alert. A heliotrope of nine inches will answer for 90 or 100 miles, for nearer distances it is much too bright to be observed through a telescope, and the light must be diminished in the following proportion. For distances of two or three miles (the usual distance of a referring mark) an aperture of 0·25 of an inch will answer, and for longer distances about 0·1 of an inch of aperture per mile of distance will suffice, viz. an inch for 10 miles, two inches for 20 miles, and so on, provided always the apparatus is carefully adjusted and the man who works it is alert and skilful.

"These apertures are cut in a board (fig. 112) which stands upon three feet, by means of which the centre of the aperture can be adjusted plumb over the station mark. This board is called a sight-vane, and stones are placed on the tail piece to prevent its being disturbed by the action of the wind. If this sight-vane be used, the wires before described are unnecessary, because cross-hairs can be fixed in the vane, and will become a substitute for the wires. The heliotrope is in this case placed two or three feet in rear of the sight-vane, and moved laterally and vertically, until the eye, applied to the centre unsilvered dot, views the observer's station and the cross-hairs in one line. The heliotrope must be secured in this position, and the means of doing so will readily suggest themselves. It is needless to say that it must be quite firm.

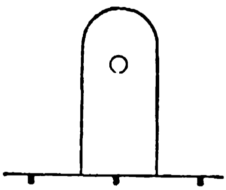


FIG. 112.

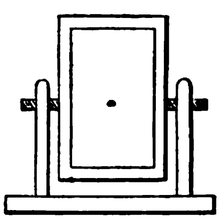


FIG. 113.

"A very good substitute for a regular heliotrope has been frequently made out of a good looking-glass with a flat surface (*vide* fig. 113). A small hole is drilled through the centre of the back board of the looking-glass, and the silvering scraped off; this aperture should be truly centrical. The looking-glass is then swung in a frame of wood in such a way that the axis of motion shall pass through the unsilvered aperture. This frame is fixed upon a vertical axis, which ought also to coincide with the unsilvered dot in the mirror. Finally the vertical axis is planted on a board, with 3 foot-screws for adjustment. They have been frequently used with success, on the subordinate series of the

Great Trigonometrical Survey as well as in the Revenue Survey, and being powerful as well as economical instruments, they will be found very useful. By means of them and vase lights, work can be carried on with great rapidity, because the only limit to the times at which observations can be made will be from 9 o'clock A.M. to 2½ or 3 o'clock P.M. But the heliotrope is more particularly recommended for the purpose of taking vertical angles with certainty between the hours of 2¾ and 3¾ afternoon, which is the time of minimum refraction, because verticals taken at any other time are subject to great irregularities, whereby heights deduced from them are nearly worthless. Luminous objects are much more correctly, rapidly, and comfortably observed than opaque ones, which, if distant, are always faint, and disappear when brought near the wire of the telescope."

Observing Angles by 'Repetition' and 'Reiteration.'

The adjustment of the theodolite and its general use, have been already described. It will be assumed therefore that before commencing the observation of angles, it is practically in perfect adjustment. There are two methods of observing angles, by which more correct values can be obtained than is possible with one single measurement. These are '*repetition*' and '*reiteration*.'

In 'repetition,' the procedure is as follows. With the two plates clamped to zero, unclamp the 'lower plate' and direct the telescope to one of the objects, (the angle between which is to be measured), say, that on the left. Clamp the 'lower plate' and make the bisection with the lower tangent screw. Now unclamp the 'upper plate' and bisect the right-hand signal. The verniers now read the desired angle. Now unclamp the 'lower plate,' keeping the upper and lower plates firmly clamped together, direct to the left-hand object, bisecting it by means of the lower tangent screw. Now unclamp the 'upper plate,' and bisect the right-hand signal. The verniers now should read exactly twice the desired angle. Proceed in the same manner till the angle has been measured many times. The final reading of the vernier, divided by the number of readings, will be the value of the desired angle, and will be much more correct than any one reading. This method is due to Borda. It was at first thought that by a sufficient number of repetitions, the true value of an angle could be obtained to a fraction of a second, with a small and inferior instrument. Experience however shows that this is not the case. There is a limit beyond which repetition produces no improvement. Errors due to slip and flexure, accumulate and prevent extreme accuracy, though a moderate number of repetitions reduce graduation error, by measuring the angle at different parts of the circle. To some extent it reduces errors in bisecting the objects aimed at, for it is reasonable to suppose that the cross-wire will not be always laid on the same side of the true centre. Repetition, however, does not eliminate the most serious source of error of all, namely displacement of the signal observed to. If, as is very often the case, in broken countries, the top of a signal 10 or 15 feet high can alone be seen, it is quite possible that the centre of the signal at its top, may be as much as 3 inches out of the perpendicular line through the centre of the trigonometrical point, either from its being incorrectly set up in the first instance, or blown aside by the wind. If the displacement of 3 inches were at right angles to the line of vision, it would subtend at 2500 feet, an angle of about 20 secs. It is clearly useless to attempt to measure angles to a single second by repetition, when a 'periodical' error, that is to say, an error occurring in the same direction in every observation, and amounting to several seconds, may exist.

For the purposes of a minor triangulation, 'repetition' is far too laborious. Amply sufficient accuracy may be obtained by 'reiteration.' In observing by 'reiteration' the leading vernier is set approximately to zero, and the plates are clamped together. The whole instrument is turned round, and some one signal or mark, called the 'referring object' or R. O. is bisected. All the verniers are then read and recorded. Then, the limb remaining stationary, the several signals are bisected in succession, and the reading of each successive point round the circle is recorded, after the manner of 'bearings.' Lastly the 'referring object' is again bisected. This reading should coincide with the first. If it does not, then some slip or torsion of the stand has taken place, or the referring signal has moved. If the second reading of the 'referring object' differs seriously from the first, the round is worthless and must be re-observed. If the difference does not exceed, say a minute, both readings should be recorded and their mean used in subsequent calculation. The reading of the 'referring object' must not be assumed to be zero.

The plates are then unclamped, and the leading vernier is set to some other angle, say approximately 60° or 90°, and the operation is repeated, and so on. The minutes and seconds should be nearly identical with those of the first readings, the degrees varying by the constant number of degrees. It is therefore merely necessary in the second and subsequent reiterations, to read the minutes and seconds.

The mean of all the readings of all verniers is then worked out to the nearest second, and the angles are obtained by successive subtraction.

For the purposes of a 'minor triangulation,' two 'reiterations' usually suffice, provided that the instrument be well graduated, and solidly constructed. If the instrument be of the 'Transit' or 'Everest' type, the first round should be taken 'face left' (that is with the vertical arc on the left of the observer), and the second 'face right.' If the theodolite be provided with three verniers it will not be necessary to alter the position of the graduated arcs. Suppose that the first round were taken 'face left' with the leading vernier A set to zero, B to 120°, and C to 240°, then transit the telescope and turn it through 180°, so as to bring it 'face right,' and again bisect the zero station, A vernier will read 180°, B 300°, and C 60°. Thus, each angle will be read on six parts of the limb.

If on the other hand there be two verniers only, then the zero station should be bisected for the second round, with the leading vernier set to some other angle such as 60° or 90°.

If a cradle theodolite be used, then the effect of 'face right' and 'face left' may be attained by shifting the telescope end-for-end in the Ys and turning it upside down, so that the level, normally beneath the telescope, is above it.

It is well also to observe the second round of angles, in the opposite direction to the first. That is to say, if the telescope has been, on the first round, directed to the successive points from left to right, with the hands of the clock, then for the second round, it should be moved in the opposite direction. This tends to eliminate slip or torsion. It is also well to bring the cross-hairs on to each signal by the tangent screw in the opposite direction on the second round, to that on the first. That is to say, that for the first round, the cross hairs should be brought by hand, before the plates are clamped, slightly to the left (say) of the object, and after clamping, the bisection should be made by the tangent screw, moving the telescope from left to right, whilst, on the second round the bisection should be made moving it from right to left. This tends to eliminate spring, and personal bias in bisection.

The readings of all verniers should be recorded to the nearest amount visible, without bias, or any attempt to obtain agreement. The surveyor must expect appreciable discrepancies, due partly to defective graduation, partly to errors of bisection. Indeed the whole object of 'reiteration' is to eliminate such errors. The second, as well as the first reading to the 'referring object,' should be recorded for each round, and the mean of the two means used, in computing the angles. If the second reading differs materially from the first, say by a minute or more, something has gone wrong and the round must be repeated, the stand has perhaps been kicked or the wrong tangent screw touched by mistake.

It is scarcely necessary to say, that the theodolite must be accurately centred over the station point, by means of the plumb-bob. It must also be carefully levelled, using the large level attached to the telescope or vertical arc, and not the small levels on the upper plate. The levelling should be tested and corrected before and after each round.

The stand must be firmly planted on the ground. If this be at all soft, stakes about four inches square should be driven down firmly, and cut off with a notch in each, to receive the point of the legs.

Principal sources of error in Observing.

The principal sources of error in observation are as follows.

Error in Centering.

1. Error in centering the theodolite at the station of observation. A little care will reduce this to a negligible quantity.

Error in Levelling.

2. Error in levelling, i.e. the error due to the axis not being truly vertical, by which the angles are measured not in a horizontal plane, but in one slightly inclined to the horizon. If the instrument be in good adjustment, carefully and firmly set up, and levelled with the large level, this will also be a negligible quantity.

Error due to dislevelment of the cross Axis of the Telescope.

3. Error due to (*structural*) dislevelment of the cross axis of the telescope. This axis, though the limb be horizontal, might be higher at one end than the other, and consequently the line of collimation of the telescope would sweep out, a slightly inclined, instead of a vertical plane. A little consideration will show that, if all the signals observed were in one horizontal plane, no error would be produced. In good instruments well adjusted, the dislevelment of telescope axis is very small, so that appreciable error is only

produced, when the signals are at very different elevations. Any error from this cause is entirely eliminated by face-left and face-right 'reiterations.'

Error in Collimation.

4. Error in collimation, i.e. when the line of sight of the telescope, as marked by the cross-wires, is not at right angles to the axis, and consequently as the telescope revolves the line of sight sweeps out, not a plane as it should do, if at right angles, but a cone. Collimation error has no effect when the signals are all in the same plane, and is entirely eliminated by face-left and face-right 'reiteration.' For this reason and for the elimination of axis dislevelment error, 'reiterations' should be made in pairs 'face right' and 'face left.' If two 'reiterations' are insufficient, then four or six should be made.

Error in Graduation.

5. Error in graduation. If the instrument be divided with a good dividing engine, this should not be great. It is, to a large extent due to imperfect centering of the limb, on the dividing engine. Partly also to shake in the tool which cuts the strokes. In a well-divided limb, it is rare to find any one graduation perceptibly out of position, with regard to those adjacent to it. The tendency is rather for the graduations to be slightly closer together at one part of the limb, and wider apart at another, the change being gradual. If the limb be tested by 'repetition,' the successive angles should vary progressively and gradually, *increasing* and then *diminishing*. Irregular changes in value would indicate a bad limb, or careless observation. Graduation error is to some extent eliminated by 'reiteration' but not entirely. The best plan is to purchase the instrument direct from a maker who possesses a good dividing engine. There are many instruments in the market made by persons who do not possess dividing engines, but get the dividing done by some one else. Such an instrument may or may not be well divided.

Elimination of error from bad Centering of Instrument's Axis.

6. Errors due to bad centering are eliminated by 'reiteration.'

Error due to displacement of Signal.

7. Error due to displacement of the signal. In a triangulation, this is probably the *greatest* and *most common* source of error. The remarks made above on this point, under 'Traverse-surveying' apply now with equal force. When possible the observation should be made to the *point* of a staff, held on the station point, or at least *as low down as possible*. Unfortunately this cannot always be done, since observations have often to be made to the summit of a signal, perhaps fifteen or twenty feet high. This error cannot be eliminated by 'reiteration' or 'repetition.' There is nothing for it, therefore, but to take the utmost care in setting up, and securing the signals, so that the summit observed, is, and remains, perpendicularly above the station point. If the signal is 30 feet high or more, it will be best to use the theodolite to set it up, as follows. Place the foot of the pole, when a tripod is not used, a little to one side of the station mark and then set up, and adjust, the theodolite some 50 feet or more from the station mark. Bisect the station mark, and clamp the limbs, elevate the telescope, and by means of guy-ropes make the apex of the pole coincide with the cross-wires. Make fast the guys. Repeat this operation in a

plane at right angles to the first, using two other guys. The summit is now perpendicularly above the station point.

Error of Bisection.

8. Error of bisection, i.e. the cross-hairs are not directed exactly to the centre of the signal. With care and with the short sides used in minor triangulation, this error should not be serious. It depends upon several factors, viz. the accuracy of focussing, the eye of the observer, the defining-power of the instrument, the definiteness of the signal, and the state of the atmosphere. If the telescope be not accurately focussed, so that the image of the signal is not in the plane of the cross-hairs, the object will flit about, if the observer moves his eye slightly up and down, or from side to side. The object-glass and eye-piece must be carefully adjusted, so that the object remains steadily bisected, in all positions of the eye. As regards the eye of the observer, the only thing that can be done is to use properly made spectacles, if his sight be imperfect. It often happens that the eye-piece of the telescope cannot be pushed near enough to the cross-hairs, for distinct vision by a short-sighted person, but this can easily be remedied by filing down the end of the tube into which the eye-piece fits. If the object-glass be indifferent, the image of the signal will be indistinct and blurred, and accurate bisection cannot be expected. A bad telescope is a nuisance, for in addition to difficulty of bisection, it is difficult to recognise distant signals. The following rough tests are useful. A well-defined distant object should come rapidly into, and out of focus, and a very little motion of the adjusting pinion should make the difference between distinctness and indistinctness. If the object keeps fairly distinct, through a considerable range of motion of the object-glass, the figure of the object-glass is imperfect, i.e. different parts of the object-glass have different lengths of focus.

On examining a bright fixed star it should appear as a point surrounded with rays of equal length. If there is a blurred 'wing' thus * the glass is defective.

Testing of Instruments at Kew Observatory.

Theodolites and sextants can be tested at the Kew Observatory, for a very moderate fee, in all points, such as graduation, quality of telescope, &c. When ordering from abroad, it is well to ask for a Kew certificate. This should never be omitted in the case of sextants, which cannot be tested by the surveyor, and of which the graduation errors are often much greater than would be supposed.

Magnifying Power of the Telescope.

The magnifying power of the telescope is usually sufficient for the power of the limb. Owing to its internal construction, a normal eye can separate two small objects at such a distance from each other, as to subtend about one minute of arc. If therefore the telescope magnify ten times, then, if optically good, an object should be observed to one-tenth of that amount, or 6 secs., which is less than the amount that can be read on an ordinary theodolite limb. For ordinary purposes, therefore, clearness and good definition are more important than very high magnifying power.

Perturbation of Atmosphere in Hot Climates.

In hot climates the atmosphere often 'boils' at certain times of the day. When this takes place it is useless to observe, the signal will dance about and become distorted in so erratic a manner, that accurate work is impossible. To attempt to observe under these conditions merely amounts to loss of time and temper.

TABLE F.

Horizontal Angles Observed at Station N.

Date. *Observer.*

Station Observed to	Face Left Vernier A. °	′	″	Vernier B. ′	″	Vernier C. ′	″	Face Right Vernier A. °	′	″	Vernier B. ′	″	Vernier C. ′	″	Mean Bearing. °	′	″	Included Angle. °	′	″
A	81	38	19	38	21	38	26	261	38	19	38	20	38	15	81	38	23			
M	124	4	51	5	2	4	52	304	4	49	4	42	5	2	124	4	53	42	26	30
L	203	35	14	35	29	35	27	23	35	31	35	19	35	18	203	35	23	79	30	30
O	284	30	43	30	50	31	10	104	30	49	30	59	30	59	284	30	55	80	55	32
R	335	49	2	48	58	49	21	155	49	8	49	13	49	6	335	49	8	51	18	13
B	26	12	57	13	6	12	42	206	12	46	12	54	12	53	26	12	53	50	23	45
A	81	38	21	38	23	38	28	261	38	30	38	29	38	25	..	..	..	55	25	30

Form of 'Field-Book' and entries therein.

The annexed (Table F) is a convenient form of 'Field-Book' for recording observations, and refers to fig. 102.

The 'mean bearings' having been worked out in the 'Field-Book,' the included angles from station to station are next computed by successive subtraction, deducting each 'bearing' from the next greater. The observed value of the several angles of each triangle, should then be inscribed each in its position on the 'diagram of triangulation,' (made during the location of the points), or on a rough diagram made by plotting the angles with a common protractor. With this before him, the surveyor now enters the 'mean observed angles,' polygon by polygon, in the 'correction-sheet,' (described above, page 180 *et seq.*), and proceeds with the correction as already explained.

Computation of the Sides in a Triangulation.

All is now ready for the computation of sides. The attached table (G) is a convenient form, being that used in the Topographical survey of India. The angles, &c., are those of the same triangulation as that of the 'correction sheet' just given.

Form of 'Computation Sheet.'

First, the observed angles are entered from the 'diagram of triangulation.' (In copying from document to document, always go back to the primitive document, in this case the 'diagram of triangulation.) Then by comparing the angles inscribed in the '*computation sheet*,' with those of the '*correction sheet*,' any error may be detected in time. Next the '*corrections*' are entered from the '*correction sheet*,' and being applied, the '*corrected angles*' are entered in the column provided for them. The three angles of each triangle are entered one above the other, placing the angle subtending the side known, by previous computation, in the middle. The several triangles are arranged in chains or circuits, starting with the base, or a side known from previous computation, and closing again upon it, or running from one known side, to close on another known side. The best way of doing this will be apparent from the 'diagram of triangulation.' The log sines of the angles are then looked out to single seconds and written in their proper place. If the surveyor has no one to check him, it will be well at this stage to compare the log sines just obtained, with those of the 'correction sheet.' Adding to, or deducting from the latter, the 'log differences' corresponding to the 'corrections' adopted, the two sets of log sines should agree.

Next, the middle log sine is subtracted from the one above it, and from that below it, and the remainders are entered in the column headed 'log differences.' When this operation is complete for the whole series, the log of the known side is entered in the column 'logs of sides,' in the middle line, against the angle which is opposite to it. It is then added to the 'log differences,' and the 'sums' are entered in the column 'logs of sides.' The log of the side common to the first and second triangles is carried into the middle line of the second triangle, and so on till the 'log sines' of all sides are obtained. Their values can now be looked out.

If the summation of the angles of each triangle, and the central angles, have been corrected to the last second of arc, and if the summation of 'log sines' has been corrected to the sixth figure from the left of the sum, then the computed value of last side of the series or chain of triangles should agree with the original value, to at least five significant figures from the left. If it do not, then an arithmetical error has been committed.

Other chains of triangles are now computed in like manner, until the sides of all the triangles have been determined. At this stage of the proceedings, the differences' between two values of any side should be so small as to be negligible.

Computation of Co-ordinates. The next, and final operation, is the computation of the 'co-ordinates' of the trigonometrical points. The 'bearing' of some long line, preferably near to the middle of the survey, is determined astronomically. The point at which the bearing is determined, becomes the true origin of the survey, and the 'meridian' through that point is the 'prime meridian' for the whole survey, as in traverse-surveying. For the purpose of computation it is well to assume an auxiliary origin wholly outside the survey so that all the 'meridional distances' and 'perpendiculars' will be of the same name. If the 'diagram of triangulation' has been made to scale, it will be easy to see the value that must be assumed for the 'co-ordinates' of the auxiliary origin, to attain that end.

Taking the known 'bearing' of the primitive side, a series of closed traverses are set up, which include every point on the survey. The 'bearings' of the sides are now computed from the corrected angles of the 'computation sheet,' by the precept given under traverse-surveying, working to single seconds (as the angles will be internal angles when summed, they should be arranged in series against the sun). This operation will be facilitated by inscribing the 'corrected angles' in red ink on the skeleton triangulation. As the 'corrected angles' are in every case computable as for a closed polygon, there should be no angular error. Next, the 'northings,' 'southings,' 'eastings,' and 'westings,' are computed in the usual manner, excepting that as single seconds are to be worked to, the computation should be made *logarithmically* and *not* with the *traverse table.*

The error in summation of N. S. E. W., should be exceedingly small, so much so, that it should be adjustable by a very slight correction. This done, the 'co-ordinates' are computed in the same manner as for ordinary traverse-surveying.

A convenient form of computation is shown in the accompanying table (H), being that of the triangulation already mentioned.

Arrangement of sheets for plotting. The next step is to arrange the sheets, on which the survey is to be plotted. The 'diagram of triangulation' will be useful for this purpose. First, lay off the 'prime meridian,' the 'auxiliary meridian' and the 'origin.' Then draw a number of squares or rectangles, so that the sides represent a round number of feet, according to the scale that is to be used. Each sheet should have at least three trigonometrical points upon it. It is, in every way desirable, that the sheet should not be too large. The smaller the sheet, the greater the number of draughtsmen that can be employed on the plan at once. It is well to provide a margin or overlap round each sheet say 0·1 foot wide, common to the adjacent sheets. This will be found exceedingly handy when filling in detail. It is moreover most convenient, when several sheets have to be consulted together. A point on the common margin is also most useful when making a measurement involving two

	R		179	59	40	+20	180	00	00				
5.	C (S, B)	S C B	85 46 47	24 50 45	53 45 05	−25 −23 +05	85 46 47	24 50 45	28 22 10	9·9986036 9·8629894 9·8693788	0·1356142 .. 0·0063894	3·7345874 3·5589732 3·6053626	B C B S S C
			180	00	43	−43	180	00	00				
6.	D (S, C)	S D C	57 49 73	34 13 12	05 18 15	+16 −20 +26	57 49 73	34 12 12	21 58 41	9·9263788 9·8791984 9·9810830	0·0471804 .. 0·1018846	3·6525430 3·6053626 3·7072472	C D S C S D
16.	K (O, J)	O K J	54 60 65	13 15 30	38 28 58	+12 +14 −30	54 60 65	13 15 30	50 42 28	9·9092220 9·9386697 9·9590498	$\bar{1}$·9705523 .. 0·0203801	3·7090763 3·7385240 3·7589041	J K J O O K
			180	00	04	− 4	180	00	00				
15.	L (O, K)	O L K	45 65 69	40 10 9	52 00 40	+ 2 −06 −28	45 65 69	40 9 9	54 54 12	9·8545910 9·9578567 9·9705961	$\bar{1}$·8967343 .. 0·0127394	3·6556384 3·7589041 3·7716435	K L O K O L
			180	00	32	−32	180	00	00				
8.	N (L, O)	L N O	60 80 38	04 55 59	53 32 35	−18 0 +18	60 80 38	04 55 59	35 32 53	9·9378644 9·9945302 9·7968536	$\bar{1}$·9433342 .. $\bar{1}$·8043234	3·7149777 3·7716435 3·5759669	O N O L N L
			180	00	00	00	180	00	00				
7.	M (N, L)	N M L	79 43 57	30 28 0	30 50 12	+ 2 + 4 +22	79 43 57	30 28 00	32 54 34	9·9926786 9·8376657 9·9236379	0·1550129 .. 0·0859722	3·7309798 3·5759669 3·6619391	L M N L N M
			179	59	32	+28	180	00	00				

Cardinal Points.	Vestings.	Co-ordinates. North.	Co-ordinates. West.	Point.
N.W.	..	..	..	..
N.E.	..	49854·0	58723·0	B
N.W.	805·2	55277·8	58526·8	C
S.W.	224·1	57666·9	62332·0	D
S.W.	433·1	54964·6	64556·1	E
S.W.	859·7	53906·0	70989·2	F
S.W.	186·9	51345·3	71848·9	G
S.E.	..	48431·0	73035·8	H
S.E.	..	43372·7	71268·6	J
N.E.	..	40156·6	67287·7	K
N.E.	..	41070·5	62855·7	L
N.W.	590·3	41949·6	57545·5	M
N.W.	587·3	44994·5	58135·8	A
	686·6	49854·0	58723·1	B
S.E.	..	..	..	..
N.E.	..	43372·7	71268·6	J
N.E.	..	45822·7	66370·4	O
N.E.	..	48943·8	63333·5	R
N.W.	991·5	52628·9	61564·6	S
S.W.	..	54964·7	64556·1	E
..	..	..	..	..
N.W.	..	..	..	..
N.W.	841·5	49854·0	58723·0	B
S.W.	999·1	52628·9	61564·5	S
S.W.	981·7	51152·8	66563·6	Q
S.E.	..	48385·0	67545·3	P
S.E.	..	45822·7	66370·2	O
N.E.	..	44522·4	61348·1	N
N.W.	587·3	44994·5	58135·7	A
	409·6	49854·0	58723·0	B

sheets. The sheets are prepared, and the points plotted, in the manner indicated for traverse-surveying.

Use of Satellite Stations explained.

It is sometimes desirable to observe *to* an object, *at* which a theodolite cannot be put up, although it can be placed near to it. For example, the spire of a church or a large flag-staff. In the former case though the theodolite cannot be put over the spire, a place may be found for it on the tower. The spire is observed *to*, from the surrounding objects, but they, in their turn, are observed *from* the tower with the instrument placed, perhaps, in four different positions at its corners. The problem is, to reduce the angles observed at the lateral or 'satellite' station, to what they would have been if observed at the central point *observed to*.

The data for the reduction are as follows: the 'measured distance' and 'bearing' from the Satellite to the Central Station, and the lengths of the sides A C, C B (fig. 114), computed from the side A B, (to sufficient accuracy for this purpose). Let C be the central station, observed *to* from A, and B. Let S be the satellite station from which the angles to A and B have been measured. It is desired to find the value of the angle A C B.

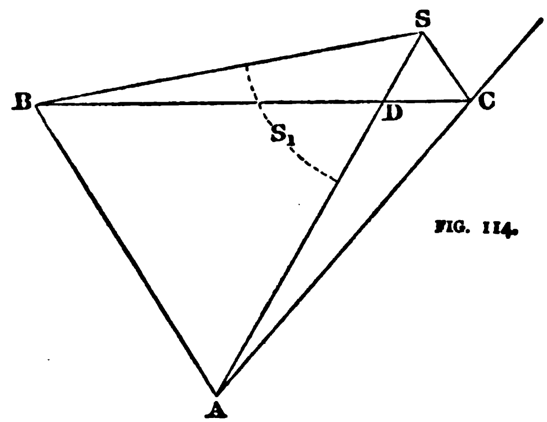


FIG. 114.

The distance S C is measured, and the angle A S C or B S C is observed. From the two observed angles at A and B, the angle at C may be computed by deducting the sum from 180°, and the lengths A C and B C computed accurately enough for the present purpose. Even, values obtained graphically by protraction would suffice.

Now the angles read at the satellite station, may be considered as 'bearings,' and the line to the central station as a 'meridian.'

Let S C O (fig. 115) (O being zero or north), represent this 'meridian,' then the field-book angles may be reduced to 'bearings,' simply by adding to, or deducting from, each angle as the case may be, some constant angle. The 'bearings' so obtained will be called 'satellite bearings,' and will be called θ_1 θ_2, &c. Draw C A_1 parallel to S A, C B_1 parallel to S B, &c.

Now it is clear that the central bearing of A or the angle O C A differs from the observed satellite bearing, by the small angle A C A_1, which we will call Δ A.

This angle is in turn equal to the angle C A S and so on, for the other points, the correction being added in the first two quadrants and deducted in the third and fourth. The length of the rays being known approximately,

$$\frac{\operatorname{Sin} \Delta A}{\operatorname{Sin} \theta_1} = \frac{SC}{CA},$$

$$\therefore \quad \operatorname{Sin} \Delta A = \frac{SC}{CA} \sin \theta_1 .$$

With the side C B and θ_2 calculate the angle Δ B and so on. In this manner a correction may be computed for each observation at S, which, when applied with the proper sign, to the original angles of the field-book, will reduce them to the central station C. Deducting each central 'bearing' successively in the usual manner, the central 'angles' at C are obtained, which may be entered on the

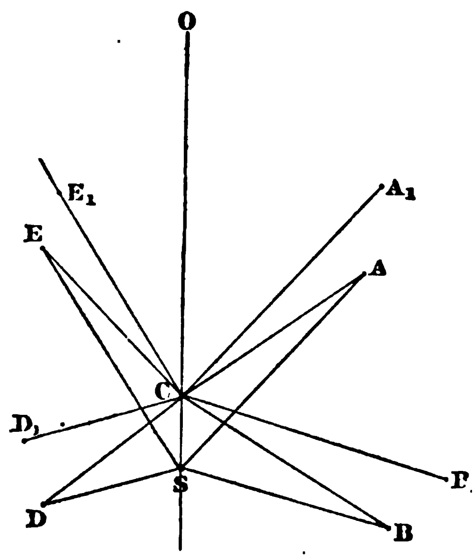


FIG. 115.

'diagram of triangulation,' and further dealt with as if they were observed angles. This procedure is rigorously correct, but is laborious. Satellite stations should therefore be avoided, though cases may occur when they have to be used.

Intersected Points. It is often convenient to determine intermediate points by observing *to them* from 'trigonometrical points,' without observing *at them*. Such points are called 'intersected points.' They should be determined from at least two triangles, and being inferior in precision to 'trigonometrical points' they should not be used for any further work of triangulation. The position of 'intersected points' can be determined with a degree of accuracy, superior to chain-measurement or traversing. They are especially convenient when the plan, or part thereof, has to be plotted to a scale so large, that a sheet will not contain a sufficient number of 'trigonometrical points,' or where the latter are not near enough together to prevent an accumulation of unavoidable error, appreciable in plotting. They also serve to fix the position of natural objects, such as spires, mill chimneys, &c. The computation of the 'intersected points' is simple.

The *observed* values of the angles O A B, O A C (fig. 116) may be obtained in two ways from the field-book, by subtracting the mean bearing of the ray A B from that of A O, or A O from A C, and of B C from B O, or B O from B A. But though the sums of the two pairs of angles will equal the *observed* angles A B C and B A C, they will not necessarily be equal to the *corrected* angles adopted in triangulation. The difference between the sum of the two observed, and the corrected angles, must be taken, and half the difference taken as the correction of each of the two angles, adding or subtracting so that the sum of the two corrected angles may be equal to the whole angle.

A complete correction might be effected by treating the triangle A B C as a polygon. To the ordinary equations of condition, a fourth must be added namely, that the sum of any pair of adjacent right and left-hand angles must be equal to the corresponding 'corrected angle' of the triangle A B C. Such an elaborate correction is rarely requisite. If the point be one of importance, it would be still better to resort to a 'satellite station' near to the object.

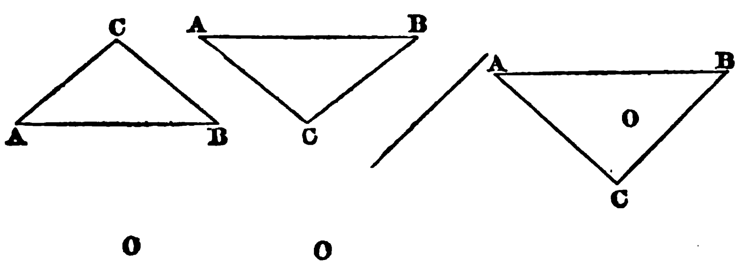


FIG. 116.

Generally, it will suffice to correct the angles so that they sum correctly. Then, deducing the value of the central angle at O (by deducting from 180°), determine O A, O B, and O C, from the two best-conditioned triangles. Compute the 'bearings' of these lines, and determine the 'difference latitude' and 'departure' of O from A, B, and C, respectively. Then, correct the 'difference latitude' and 'departure' found, by small proportional corrections, so that the sum or difference of the 'difference latitude' or 'departure' of the 'intersected point' from each 'pair of points,' agrees exactly with the 'difference latitude' of each pair of points, and adopt the mean position so obtained.

Fixing the position of a Point, from observations to three known Points.

If the position of three points is known, the position of a fourth may be obtained, by observing at it, the angles which the three points subtend.

It is necessary that a surveyor should know how to deal with this problem, in case of accident, or from having no other data, and from the necessity sometimes of bringing up the work of others. As a *system*, however, observations to three points are unsatisfactory and lazy, the method is not susceptible of minute accuracy, and there is no check, unless four points are observed, also large errors may creep in from mistakes in record, or in mistaking the stations. The observer has only to go up to one of the three known points, and observe back to the station requiring to be fixed, and the case then becomes an affair of simple triangles, checked by common sides, and this would always be done by a careful worker.

This problem may, however, often be of use in exploration work in hilly countries. For instance, having fixed the position of three or four conspicuous peaks, the surveyor may then fix the position of villages, and other points, as he proceeds on his journey. It is also useful in marine surveying to determine the position of points afloat.

Let P_1 P_2 P_3 (fig. 117) be the three points known by their ordinates, P_0 the unknown point at which the angles $P_1 P_0 P_2 = \theta_1$, and $P_2 P_0 P_3 = \theta_2$, are observed.

Bisect $P_1 P_2$, $P_2 P_3$ by perpendiculars Q R, and S T. At P_1 or P_2 lay off by protractor or natural tangent $< Q P_1 O_1 = 90 - \theta_1$.

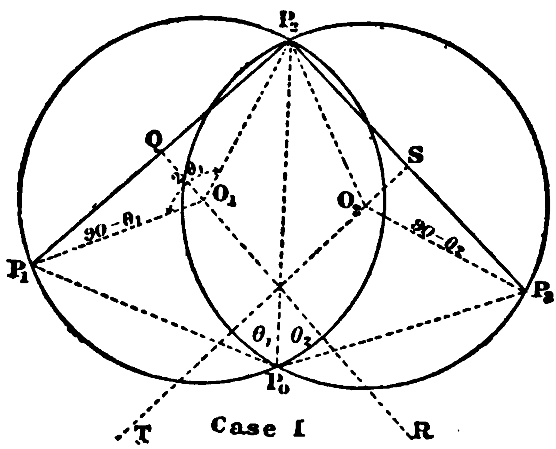


FIG. 117.

Then O_1 is the centre of a circle passing through P_1 P_2 and P_0,

For $< Q P_1 O_1 = 90° - \theta_1$, and $< P_1 Q O_1 = 90°$ $\therefore$ $< Q O_1 P_1 = \theta_1$ and $< P_1 O_1 P_2 = 2 \theta_1$.

But the angle at the centre of the circle is twice the angle at the circumference $\therefore$ $< P_1 P_0 P_2 = \theta_1$. Hence, any point on the circumference of a circle with centre O_1 and radius $O_1 P_1$ or $O_1 P_2$ subtends in the proper segment, the angle θ_1 at any point of the circumference.

Proceed similarly with P_2 and P_3, drawing through these points a segment of a circle containing the angle θ_2.

The intersection of these circles is the point sought P_0.

It will be seen that in all three cases a third circle may be drawn as a check through P_1 and P_3, using in cases I. and II. $\theta_1 + \theta_2$, and in case III. $360 - (\theta_1 + \theta_2)$, as the angle to be laid off.

There are, therefore, two ways of solving this problem in each case, and the merit of this mode of construction is that it shows at a glance, whether the data can give an accurate result, and also, which of the two solutions is that which will give the best determination.

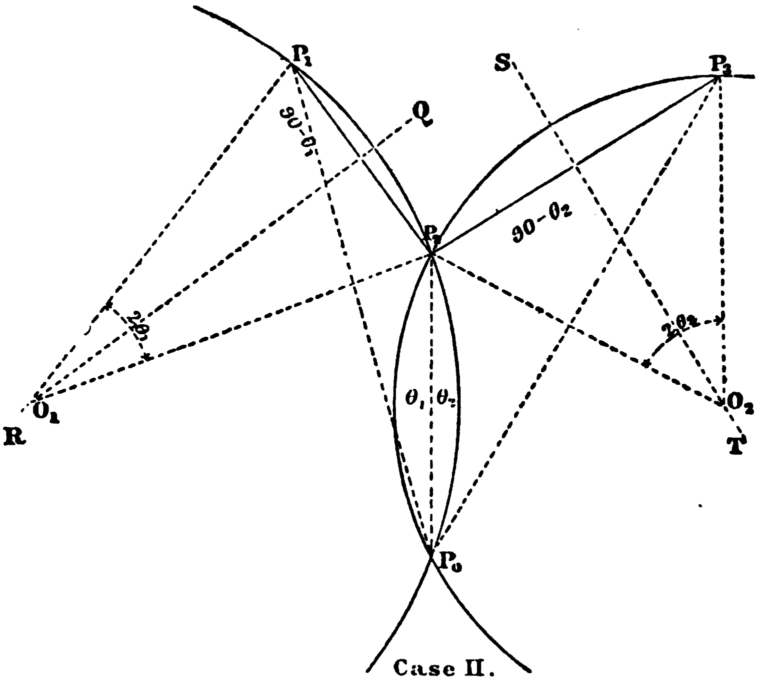


FIG. 118.

Take case I. (fig. 117). If P_1 P_2 P_3 and P_0 were so situated as to fall upon the circumference of a single circle, the problem is indeterminate, this can only

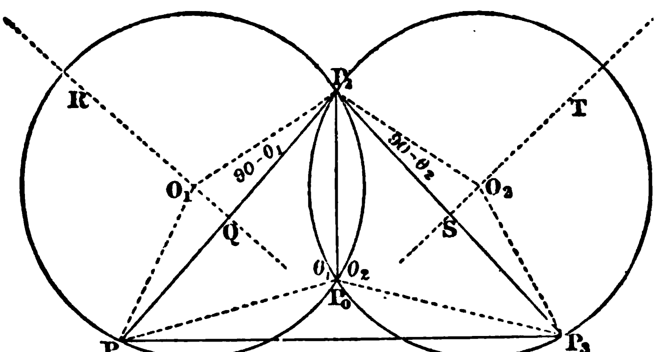


FIG. 119.

happen when the points are so situated somewhat as in case I. It cannot occur when the points are situated as in cases II. and III. (figs. 118 and 119).

Again, if the circles intersect badly, then the calculation will give results open to a range of errors, a small error in an observed angle giving a large error in the result.

In the positions sketched in case I. the intersection is fair, or at least as good as any other.

In the position of the points, shown in case II., it so happens that the circle through P_1 P_0 and P_3 cuts either of the others more nearly at right angles than they cut each other. It would therefore be better to compute with the sides P_1 P_2 and P_1 P_3, or P_1 P_3 and P_2 P_3, than with those used in the construction.

Similarly in case III. as drawn, the sides P_1 P_2, P_1 P_3 give the best intersection.

This consideration, however, really affects the selection of the final triangle to be used for the determination of the point P_0.

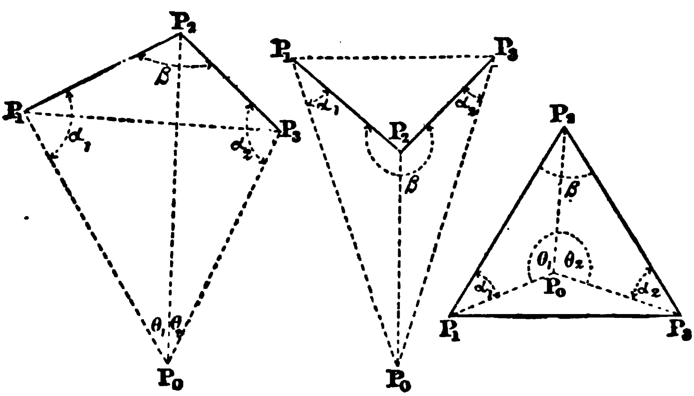


FIG. 120.

Thus, in case I. (as drawn), both the triangles P_1 P_2 P_0, and P_2 P_3 P_0, are well conditioned.

In case II. (as drawn) the triangle P_1 P_3 P_0 is better than either of the others.

In case III. (as drawn), all these are pretty much the same, but on the whole it would be better to work from P_1 P_2 and P_1 P_3.

When the co-ordinates of three points are known, the sides and angles of the triangles formed by joining them, can be computed by the ordinary rules. Generally they can be obtained direct from the triangulation sheets.

Call the angle P_1 P_2 P_3 (fig. 120) measured as shown β.

$$< P_2 P_1 P_0 = \alpha_1.$$
$$< P_2 P_3 P_0 = \alpha_2.$$

Then

$$\theta_1 + \theta_2 + \alpha_1 + \alpha_2 + \beta = 360°.$$

Therefore

$$\alpha_1 + \alpha_2 = 360 - (\theta_1 + \theta_2 + \beta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$= \text{a known quantity.}$$

If $\alpha_1 - \alpha_2$ be determined, then the value of either α_1 or α_2 can be found, but $P_1 P_2 : \sin \theta_1 : : P_0 P_2 : \sin \alpha_1$,

$$\therefore \frac{P_0 P_2}{\text{Sin } \alpha_1} = \frac{P_1 P_2}{\sin \theta_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Similarly,

$$\frac{P_0 P_2}{\text{Sin } \alpha_2} = \frac{P_2 P_3}{\sin \theta_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Divide (2) by (3).
Then

$$\frac{\text{Sin } \alpha_2}{\text{Sin } \alpha_1} = \frac{P_1 P_2 \sin \theta_2}{P_2 P_3 \sin \theta_1} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

the right-hand side involves only known quantities.
Let

$$\frac{P_1 P_2 \sin \theta_2}{P_2 P_3 \sin \theta_1} = \tan \phi,$$

$$\therefore \frac{\text{Sin } \alpha_2}{\text{Sin } \alpha_1} = \tan \phi.$$

Adding and subtracting,

$$\frac{\text{Sin } \alpha_1 + \sin \alpha_2}{\text{Sin } \alpha_1 - \sin \alpha_2} = \frac{1 + \tan \phi}{1 - \tan \phi};$$

but by the rules of trigonometry

$$\frac{\text{Sin } \alpha_1 + \sin \alpha_2}{\text{Sin } \alpha_1 - \sin \alpha_2} = \frac{\tan \frac{1}{2} (\alpha_1 + \alpha_2)}{\tan \frac{1}{2} (\alpha_1 - \alpha_2)},$$

generally by trigonometry

$$\text{Tan } (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B},$$

remembering that $\tan 45° = 1{\cdot}0000$. We may make $A = 45°$, and $B = \phi$ then

$$\text{Tan } (45° + \phi) = \frac{\tan 45° + \tan \phi}{1 - \tan 45° \tan \phi},$$

or

$$\text{Tan } (45° + \phi) = \frac{1 + \tan \phi}{1 - \tan \phi}.$$

Hence

$$\frac{\text{Tan } \frac{1}{2} (\alpha_1 + \alpha_2)}{\text{Tan } \frac{1}{2} (\alpha_1 - \alpha_2)} = \tan (45° + \phi)$$

$$\text{Cot } \tfrac{1}{2} (\alpha_1 - \alpha_2) = \cot \tfrac{1}{2} (\alpha_1 + \alpha_2) \tan (45° + \phi)$$

Hence $\alpha_1 - \alpha_2$ may be computed.
Comparing this with the value of $(\alpha_1 + \alpha_2)$ in equation (1), α_1 and α_2 are known.

Hence in each triangle we have two angles and one side, whence the remaining sides may be computed.

Again by computing the remaining angles at P_1 and P_2 in the triangle $P_1\ P_2\ P_3$, the three angles of the triangle $P_1\ P_3\ P_0$ may be computed, as may be most expedient.

Example:—

Let the ordinates of three points (fig. 121) be as follows:—

$P_1 = 5534 \cdot 0$ N	$1845 \cdot 0$ E
$P_2 = 2486 \cdot 7$ "	$1485 \cdot 0$ "
$P_3 = 0000 \cdot 0$ "	$0000 \cdot 0$ "

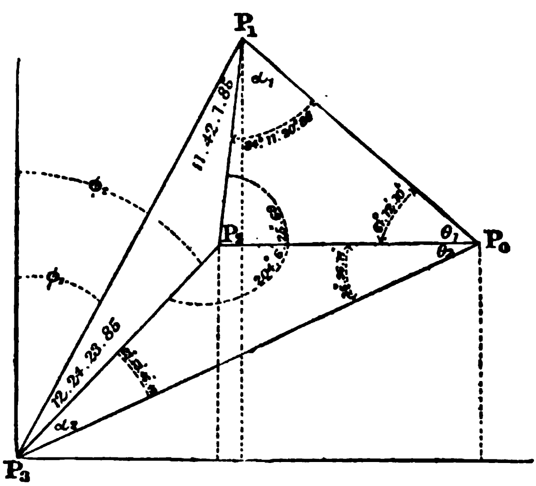


FIG. 121.

Let the following angles be observed at the point sought P_0.

	°	′	″
$P_1\ P_0\ P_2 = \theta_1 =$	61	12	10
$P_2\ P_0\ P_3 = \theta_2 =$	36	36	17
$\theta_1 + \theta_2 =$	97	48	27

The first step is to compute the bearings

ϕ_1 of P_1 from P_3

ϕ_2 of P_2 from P_3

ϕ_3 of P_1 from P_2.

Now

$$\text{Tan } \phi_1 = \frac{1845 \cdot 0}{5534 \cdot 0}, \text{ tan } \phi_2 = \frac{1485 \cdot 0}{2486 \cdot 7}.$$

$$\text{Tan } \phi_3 = \frac{1845 \cdot 0 - 1485 \cdot 0}{5534 \cdot 0 - 2486 \cdot 7} = \frac{360 \cdot 0}{3047 \cdot 3}$$

$$\text{Log } 5534{\cdot}0 = 3{\cdot}7430392$$

$$\text{Colog } ,, = \bar{4}{\cdot}2569608$$
$$\text{Log } 1845 = 3{\cdot}2659964$$

$$\therefore \text{ Log tan } \phi_1 = \text{log tan } 18^\circ\ 26'\ 17'' = 9{\cdot}5229572$$

$$\text{Log } 2486{\cdot}7 = 3{\cdot}3956234$$

$$\text{Colog } ,, = \bar{4}{\cdot}6043766$$
$$\text{Log } 1485{\cdot}0 = 3{\cdot}1717265$$

$$\therefore \text{ Log tan } \phi_2 = \text{log tan } 30^\circ\ 50'\ 40{\cdot}85'' = 9{\cdot}7761031$$

$$\text{Log } 3047{\cdot}3 = 3{\cdot}4839152$$

$$\text{Colog } ,, = \bar{4}.5160848$$
$$\text{Log } 360 = 2{\cdot}5563025$$

$$\therefore \text{ Log tan } \phi_3 = \text{log tan } 6^\circ\ 44'\ 15{\cdot}16'' = 9{\cdot}0723873$$

Hence $\beta = 180 + (30^\circ\ 50'\ 40''\ 85) - (6^\circ\ 44'\ 15{\cdot}16'')$
$\beta = 204^\circ\ 6'\ 25''\ 69$

Also,

	°	′	″
$< P_1 P_2 P_3 =$	155	53	34·31
$< P_1 P_3 P_2 =$	12	24	23·85
$< P_3 P_1 P_2 =$	11	42	1·84
	180	0	0·00

$$P_1 P_3 = 5534 \sec 18^\circ\ 26'\ 17{\cdot}00.''$$

$$\text{Log cos } 18^\circ\ 26'\ 17''{\cdot}00 = 9{\cdot}9771134$$

A.C. $0{\cdot}0228866$

$$\text{Log } 5534 = 3{\cdot}7430392$$

$$\text{Log } P_1 P_3 \quad . \quad . \quad = 3{\cdot}7659258$$

$$\text{Log cos } 30^\circ\ 50'\ 40''{\cdot}85 = 9{\cdot}9337708$$

$0{\cdot}0662292$

$$\text{Log } 2486{\cdot}7 = 3{\cdot}3956234$$

$$\text{Log } P_2 P_3 \quad . \quad . \quad = 3{\cdot}4618526$$

$$\text{Log cos } 6^\circ\ 44'\ 15{\cdot}16'' = 9{\cdot}9969904$$

$0{\cdot}0030096$

$$\text{Log } 3047{\cdot}3 = 3{\cdot}4839152$$

$$\text{Log } P_1 P_2 \quad . \quad . \quad = 3{\cdot}4869248$$

Proof :—

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

		°	′	″	
Log sin A = log sin		155	53	34·31	= 9·6111326
Log a = log $P_1 P_3$		.	.		= 3·7659258
					5·8452068
Log sin B = log sin		12	24	23·85	= 9·3321319
Log b = log $P_1 P_2$		.	.		= 3·4869248
					5·8452071
Log sin C = log sin		11	42	1·84	= 9·3070595
Log c = log $P P_3$		.	.		= 3·4618526
					5·8452069

Actual computation,

	°	′	″
$\beta =$	204	6	25·69
$\theta_1 + \theta_2 =$	97	48	27·00
	301	54	52·69
	360	0	0·00
$a_1 + a_2 =$	58	5	7·31
$\frac{a_1 + a_2}{2} =$	29	2	33·65

	°	′	″	
Log sin θ_1 = log sin	61	12	10	= 9·9426677
		Log $P_2 P_3$		= 3·4618526
				13·4045203
		Colog		6·5954797
Log sin θ_2 = log sin	36	36	17	= 9·7754583
		Log $P_1 P_2$		= 3·4869248
Log tan ϕ = log tan	35	47	13·44	= 9·8578628
	45	0	0· 0	
Log tan $(\phi + 45)$ = log tan	80	47	13·44	
Log tan $(\phi + 45)$ = log tan	80	47	13·44	10·7899590
Log cot $\frac{a_1 + a_2}{2}$ = log cot	29	2	33·65	= 10·2554855
∴ Log cot $\frac{a_1 - a_2}{2}$ = ,,	5	8	47·23	= 11·0454445

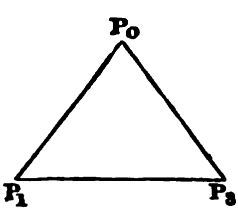
$$\begin{aligned} & \quad\ ^\circ \quad\ ' \quad\ '' \\ a_1 - a_2 &= 10 \quad 17 \quad 34{\cdot}46 \\ a_1 + a_2 &= 58 \quad\ 5 \quad\ 7{\cdot}31 \\ \hline 2\,a_1 &= 68 \quad 22 \quad 41{\cdot}77 \\ a_1 &= 34 \quad 11 \quad 20{\cdot}88 \\ a_2 &= 23 \quad 53 \quad 46{\cdot}42 \end{aligned}$$

In the two triangles there are two angles and one side given, whence the other sides and the ordinates of P_0 may be computed as usual. In the present case it would however be better to calculate from the large and well-conditioned triangle $P_1\,P_0\,P_3$ of which the side $P_1\,P_3$ is known.

$$\begin{aligned} \text{The} < P_1\,P_3\,P_0 &= (23^\circ\ 53'\ 46{\cdot}42'') + (12^\circ\ 24'\ 23{\cdot}85'') = 36^\circ\ 18'\ 10{\cdot}27'' \\ \text{The} < P_3\,P_1\,P_2 &= (34\ 11\ 20{\cdot}88) + (11\ 42\ \ 1{\cdot}84) = 45\ 53\ 22{\cdot}72 \\ \text{The} < P_1\,P_0\,P_3 &= \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 97\ 48\ 27{\cdot}00 \\ & \qquad\qquad\qquad\qquad\qquad\qquad\quad \text{Sum } 179\ 59\ 59{\cdot}99 \end{aligned}$$

$$\begin{aligned} & \quad\ ^\circ \quad\ ' \quad\ '' \\ P_1 &= 45 \quad 53 \quad 22{\cdot}72 \\ P_0 &= 97 \quad 48 \quad 27{\cdot}00 \\ P_3 &= 36 \quad 18 \quad 10{\cdot}27 \\ \hline & \ 179 \quad 59 \quad 59{\cdot}99 \end{aligned}$$

COMPUTATION OF TRIANGLE.

Triangle.	Angles.	Corrected Angles.	Log Sin.	Difference.	Sum.	Sides in Feet.	
		° ′ ″					
	P_1	45 53 22·72	9·8561247	$\bar{1}$·8601694	3·6260952	4227·61	$P_3\,P_0$
	P_0	97 48 27·00	9·9959553	..	3·7659258	5833·45	$P_1\,P_3$
	P_3	36 18 10·27	9·7724468	$\bar{1}$·7764915	3·5424173	3486·72	$P_1\,P_0$
		179 59 59·99					

$$\begin{aligned} & \qquad\qquad\quad\ ^\circ \quad\ ' \quad\ '' \\ \text{Bearing of } P_3\,P_1 &= 18 \quad 26 \quad 17{\cdot}00 \\ \text{Add } P_1\,P_3\,P_0 &= 36 \quad 18 \quad 10{\cdot}27 \\ \hline \text{Bearing of } P_3\,P_0 &= 54 \quad 44 \quad 27{\cdot}27 \end{aligned}$$

$$\begin{aligned} \log\sin 54^\circ\ 44'\ 27{\cdot}27'' &= 9{\cdot}9119828, \ \log\cos = 9{\cdot}7613826 \\ \text{Log } P_3\,P_0 &= 3{\cdot}6260952 \qquad\qquad\quad 3{\cdot}6260952 \\ \hline & \quad\ 3{\cdot}5380780 \qquad\qquad\quad 3{\cdot}3874778 \end{aligned}$$

∴ Dep. = 3452·06, and diff. lat = 2440·49

P_0 = 2440·49 N. 3452·06 E.

	°	′	″
Bearing of $P_1 P_8$ =	198	26	17·00
Deduct $P_8 P_1 P_0$ =	45	53	22·72
Bearing of $P_1 P_0$ =	152	32	54·28

log sin 152° 32′ 54″·28 =	9·6637022, log cos =	9·9481197
Log $P_1 P_0$	= 3·5424173	3·5424173
	3·2061195	3·4905370

Departure = 1607·38 E, diff. lat = 3094·12 S
P_0 = 2439·88 N. 3452·38 E.

It is to be observed, that when the sides and angles are given from the 'triangulation sheets,' the computation is not a formidable one. This method may therefore be useful in fixing intersected points from an existing triangulation.

The formula given is the neatest, and is suitable for every case, it is derived from 'Adjustment of Observations,' by T. W. Wright, B.A.

Observations from two unknown to two known points, and to each other.

There is another problem which may be useful in taking up secondary points. Angles are observed from two unknown points to two known, and to each other.

To fix the unknown points.

Let A and B be the known points, X and Y the unknown points (fig. 122).

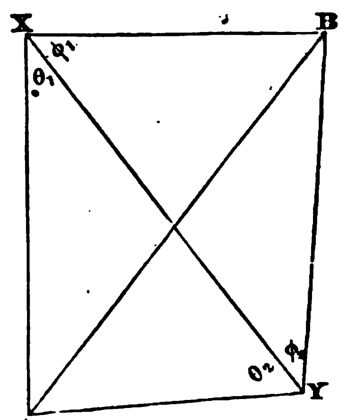


FIG. 122.

Let the angles A X Y = θ_1, B X Y = ϕ_1 be observed, as also A Y X = θ_2, B Y X = ϕ_2. Assume some length for X Y, say 1·000000 or 1000·0.

In the triangle X Y A, the angles θ_1 θ_2 are given, and the base X Y is assumed to be given as unity.

Compute A X, A Y in terms of X Y, also in the triangle X Y B compute B X and B Y also in terms of X Y.

Now in the triangle A X B you have two sides A X, B X, known, in terms of the assumed values of X Y, also the included angle $\theta_1 + \phi_1$. Hence calculate A B in terms of X Y, also calculate the angles X A B and X B A.

Do the same in A Y B.

To get the *real value* of the sides A X and A Y, &c., you have merely to say 'as the calculated value of A B is to its true value, from triangulation, so is the assumed value of X Y to its real value.' So for the other sides. That is to say, you have to add to the logarithmic value of their computed lengths, the constant.

$$\text{Log}\left(\frac{\text{true value of A B}}{\text{calculated value of A B}}\right).$$

Use of two Trig. points from any existing Grand Δn. in lieu of a Base Line.

Hitherto it has been asssumed that a base line has been measured, specially, for the survey. It may, however, happen that a Grand Triangulation exists, and that the survey in question is undertaken for the purpose of completing the map. Any two points A and B (fig. 123), of the great trigonometrical network, will serve as a base line, and it may be assumed that the distance between them is known far more accurately than any measurement can be made by the appliances obtainable by the surveyor.

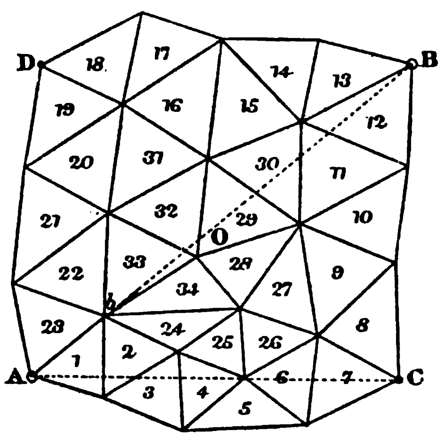


FIG. 123.

Then by measuring a base A *b*, approximately, to serve as the known side for triangle No. 1 of the minor triangulation, and working round to the point B, we have only to get the length of A B from the minor triangulation in terms of the measured base A *b*. Then, as the value of A B, so found, is to its true value from the main triangulation, so is the measured value of the base A *b*, to its true value and all the calculated sides of the minor triangulation must be adjusted in the same proportion, the adjustment being made, as in the last case, by the addition or subtraction of a constant logarithm. Other points C and D of the great triangulation may be used as checks.

Selection of Size of Theodolite.

The size of theodolite to be employed in a minor triangulation (setting aside money considerations), depends a good deal on the nature of the country to be surveyed. In any case an 8-inch theodolite of the transit pattern, would suffice for every purpose. With such an instrument 'azimuths' and 'latitudes' could be obtained with a considerable degree of accuracy. If a telegraph station were at hand, whereby time signals could be obtained from some standard timepiece, 'longitudes' could also be obtained with some degree of precision.

An 8-inch theodolite in open country, where transport is easy, is very suitable. In rough country a smaller and lighter instrument would be preferable. The larger and therefore the heavier the instrument, the more liable it is to injury in transport, and the greater is the cost and delay, in carrying it from place to place, and in setting it up. Generally, and especially in rugged country, the lightest instrument that will give the desired results, should be used. These can be obtained, (in a triangulation having one mile sides) by means of a 5-inch theodolite. In a triangulation of this description which the writer once inaugurated, and for some time directed, an old fashioned 5-inch cradle theodolite was used, by persons who had little previous experience of the work. The means available for erecting signals and the like were limited. Yet the work was so far satisfactory that in adjusting the polygons a maximum correction of 25 seconds was rarely needed. As the surveyors obtained more experience and skill, still better results were obtained.

With the perfect graduation which is now attainable, precision depends rather on design and workmanship, than on size. In the great triangulation connecting Natal with the Cape Colony, better results were obtained with a 10-inch theodolite of special construction than with an 18-inch of more ordinary construction. Moreover the results attained by means of these relatively small instruments, were superior to those obtained with the 3-foot circle in the earlier days of the Indian Survey.

Five-inch theodolites are now made by Troughton and Simms and doubtless by others, in which micrometer microscopes are substituted for verniers. By this means readings to 10 seconds and less can be obtained with ease. Setting aside the question of accuracy, it is the universal opinion, that the angles can be read by micrometers more quickly, easily, and pleasantly, than by means of verniers and that there is less liability to error. Such an instrument would doubtless be most useful for all survey work.

Methods of filling in Details.

The method to be used in filling in the detail of a triangulation, depends upon the scale of the map or plan which is to be produced. In the case of the Topographic Map of India, published on the scale of 1 inch to 4 miles, a plane table of the simplest character was used. Traversing was not resorted to, except in the case of important boundaries. The plane-table work was surveyed in the field on a larger scale and subsequently reduced by photography.

If a map on the scale of $\frac{1}{60,000}$ (a little larger than 1-inch to 1 mile), has to be produced, the plane-table, compass and chain, or even compass and pacing may be used. In this case also it would be well to plot and fill in, to a larger scale, say $\frac{1}{10,000}$, and reduce afterwards by photography. With so small a scale as $\frac{1}{60,000}$ many features such as roads and paths have to be conventionalised.

These simple methods will not suffice in the case of a plan on the scale of $\frac{1}{2500}$ or even of $\frac{1}{10,000}$. The English Ordnance Survey practice, is, to proceed at once to chain surveying, measuring the sides of triangles, taking up detail on the way, and subdividing each main triangle, by as many more lines as may be necessary to delineate detail. Traversing is avoided to the utmost, why, the writer is a loss to understand, unless indeed it be owing to the fact that at the beginning of the century, when the Ordnance Survey was inaugurated, computation of traverses, and plotting them by co-ordinates, was not generally known. The uncertain protractor was then the only means available.

As a fact, traverse-surveying may easily be conducted, with greater accuracy than general chain-surveying, for the simple reason that the line along which the measurements are made, is usually a road or path, which is favourable to accuracy, and there is every possible check.

In the survey of Malta to which reference has been made, $\left(\text{the scale being } \frac{1}{2500}\right)$ the following procedure was followed. The roads, lanes and paths were traversed, the measurements being made with steel bands. Every detail within the range of a tape, was accurately taken by offsets set out with an optical square. All these measurements were made with a degree of accuracy, which fitted them for the production of a plan on the scale of $\frac{1}{500}$, the scale that was used for detail plans of villages.

The traverses were closed on, and adjusted to the trigonometrical points. The interior details of fields and gardens, which were excessively complicated, were then filled in by means of a plane-table, not the primitive instrument described in English works on surveying, but one of the more perfect contrivances used on the Continent. It was provided with a tacheometer telescope, mounted on the sight rule, whereby distances could be read directly off a staff, held at the point to be fixed. The construction and use of this instrumemt will be described in a chapter on tacheometry (see also p. 237).

Examination of completed Surveys in the Field.

Lastly, the finished sheet should be examined, and corrected in the field. For this purpose a plane-table of simple construction will be found useful.

No important survey should be published until it has been so verified. This process, though important, is simple enough, after a little practice. The survey checker is provided with a finished sheet, or a tracing thereof, a simple plane-table (mainly to act as a drawing-board), a tape, scale, and dividers. Going to some commanding point he examines the country round, to see that no fences or buildings have been omitted. He then perambulates the roads, seeing that no buildings, or bends, in the fences have been omitted.

Sometimes an offset is overlooked, and a crooked fence plotted as though it were straight, sometimes the frontage of a building is omitted, &c. As he goes along he notes each cross-fence and its alignment, as also intersections of lines marked by natural objects. He finds that, standing in the road, with the angles of two buildings in line, he is so many feet from another building. He lays his ruler through the same objects on the plan and sees whether the distance from the outer section to the building, as given by the plan scales correctly. Again he observes that standing in the road, at a certain distance from a cross-fence, the tangent to a curve in the fence is on with the corner of a tree, and so forth. He at once makes any necessary correction, inking it in with red ink. With a little ingenuity and energy, the checker may satisfy himself that the plan is fit for publication.

Reduction of Measurements to M. S. Level.

In all geodetic operations, it is usual to give the distances between points, not merely as measured on a horizontal surface, but reduced to one horizontal surface, namely '*mean sea level*.'

Now the surface corresponding to 'mean sea level' is spherical, viz. that which would be formed by the sea, were the land removed. Fig. 124 represents a section of the terrestrial sphere through its centre. A and

B are two lofty mountains. The arc D E represents 'mean sea level.' If a canal were cut from sea to sea, the mean level of the water in the canal would correspond to the arc D E. With the same centre let arcs A C and B F be drawn parallel to D E.

It is evident that the arc F B is longer than A C, which in turn is longer than D E. In extensive surveys, it is usual to reduce all distances to the 'mean sea

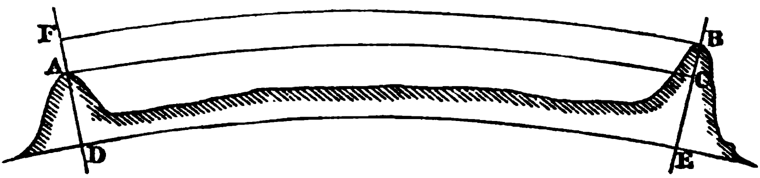


FIG. 124.

level,' and so reduced they are called 'geodetic distances.' The distances A C and B F are called the 'horizontal distances,' at the level of A and of B respectively.

To determine the corrections to be applied to reduce a measurement to M.S.L. Let A D = h_1, B E = h_2, and r = radius of sphere, or radius of the arc D E, which represents 'mean sea level.'

Then

$$\frac{AC}{DE} = \frac{r + h_1}{r},$$

and

$$\frac{FB}{DE} = \frac{r + h_2}{r},$$

also,

$$\frac{FB}{AC} = \frac{r + h_2}{r + h_1}.$$

Now as $r = 20{,}900{,}000$ feet (nearly) it is evident that, in most ordinary cases, the 'geodetic distance' differs from the 'measured distance' by a minute fraction only, less than the probable unavoidable error of measurement. Thus if h were 1000 feet then $\frac{AC}{DE} = \frac{20{,}901{,}000}{20{,}900{,}000} = \frac{100{,}005}{100{,}000}$. Or, at the rate of 5 parts in one hundred thousand.

In all triangulation work the angles measured are indeed spherical angles, and if observed with absolute correctness, would sum, in each triangle to more than two right angles. In the process of adjustment above described, not merely have unavoidable errors of observation been eliminated, but this spherical excess also, for the three observed angles of each triangle have been made to sum to two right angles. The result is that the computed co-ordinates give the relative positions of the points, as though they were on a plane, but coinciding as nearly as possible with their relative positions on the sphere. The distances are all in terms of the base. If the base line were measured at sea level, 'geodetic distances' would be determined, hence, if the base be measured at some elevation above the sea its measured length should be reduced to its 'geodetic length' by the rule given above. This done, all other 'distances' will be 'geodetic.' Beyond this 'base reduction to sea level,' no further notice of the spherical form of the globe need be taken in minor 'triangulation,' as far as horizontal measurements are concerned.

Effect of 'Spherical form of the Earth' on Vertical Measurements.

In determining the relative altitudes of trigonometrical points by vertical angles, the curvature of the earth due to its 'spherical form' must be taken into account, as it is appreciable even in short distances.

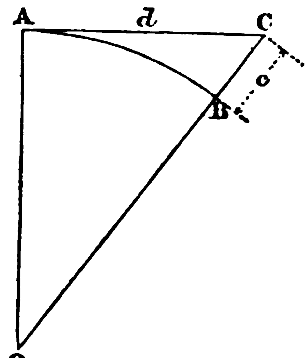


FIG. 125.

The 'curvature of the earth' may be estimated with sufficient accuracy, for practical purposes, by the following approximate method.

Let A and B be two points of equal altitude on the earth's surface (fig. 125). Let O be the centre of the sphere. Let A C be a horizontal line, drawn through the point A, that is to say a line perpendicular to the radius O A. It will cut the radius O B prolonged in some point C. The length B C = c may be called the 'curvature' for the arc A B. Let r = the radius O A = O B. Now since the angle O A C is a right angle, $(OC)^2 = (OA)^2 + (AC)^2$.

Let

$$BC = c \text{ (the curvature), and } AC = d.$$

Then

$$(r + c)^2 = r^2 + d^2$$

Hence

$$r^2 + 2rc + c^2 = r^2 + d^2.$$

Therefore,

$$c = \frac{d^2}{2r + c}.$$

But $2r + c$ differs but little from $2r$, c being always a very small quantity compared with r. Also the arc A B may be taken as equal to A C = d. Then we have the general rule. Curvature $= \frac{(\text{distance})^2}{2 \times \text{radius of earth}}$.

With the assumed mean radius of the earth the curvature for a distance of 1000 feet is 0·0239 feet.

Hence

Distance	1000 feet	curvature	= 0·0239
"	2000	"	= 0·0936
"	3000	"	= 0·2091
"	4000	"	= 0·3824
"	5000	"	= 0·5975

Curvature for One Mile.

The curvature for 1 *mile is* 8 *inches, increasing in proportion to the square of the distance in miles.* (See Table K, p. 226.)

Calculation of Vertical Measurements.

The rigorous determination of the 'difference of level' of two points by means of 'vertical angles' taking into account the curvature of the earth is performed as follows. Assume for simplicity that two theodolites are simultaneously set up at two points, on the earth's surface, and that the telescopes are respectively directed, so that the intersection of the cross-hairs of each is directed to the horizontal axis of the other. Then from the vertical angles observed, and the known 'geodetic distances' between the two points, it is required to determine the difference of level between the horizontal axes of the theodolites. Then by adding or subtracting the difference of the heights of the axes of the theodolites to determine the difference of level, of the points themselves. The following two cases may occur.

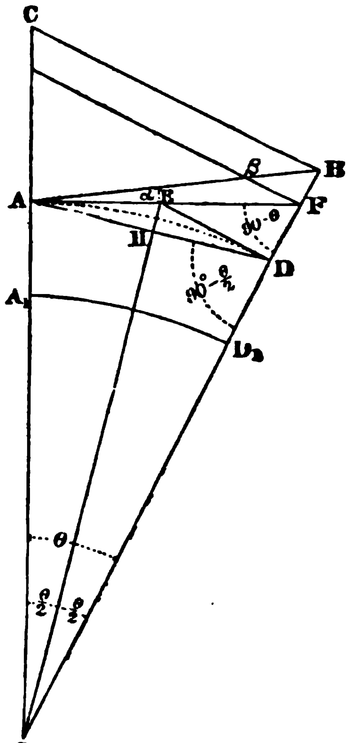


FIG. 126.

1. The difference in level is *so great*, that *one* observed angle is an '*elevation*,' the *other* a '*depression*.'

2. The difference of level being small, *is such*, that *both* angles are '*depressions*.'

Case 1. *Elevation and Depression.*

Let the angle of elevation $F A B = \alpha$ be observed at A (fig. 126), and the angle of depression $C B A = \beta$ at B. Knowing the 'geodetic distance' $A_1 D_1$ the angle A O B, A O D_1 subtended at the centre of the earth by the arc A D or $A_1 D_1$ may be computed, by dividing the 'geodetic distance' by the length of one second of arc with the radius of the earth proper to the place. Knowing the level of the point A, the horizontal distance A H D may be computed. Let the $\angle A O B = \theta$. Draw D E perpendicular to O B cutting A F in E. Join O E. Then A O B is bisected by O E, and A E = D E.

Then the exterior angle $A F O = \angle B A F + \angle A B O$

or,

$$(90° - \theta) = \alpha + (90° - \beta),$$

$$\therefore \quad \theta = \beta - \alpha. \quad \quad (1)$$

Next in the triangle A B D the exterior angle

$$A D O = \angle D A B + \angle A B D,$$

$$\therefore \quad \angle D A B = \angle A D O - \angle A B O,$$

but

$$\angle ADO = \left(90 - \frac{\theta}{2}\right) \text{ and } \angle ABO = (90 - \beta).$$

$$\therefore \quad \angle DAB = \left(90^\circ - \frac{\theta}{2}\right) - (90 - \beta)$$

$$= \beta - \frac{\theta}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1)

$$\theta = \beta - a,$$

or

$$\frac{\theta}{2} = \frac{\beta - a}{2}.$$

$$\therefore \quad \angle DAB = \beta - \left(\frac{\beta - a}{2}\right) = \frac{\beta + a}{2}.$$

But

$$\frac{BD}{AD} = \frac{\sin DAB}{\sin ABD} = \frac{\sin \frac{\beta + a}{2}}{\sin (90 - \beta)}.$$

$$= \frac{\sin \frac{\beta + a}{2}}{\cos \beta}.$$

Let Δh = difference in height between A and B, then

$$BD = \Delta h = AD \frac{\sin \frac{\beta + a}{2}}{\cos \beta}.$$

$$\Delta h = 2 r_1 \sin \frac{\theta}{2} \frac{\sin \frac{\beta + a}{2}}{\cos \beta}.$$

Where r_1 = the radius of the earth O A_1, augmented by the height of the lower point A above M.S.L.

Whence, setting aside for the present, refraction, (which will be considered later on), Δh may be computed rigorously. If one angle only were observed, the other may be computed by equation (1).

Case (2), both angles being depressions.

Let A and B be the points (fig. 127). Let the depression angle at A, be C A B = β_1, and at B, C B A = β_2. Let the $\angle$ A O B = θ as before. Draw D E perpendicular to O D, cutting A C in E. D E is parallel to B C, and consequently the $\angle$ A E D = $\angle$ A C B. Join E O. Then $\angle$ A O B is bisected by O E. The four angles of the quadrilateral O A C B must sum to 360°.

The angles at A and B are right angles.

Consequently

$$\angle AED = \angle ACB = 180^\circ - \theta.$$

Hence in the triangle A B C

$$(180^\circ - \theta) + \beta_1 + \beta_2 = 180^\circ,$$

$$\theta = (\beta_1 + \beta_2) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Since D E is parallel to B C
The $\angle$ A E D = $\angle$ A C B = $(180^\circ - \theta)$
And E O obviously bisects the $\angle$ A E D

$$\therefore \quad \angle \text{AEO} = \angle \text{DEO} = \left(90 - \frac{\theta}{2}\right).$$

But A E = E D so that

$$\angle \text{EAD} = \angle \text{EDA} = \frac{\theta}{2}.$$

Hence, in the triangle A B D the $\angle$ B A D = $\angle$ C A D − C A B = $\frac{\theta}{2} - \beta_1$.

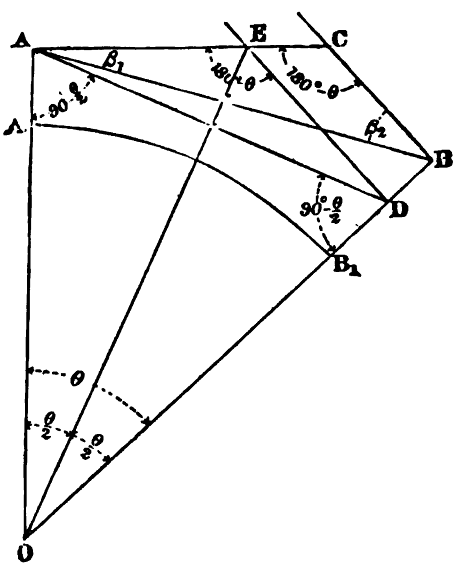


FIG. 127.

But since $\theta = (\beta_1 + \beta_2)$. . . . . from (3)

$$\frac{\theta}{2} = \frac{\beta_1 + \beta_2}{2}.$$

$$\therefore \quad \angle \text{BAD} = \frac{\beta_1}{2} + \frac{\beta_2}{2} - \beta_1 = \frac{\beta_2 - \beta_1}{2}.$$

So that in the triangle B A D the following relations obtain,

$$\frac{\text{BD}}{\text{AD}} = \frac{\sin \text{BAD}}{\sin \text{ABD}}.$$

$$\therefore \quad BD = \Delta h = AD \frac{\sin \frac{\beta_2 - \beta_1}{2}}{\sin 90 - \beta_2}.$$

$$\therefore \quad \Delta h = 2 r_1 \sin \frac{\theta}{2} \frac{\sin \frac{\beta_2 - \beta_1}{2}}{\cos \beta_2}. \qquad (4)$$

Where r_1 = mean radius of earth augmented by the elevation of the point A.

Neglecting refraction, the difference of level between two distant points may be calculated for case (1) by equations (1) and (2), and for case (2) by equation (3) and (4).

The angle θ, subtended at the earth's centre by the two stations, is called the 'subtended angle.'

It is equal to

The *sum* of two angles of *depression*, or

The *difference* of the angle of *elevation* and *depression*.

Hence, if one angle only be correctly observed, the second can be calculated by adding or subtracting it from the known subtended angle.

Half the difference between two depressions, or half the sum of an elevation and a depression, is called the 'contained angle.'

The factor $2 r_1 \sin \frac{\theta}{2}$ may in all practical cases be safely replaced by d, the horizontal distance. Even if d were 60 miles, the difference would scarcely be appreciable.

The ratio of the two factors for 60 miles is 1·000013, making therefore, an error of 0·13 feet in the height of a mountain 10,000 feet high, corresponding, with an error of less than one-tenth of a second of arc in the observed angle.

The rule is,

Multiply the horizontal distance by the sine of the contained angle, and divide the product by the cosine of the depression, or by the cosine of the greater depression, if both be depressions.

Or in logarithms.

Add,
Log horizontal distance.
Log sine contained angle.
Log secant greater depression.

A rule has already been given for calculating the 'geodetic distance' from the 'horizontal.' As we have to operate with the logarithm of the 'geodetic distance,' the operation may be simplified by the further use of logarithms.

Let

g = 'geodetic distance' measured at 'mean sea level.'
d = 'horizontal distance' at some height above 'mean sea level.'
h = height above M. S. L.
r = radius of earth at M. S. L.

Then

$$\frac{d}{g} = \frac{r + h_1}{r} \text{ (as before).}$$

The mean value of r is 20,889,000 feet.
The logarithm of g is given from the triangulation.
Putting the above in logarithms,

$$\text{Log } d = \log g + \log (r + h) - \log r.$$

Now Log r = log 20,889,000 = 7·319 9176
and log 20,899,000 = 7·320 1255

Difference = 0·000 2079

Also, between these two limits corresponding with an interval of 10,000 feet, we may obtain the logarithm of $r + h$ by proportion, as one interpolates in ordinary working.

We may say $\log (r + h) = \log r + \frac{0{\cdot}0002079\, h}{10{,}000}$.

Putting this for $\log (r + h)$

$$\text{Log } d = \log g + \log r + \frac{({\cdot}0002079 \times h)}{10{,}000} - \log r$$

$$= \text{Log } g + \frac{(0002079 \times h)}{10{,}000}.$$

The multiplication in the bracket may be performed logarithmically with ease.

Log ·0002079	=	$\bar{4}$·317 8545
Log 10,000	=	4,000 0000
Constant log	=	$\bar{8}$·317 8545

Then, *to the logarithm of the height of the station above 'mean sea level,' add the constant log* $\bar{8}$·1378, *the natural number corresponding with the sum will be the correction to be added to log g to convert it into log d.*

Example.

h = 1,463·3 feet log = 3·1653
Constant log = $\bar{8}$·3178

Natural number of correction 0·0000304 = $\bar{5}$·4831
Adding log g = 5·1346620
log d = 5·1346924

'Eye and Object' correction explained. Hitherto it has been assumed that the height of the signal observed to, above its station point, is equal to the height of instrument above the station point at which the observation is made, in other words, that the angles are truly reciprocal. This will rarely be the case, for the height of the signal will usually be greater than the height of the instrument. A correction must therefore be applied, known as the 'eye and object' correction, to reduce the observed angles to true reciprocity.

The 'eye and object' correction can be computed rigorously, taking into account the curvature of the earth. This process is laborious, and in all practical cases, unnecessary. For most purposes the following simple rule will suffice.

Take the difference between the height of the signal observed to, and that of the instrument at the point of observation. Divide by the 'horizontal distance.' The quotient is the 'eye and object' correction, expressed in circular measure.

Or in logarithms,

To constant log (*vide infra*, p. 222) . . .	4,6856
Add log 'horizontal distance'	
Sum . .	
The sum deducted from zero or arithmetical complement	
Add log difference of height	
Sum log correction in seconds	

Example.

Let	Log horizontal distance	=	3·7964
	Height of instrument	=	5·24 feet
	Height of signal	=	15·62 „
	Observed angle of depression . . .	=	− 1° 17′ 15″

Then	Log horizontal distance		3·7964
	Constant log (*vide infra*)		4·6856
	Sum . .		8·4820
	Arithmetical complement. . . .		1·5180
	Height of signal . . .	− 15·62 feet	
	Height of instrument . .	+ 5·34 „	
		− 10·28 log =	1·0120
		0° 5′ 39″ = 338″·8 log =	2·5300

The observed angle is a depression and therefore the minus sign must be applied.

		° ′ ″
Observed angle	−	1 17 15
The eye and object correction . .	−	0 5 39
		1 22 54

If we call altitude +
„ depression −
„ height of signal −
„ height of instrument +

and perform all additions algebraically there will be no difficulty in knowing the manner in which the correction is to be applied.

Example 2.

Let	Height of instrument	= 8·36 feet
	Height of signal	= 3·96 „
	Log horizontal distance	= 3·6483
	Observed angle of altitude . . .	= 2° 37′ 45″

Constant log (*vide infra*)		4·6856
Log horizontal distance		3·6483
Sum . .		8·3339
Arithmetical complement		1·6661
Height of signal . . .	− 3·96 feet	
Height of instrument . .	+ 8·36 „	
Algebraic sum . . .	+ 4·40 log =	0·6435
Correction in seconds = + 0° 3′ 24″ = 204″ log =		2·3096
Observed angle of altitude	+ 2° 37′ 45″	
Corrected angle	+ 2 41′ 09″	

It must be noted that the constant log 4·6856 is the logarithm of one second of arc, expressed in circular measure, increased by 10·0 as usual. Now the sines and tangents of small arcs are (up to a degree or so), very nearly equal to the arc in seconds multiplied by the sine, or tangent, of one second.

Thus,

Log sin 60″	= 6·4637261	Log tan 60″	= 6·4637261
Log 60″	= 1·7781513	Log 60″	= 1·7781513
Log diff. = $\log\left(\frac{\text{sin arc}}{\text{arc}''}\right)$	= 4·6855748	$\text{Log}\left(\frac{\text{tan arc}}{\text{arc}''}\right)$	= 4·6855748
Log sin 1° 0′ 0″	= 8·2418553	Log tan 1° 0′ 0″	= 8·2419215
Log 3600″	= 3·5563025	Log 3600″	= 3·5563025
	4·6855528		4·6856190
Log. diff.	0·0000218	Log. diff.	0·0000442

so that for all practical purposes, the log sine or tangent of a small angle, less than a degree, may be obtained correct to four places of decimals, by adding the constant logarithm 4·6856 to the logarithm of the angle, reduced to seconds. Conversely, we may obtain the angle corresponding to a small log sine or tangent by deducting from the said logarithm the constant logarithm 4·6856, or better, by adding its arithmetical complement 5·3144, and subsequently deducting 10·0000 from the sum. To provide for cases in which the highest accuracy is required, (extending to the second place of decimals of a second), tables are provided, in most extensive logarithm tables, giving to seven places of decimals, the value of $\log\left(\frac{\text{sin arc}}{\text{arc in secs.}}\right)$, and $\log\left(\frac{\text{tan arc}}{\text{arc in secs.}}\right)$, for every minute of arc from 0″ to 3600″. To obtain log sine or log tangent of a small arc, it is only necessary to add to the logarithm of the arc expressed in seconds, the logarithm proper to its nearest value in minutes. Conversely, to find the angle corresponding to a given log sin or log tan, find the value in the ordinary way to the nearest minute, and deduct the logarithm appropriate to that minute.

In this way small angles may be dealt with much more accurately than by interpolation, especially if the tables at hand give log sines and log tangents to one minute only.

When the difference in height of the instrument and signal is great, compared with the distance, and where the vertical angle is considerable, a closer approximation may be obtained by multiplying the 'eye and object' correction, found as above, by the square of the cosine of the angle of inclination.

The limits beyond which it would be desirable to apply this second consideration, may be seen by an inspection of the following table. In the first column are the various angles of 'elevation' or 'depression.' Entering the table with the nearest vertical angle, the error that would be produced by the neglect of the correction of the 'cosine squared' of the vertical angle will be found under the column which is headed with the eye and object correction, as determined by the general rule.

TABLE I.—GIVING 'EYE AND OBJECT' CORRECTIONS.

Correction as calculated by general rule.										
h/d.	·001	·002	·003	·004	·005	·006	·007	·008	·009	·01
Corrn. in secs.	206·3	412·5	618·8	825·0	1031	1238	1444	1650	1856	2063
logm.	2·31441	2·61544	2·79153	2·91647	3·01338	3·09256	3·15950	3·21750	3·268[illegible]5	3·31441
Vertical Angle.	**Error in Seconds.**									
1°	·06	·13	·19	·25	·31	·38	·44	·50	·57	·63
2°	·25	·50	·75	1·00	1·26	1·51	1·76	2·01	2·26	2·51
3°	·56	1·13	1·69	2·26	2·82	3·39	3·95	4·52	5·08	5·65
4°	1·00	2·01	3·01	4·01	5·02	6·02	7·03	8·03	9·03	10·04
5°	1·57	3·13	4·70	6·27	7·83	9·40	10·97	12·54	14·10	15·67
7½°	3·51	7·03	10·54	14·06	17·57	21·08	24·60	28·11	31·63	35·14
10°	6·22	12·44	18·66	24·88	31·10	37·32	43·54	49·76	55·98	62·20
12½°	9·66	19·33	28·99	38·65	48·31	57·98	67·64	77·30	86·96	96·62
15°	13·82	27·63	41·45	55·27	69·09	82·90	96·72	110·5	124·4	..
17½°	18·65	37·30	55·95	74·61	93·26	111·9	130·6	..	..	..
20°	24·13	48·26	72·39	96·51	120·6	..	..	..	..	..
25°	36·84	73·68	110·5	147·4	..	..	..	..	..	..
30°	51·57	103·1	154·7	..	..	..	..	..	..	..

Having found the 'eye and object' correction by the rule given, the computer can see by entering the table with the vertical angle, whether the result obtained by the rule, is liable to an error of one, two, or more seconds, up to two minutes. He can at a glance see whether, according to the degree of accuracy which is attainable with the instrument at his disposal, it is worth while to apply the second correction. If it be, then it is only necessary to write down the log

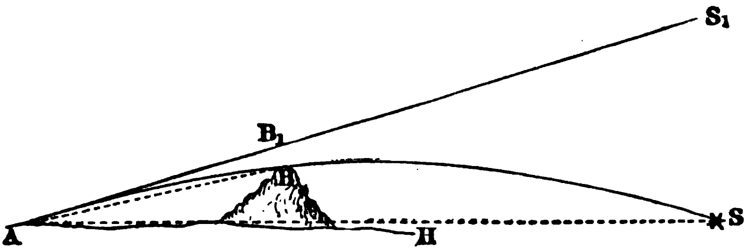


FIG. 128.

cosine of the vertical angle twice, and add up. The sum will be the corrected log sine of the 'eye and object' correction in seconds. Examples will be given later on, *vide* p. 230.

Effect of Refraction on Vertical Angles.

The effect of refraction has now to be considered. Rays of light passing through the atmosphere, nearly parallel to the earth's surface, do not travel in a straight line as they do in space (fig. 128). Owing to the varying density of the air, due to differences of pressure, and to irregular heating, they travel in a curved line. Usually, almost invariably, the path of the ray is *concave* towards the earth's surface, the effect of this being to make an object appear *higher* than it *really is*.

Thus if O be a signal and A the point of observation (fig. 129), the rays of

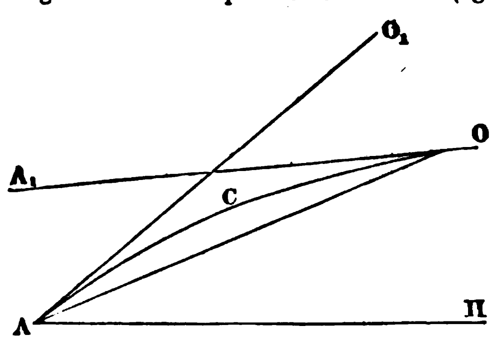


FIG. 129.

light would not proceed along the straight line O A, but along a curved line O C A. The consequence is that the rays enter the eye of the observer at a greater angle than O A H. They enter it as if O C A were a tangent at A to that curve, and consequently the object O appears as though it were at O_1, and an erroneous angle O_1 A H is observed. The angle O_1 A O is the 'refraction error.' If now the eye were removed to O and directed to A, then O A_1 will also be tangent to the curve O C A and the apparent position of A would be at A_1. Assuming that the curve is symmetrical, then the angle A_1 O A will be equal to O_1 A O.

By taking reciprocal angles, 'refraction errors' may be eliminated, by thus ascertaining their values, and applying the same as corrections.

The observed angles, corrected by applying the 'eye and object correction' may be called the 'observed reciprocal angles,' and be represented by A and D or D_1 or D_2 as the case requires.

Then

$$\left.\begin{aligned} D &= (\beta - \rho) \\ A &= (\alpha + \rho) \\ D_1 &= (\beta_1 - \rho) \\ D_2 &= (\beta_2 - \rho) \end{aligned}\right\}$$

D, A, D_1 and D_2 being observed angles corresponding to the corrected angles, β, β_1, β_2, and α.

Where ρ is the refraction error.

Then, in the two cases

$$\begin{aligned} D - A &= (\beta - \rho) - (\alpha + \rho) & (D_1 + D_2) &= (\beta_1 - \rho) + (\beta_2 - \rho) \\ &= (\beta - \alpha) - 2\rho & &= (\beta_1 + \beta_2) - 2\rho \\ &= \theta - 2\rho & &= \theta - 2\rho \end{aligned}$$

where, as before, θ = the 'subtended angle' as already defined.

Hence

$$\rho = \frac{\theta - (D - A)}{2} \text{ and } \rho = \frac{\theta - (D_1 + D_2)}{2}.$$

The rule then is, (1) *Subtract the angle of elevation from the depression angle, and subtract the remainder from the subtended angle. Half of the final remainder will then be the 'refraction error.'*

Or, (2) *Add together the two depression angles, and subtract the sum from the subtended angle. Half the difference is the 'refraction error.'*

If the refraction be fairly regular, and if the reciprocal angles be observed simultaneously, or under similar conditions of the atmosphere on different days, the refraction will be determined with sufficient accuracy for all practical purposes, for the correction for refraction is usually but small. It is generally expressed as a fraction of the subtended arc, and amounts to about one-fifteenth part thereof (*vide infra*).

If the negative sign be always ascribed to the 'refraction error' and to 'depressions,' and the positive sign to 'elevations,' the algebraic sum will be the '*corrected angle*.'

Thus, if $\rho = 10''$

$$\begin{aligned} \text{and the observed angle of elevation } A &= + \;\; 2^\circ\, 10'\, 43'' \\ \rho &= - \;\; 0 \;\; 0 \;\; 10 \\ \text{then, the corrected angle } \alpha &= + \;\; 2 \;\; 10 \;\; 33 \end{aligned}$$

$$\begin{aligned} \text{If the observed angle of depresssion } D &= - \;\; 3^\circ\, 17'\, 06'' \\ \rho &= - \;\; 0 \;\; 0 \;\; 10 \\ \text{the corrected angle } \beta &= - \;\; 3 \;\; 17 \;\; 16 \end{aligned}$$

It must not be forgotten that the correction thus obtained, includes all errors of observation.

'Refraction' has also the effect of reducing the 'curvature' already treated of. Usually it reduces it by about one-seventh of its true value. The second column of the following table, from the 'Manual of Surveying for India,' gives the combined effect of curvature and refraction, as determined in this manner.

TABLE K.—GIVING THE DIFFERENCE OF TRUE AND APPARENT LEVELS.

Distance in Chains.	Correction.		Distance in Chains.	Correction.	
	Curvature in Decimals of a Foot.	Curvature and Refraction in Decimals of a Foot.		Curvature in Decimals of a Foot.	Curvature and Refraction in Decimals of a Foot.
1	·000104	·000089	11	·012610	·010809
2	·000417	·000358	12	·015007	·012863
3	·000938	·000804	13	·017613	·015097
4	·001668	·001430	14	·020427	·017509
5	·002605	·002233	15	·023450	·020100
6	·003752	·003216	16	·026680	·022869
7	·005107	·004378	17	·030120	·025817
8	·006670	·005717	18	·033767	·028943
9	·008442	·007236	19	·037623	·032248
10	·010422	·008933	20	·041687	·035732

Now the value of error of refraction though usually small, is very uncertain. It varies at different hours of the day, being least (in the tropics) at about 3h. 45m. P.M. It varies also with temperature, and barometric pressure. Over highly heated arid plains, it becomes exceedingly irregular, the curve sometimes being actually convex towards the earth, so that objects are *depressed* and not *raised*. The mirage is an extreme case of this. The rays of light proceeding from the sky become so much bent upwards, on encountering the highly-heated stratum of air, immediately above the parched ground, that the observer, looking in a nearly horizontal direction sees, not the ground and the objects thereon, but an image of the sky, and the appearance is that of a sheet of water. Horizontal deviation is also very common. A series of rods, ranged out in a straight line at one time, have presented the appearance of a curve at another time. When disturbance is so great as this, accurate observation of any kind is obviously impracticable. At any time, however, the effect of refraction is uncertain, and accurate results can only be arrived at, by taking reciprocal angles under similar atmospheric conditions.

'Terrestrial refraction' must not be confounded with 'celestial or astronomical refraction,' that is to say, the refraction which affects the observation of heavenly bodies. 'Celestial refraction' at low angles, is greater than 'terrestrial,' because the rays from the stars traverse the whole depth of the atmosphere, whilst rays from a terrestrial object, pass through a limited section of it only, and in a comparatively horizontal direction.

Procedure when Reciprocal Angles are not obtained.

It may, however, happen that it is impossible to obtain reciprocal observations. For example, it may be required to determine the height of an inaccessible peak, the distance of which has been determined by intersection. In this case the only plan is to assume a value for the correction for refraction.

Value of Refraction as a Fraction of 'Subtended Arc.'

It is the general practice of geodetic writers to express this value as a fraction of the 'subtended arc,' that is, the angle subtended at the centre of the earth between the two stations, or the angle θ in fig. 130. Usually, this fraction is found to be about $\frac{1}{13}$. Hence it is necessary to add or deduct this fraction of the contained arc to the observed angle of 'depression' or 'elevation' as the case may be.

Method of calculating Altitudes in practice.

The rigorous method of calculating altitudes has been discussed, mainly with the view of explaining the manner in which terrestrial refraction is determined, but in a 'minor triangulation,' so operose a procedure is rarely necessary, and would not be justified.

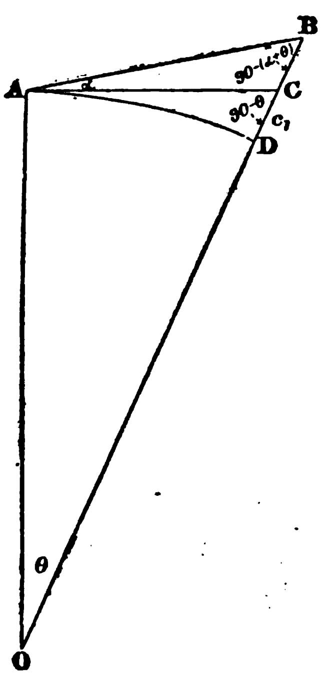


FIG. 130.

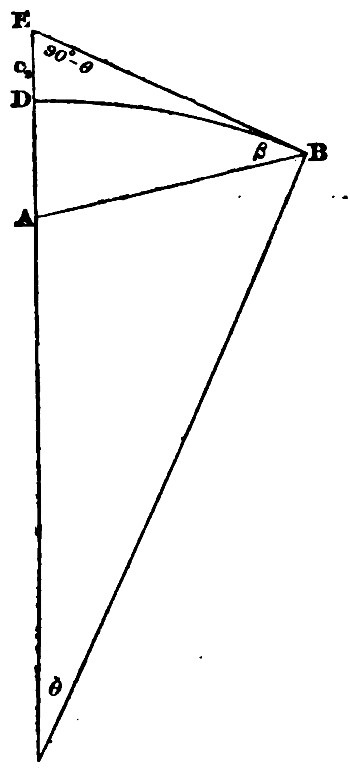


FIG. 131.

The following simplified method will suffice, as shown by investigation.

Let A and B (fig. 130) be two points of different elevation. Let the angle

B A C be observed at A. Then the difference of level B D is made up of two parts added together, namely B C and C D. Now, setting aside refraction for the present, C D is the correction for curvature, which can be computed with practical accuracy by the formula already given, or taken from a table. Rigorously, the distance B C is obtained from the triangle A B C as follows.

Let $\quad BAC = a$

$\quad ACB = 90° + \theta$

and $\quad ABC = 90° - (a + \theta)$

also $\quad r_1 = \text{radius OA, and } r_2 = \text{OB}.$

Then $$\frac{BC}{AC} = \frac{\sin a}{\sin\left(90° - (a + \theta)\right)} = \frac{\sin a}{\cos(a + \theta)}$$

and $\quad AC = r_1 \tan \theta.$

Hence $$BC = r_1 \tan \theta \times \frac{\sin a}{\cos(a + \theta)}. \qquad (1)$$

But the difference of level $\Delta h = BD = BC + CD$, where $DC = c_1$ is the curvature due to radius O A.

Again, in the case in which the angle of 'depression' is observed at B (fig. 131). The true difference of level between B and A is A D which is equal to $AE - ED$, E D being the curvature due to the radius $OB = r_2$.

Now in the triangle A B E

$$ABE = \beta$$
$$AEB = 90° - \theta$$
$$EAB = 90° - (\beta - \theta).$$

Then $$\frac{AE}{BE} = \frac{\sin \beta}{\sin\left(90° - (\beta - \theta)\right)} = \frac{\sin \beta}{\cos(\beta - \theta)}.$$

And $\quad BE = OB \tan \theta = r_2 \tan \theta.$

Hence $$AE = r_2 \tan \theta \frac{\sin \beta}{\cos(\beta - \theta)}. \qquad (2)$$

$$\therefore AD = \Delta h = r_2 \tan \theta \frac{\sin \beta}{\cos(\beta - \theta)} - c_2$$

where c_2 is the curvature due to radius O B.

Now, in the cases which will occur in practice, the subtended angle θ will rarely exceed 60″, and may be omitted from the value $\beta - \theta$, not affecting the value of the cosine in five places of decimals.

Hence the factors $\frac{\sin a}{\cos(a + \theta)}$ and $\frac{\sin \beta}{\cos(\beta - \theta)}$ may be replaced by $\tan a$ and $\tan \beta$. Again $r_1 \tan \theta$ will not differ appreciably from $r_2 \tan \theta$ and both will closely approach to the horizontal distance at the station, whose elevation is given.

Hence we may say (representing horizontal distance by H)

$$\Delta h = \text{H} \tan \alpha + c_1 \text{ or } \Delta h = \text{H} \tan \beta - c_2$$

c_1 and c_2 represent the curvature expressed in linear measure, and may be taken from the table given. They will be practically identical.

To take into account refraction, it is merely necessary to diminish the curvature by one-seventh part of its value. The results are tabulated in the second column of the above table under the heading 'curvature and refraction.'

Hitherto it has been assumed as before that the angles are truly reciprocal. A little consideration will show that the formula gives the elevation of the signal above the axis of the instrument, so that to determine the 'altitude' the height of the instrument must be $\frac{\text{added}}{\text{deducted}}$ and the height of signal $\frac{\text{deducted}}{\text{added}}$ according as the angle is an 'elevation' or 'depression.'

Hence the following rule is obtained.

Multiply the horizontal distance by the tangent of the vertical angle. The product is the difference of level between the eye and the signal.

Mark this, + if a rise, − if a fall.

Mark height of instrument always +

Height of signal, *observed to*, always −

Curvature and refraction, always +.

Sum algebraically, that is to say,

Sum the positive and the negative quantities separately, and take the lesser from the greater, the result being the difference of level between the two stations.

Proceed in the same manner at the second station, take the arithmetical mean of the two differences of level (that is, half the sum of the rise, and of the fall) as the correct difference.

The following examples show the results of computation by the 'rigorous' and 'approximate methods,' respectively. The data are ideal, and devoid of instrument error. The object of these examples is not to give an idea of the degree of accuracy that is attainable by means of vertical angles, but merely to show the comparative effect of the two methods of computation.

Example 1 :—*Calculations by 'rigorous computation.'*

The level of station A being known, to find the levels of B and C.

Data.

Level of station A	275·36 feet
Mean observed angle of elevation at A . .	2° 43′ 4″
Height of instrument above station A . .	4·32 feet
Height of signal at A	11·36 feet
Mean observed angle of depression at B to A .	2° 33′ 32″
Height of instrument at B	5·11 feet
Height of signal at B	13·93 feet
Log 'geodetic distance' at A B . . .	3·7244888
Observed angle of depression at B to C . .	1′ 0″

Height of instrument at B	4·12 feet
Height of signal at B	5·11 feet
Observed angle of depression at C to B . .	0° 0′ 15″
Height of instrument at C	4·75 feet
Height of signal at C	3·75 feet
Log 'geodetic distance' B to C . . .	4·099,0934

(*a*) To find the subtended angle A to B.

Log 'geodetic distance'	3·72449
Deduct log feet in one second . . .	2·00549
Log subtended angle in seconds = log 52″·4 .	1·71900

(*b*) To find the log of 'horizontal distance' of A B at level of A.

Log height of station A = log 275·36 feet .	2·4399
Add constant log	$\bar{8}$·3178
Sum . . .	$\bar{6}$·7577

The natural number corresponding to this logarithm is	0·0000057
Adding log 'geodetic distance' . . .	3·7244888
The sum is log 'horizontal distance' . .	3·7244945

(*c*) To find the 'eye and object correction' for angle at A.

Height of signal at B	13·92 feet
Height of instrument at A	4·32 feet
Difference . . .	9·60 feet

Log 'horizontal distance'	3·72449
Log sin 1″	4·68558
Sum . . .	8·41007

Arithmetical complement	1·58993
Log 9·6 feet (diff. signal and inst. height) . .	0·98227
Log cosine 2° 43′	9·99951
The same (to square value)	9·99951
Log 'eye and object correction' for angle at A = log 373	2·57122

Signal higher than instrument *deduct* from angle of elevation.

Note.—The 'eye and object correction,' computed by the rigorous method, agrees with the above to a fraction of a second.

(*d*) 'Eye and object correction,' for observed angle at B.

Height of signal at A . . .	11·36	
Height of instrument at B . .	5·11	
Difference . .	6·25 Log	0·79588
Log cos 2° 34′ to nearest minute . . .		9·99956
The same again (to square value) . . .		9·99956
Arithmetical complement of 'log distance,' log sin 1″ (from *c*)		1·58993
Sum log correction in seconds = log 243″ . .		2·38493

Correction 4′ 03″.

(*e*) Correction for refraction.

	° ′ ″	° ′ ″
Observed depression at B . . .	2 33 32	
Add 'E and O correction,' 243″ (*d*) = 4′ 03″	4 03	
Sum = corrected depression		2 37 35
Observed altitude at A	2 43 04	
Deduct 'E and O correction,' 373″ (*c*)	6 13	
Sum corrected altitude		2 36 51
Difference		0 0 44
The subtended angle is (*a*)		0 0 52
The difference is twice the refraction . .		0 0 8
The correction for refraction is half the above, or		0 0 4

To be *added* to 'depression' *deducted* from 'elevation angles.'

(*f*) Computation of difference of level B from A.

	° ′ ″	° ′ ″
The apparent depression . . .	2 37 35	
Add refraction	0 0 4	
Corrected depression		2 37 39
The apparent elevation	2 36 51	
Deduct refraction	0 0 4	2 36 47
Sum . . .		2)5 14 26
Half sum = contained angle.		2 37 13

Log sin contained angle = log sin 2° 37′ 13″ .	8·6600733
Log secant depression = log secant 2° 37′ 39″ .	0·0004568
Log 'horizontal distance'	3·7244945
Log 'difference of level,' 242·69 feet . .	2·3850246

(*g*) Calculation of level.

Level of station A		275·36 feet
Add rise A to *b* from (*f*)		242·69 "
Level of station B		518·05

(*h*) To find the subtended angle B to C.

Log geodetic distance . . .	4·09909
Deduct log feet in one second . .	2·00549
Log subtended angle in seconds = log 124" . . .	2·09360

(*j*) To find the log of 'horizontal distance' of B C at level of B.

Log elevation of B = log 518·05 .	2·7144	
Constant log	$\bar{8}$·3178	
	$\bar{5}$·0322	
The number corresponding to this log is . .		0·0000108
The given log geodetic distance . . .		4·0990934
The log horizontal distance B C . . .		4·0991042

(*k*) To find the 'eye and object correction' for angle at B.

Here we may proceed by the simple method of tangents.

Log horizontal distance			4·0991
Log tan 1 second			$\bar{4}$·6856
			$\bar{8}$·7847
Arithmetical complement			1·2153
Height of instrument at B . .	4·12		
" signal at C . . .	5·73		
	1·61	log	0·2068
Log of 'eye and object correction' in seconds = log 26"			1·4221

(*l*) 'Eye and object correction' for angle at C.

Arithmetical complement from B . . .			1·2153
Height of signal at B . . .	5·11		
Height of instrument at C . .	4·75		
Difference	0·36	log	$\bar{1}$·5563
Log 'E and O correction' at C = log 5·91" .			0·7716

Say 6".

(*m*) Determination of refraction from B.

	° ′ ″	° ′ ″
Observed depressions at B	0 1 0	
Add 'eye and object correction'	0 0 26	
The sum is 'apparent depression'		0 1 26
Observed depression at C	0 0 15	
Add 'eye and object correction'	0 0 6	
The sum is 'apparent depression'		0 0 21
The sum of the two depressions is		0 1 47
But the subtended angle is		0 2 4
The difference is		0 0 17

Half of which is the correction for refraction to be added to both angles.

Say 8″ and 9″ respectively.

(*n*) Calculation of difference of level.

	° ′ ″	° ′ ″
The angle of depression at B	0 1 26	
Add refraction	0 0 8	
The sum is the corrected angle		0 1 34
The angle of depression at C	0 0 21	
Add refraction	0 0 9	
The sum is the corrected angle		0 0 30
The difference is		0 1 4
The half difference is the contained angle		0 0 32

Then log 32″	1·5051500
Log sin 1″	4·6855749
Log secant 0° 1′ 34″	10·0000000
Log horizontal distance	4·0991040
Difference of level 1·95 feet	0·2898289

Level of B	518·05
Fall from B to C	1·95
Level of C	516·10

Example 2.—Calculations of same by approximate formula.

(f_1) Computation of difference in level B from A.

Angle at A = 2° 43′ 04″ log tan	8·67642
Log 'geodetic distance'	3·72449
log + 251·72 =	2·40091

		Feet.
Apparent rise A to B	+	251·72
Height of instrument at A	+	4·32
Height of signal at B	−	13·92
Curvature of refraction	+	0·58
The algebraical sum is the apparent difference of level, and as the *plus* quantities are in the majority, it is a *rise*. . . .		242·70 feet.

Computation of difference in level A from B.

Angle at B 2° 33′ 32″ log tan	8·65021
Log horizontal distance	3·72449
log − 236·97 =	2·37470

Apparent fall B to A	−	236·97
Height of instrument at B	+	5·11
Height of signal at A	−	11·36
Curvature and refraction	+	0·58
The algebraical sum = difference level B to A .		242·64
The arithmetical sum of the two differences of level is		2)485·34
Mean difference of level B above A		242·67
As against, by the 'rigorous method' . .		242·69″

The difference being 0·02 foot, represents less than one second of arc, a quantity far smaller than the probable error in the angular measurement.

Computation of difference in level C from B.

Observed angle at B 0° 1′ 0″ log tan. . .	6·46373
Log horizontal distance	4·09910
log . .	0·56283

Apparent difference of level . .	−	3·57
Height of instrument at B . .	+	4·10
Height of signal at C	−	5·73
Curvature and refraction . .	+	3·24

Algebraical sum = difference of level B to C . − 1·96

Angle at C = 0° 0′ 15″ . . .		1·17609
Log tan 1 second		4·68557
Log 'horizontal distance' . .		4·09910
log . .		9·96076
Apparent Depression . . .	−	0·91
Height of instrument at C . .	+	4·75
Height of signal at B . . .	−	5·11
Curvature and refraction. . .	+	3·24

The algebraical sum of the above is . . .	+	1·97
(As the *plus* quantities are in the majority, the difference of level is a *rise*) . . .		
The arithmetical sum of the *rise* and *fall* is .		3·93
Half this is the mean difference of level . .		1·96

A result almost identical with that obtained by the rigorous method.

Hence it appears that the more simple method of tangents, gives, in the cases which are likely to occur in the practice of 'minor triangulation,' results which differ inappreciably from those which are obtained by the rigorous, but laborious process, and the errors are less than those of observation when an ordinary 5-inch or 6-inch theodolite is used.

The actual computation indeed, is not more difficult by the rigorous methods. It is the numerous corrections that take time. If much work had to be done, tables might be prepared which would greatly facilitate work, for instance a table could be made giving by inspection, the 'subtended angle' in terms of the 'distance.'

A table on this principle is given in the Manual of Surveying for India. Nor would it be difficult to prepare a subtense-table, giving the 'eye and object correction' by inspection. If such tables were to hand, the 'rigorous method' would be as quick as the 'approximate.'

Observation of Vertical Angles.

The 'vertical angles' will be observed simultaneously with the 'horizontal angles' of the triangulation. If the vertical limb be furnished with two verniers, both should be read. A Reading 'face right' and 'face left,' should be taken, and the 'mean vertical angles' deduced in the ordinary manner.

At each station, the 'height of instrument' and 'of signal' must be recorded. If the trigonometrical point be in a position difficult of access, it will be well to establish near to hand, a bench-mark in some more convenient position, by using the theodolite as a level, or by observing an angle, and measuring a distance.

After the differences of level have been computed, they should be arranged in closed circuits, returning to the starting point. The '*rises*' and '*falls*' should be arranged in columns, similar to those of an ordinary levelling book. When the circuit is complete, the sum of the 'rises' should be equal to the sum of the 'falls.' It is not probable that the summation will be exact. Judicious corrections being made, the 'reduced levels' may be worked out as in ordinary levelling. Moreover, matters may be so arranged as to have interlacing circuits, whereby several values of each point may be obtained each from a distinct circuit. In this way a very fair average value may be obtained.

Desirability, or the reverse, of Vertical Angle determination of Heights.

The question now arises as to the circumstances under which it is desirable, or the reverse, to determine heights by triangulation. The observation of vertical angles adds very materially to the time which would be occupied, in observing horizontal angles alone. It does not, however, add materially to the total cost of triangulation, for the time actually spent in observing, is small when compared with that spent in going from point to point, in locating points, and in putting up signals, work which has to be performed in any case. The computation of the differences of level is somewhat laborious, the result being, that the levels of a number of points, distant from each other by from half a mile to a mile, are determined with approximate accuracy. The degree of accuracy will not (with the instruments likely to be used in connection with minor triangulation) be equal to that of good levelling, by which moreover a large number of intermediate points would be determined.

The question of the desirability of resorting to vertical angles, also depends upon the class of map or plan which it is desired to produce. If the triangulation be undertaken for the purpose of preparing a small-scale topographical map, and if the country be rugged, making levelling expensive, then vertical angles will be valuable, and the approximate levels of a number of points will be determined, which will be most useful as a basis for sketching in topographical detail.

If on the other hand, it be desired to produce at once, large-scale plans with the fullest detail; and if the country be open and level, so as to make levelling easy and cheap, then the observation of vertical angles is scarcely worth the trouble and cost. Levelling will give at least as accurate results, and the levels of numerous intermediate points may be determined without appreciable additional cost. The computation of levels is a matter of the utmost simplicity, and the system may be arranged so as to give complete checks on the work. Levelling, moreover, is an operation which may be deputed to persons of limited attainments. If levelling operations are deferred, until the detail plan is finished, the surface of the ground may be covered with a number of spot-levels and bench-marks at a cheap rate. It is only necessary to give the leveller a tracing of the ground, over which he is to extend his operations, and he can locate upon it in the field (often without any measurements, but in any case by most simple ones), the positions of the spot-levels which he has taken, indicating the position of the staff by means of a number on the plan, corresponding with the proper entry in the field-book.

On these principles therefore must the question of observing vertical angles or not, be determined.

CHAPTER VI.

ON THE PLANE-TABLE AND METHOD OF USING IT.

Preliminary Remarks.

THE plane-table is usually regarded in England as an instrument of secondary accuracy, ranking little above the prismatic compass, and suitable only for topographical work on a small scale. As usually constructed and described, it is not adapted to the preparation of accurate and large-scale plans. On the Continent, however, it has received many developments, and is classed as an instrument of precision, only second to the small theodolite. The uncertain climate of England is not favourable to the

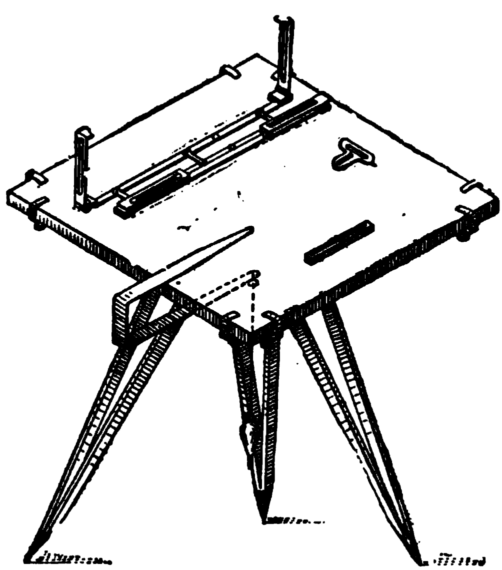


FIG. 132.

use of the plane-table, hence its neglect. In the Tropics, on the other hand, and in Continental climates generally, it is of the utmost use, and for purposes of instruction in surveying it is invaluable.

Construction of a simple form of Plane-Table.

The construction of the plane-table, in its simplest form, is so well known as scarcely to need description. A board about 1′ 6″ or 2′ 0″ square (fig. 132), made of well-seasoned wood, is supported upon a light but firm tripod stand, like that of a camera, but firmer. It is attached to the tripod by means of a central screw, with a wing-nut beneath the stand (fig. 133) and its head recessed into the board, and covered with a brass washer screwed to the same. This screw serves the purpose of a vertical axis, allowing the table to be turned independently of the stand. By means of the wing-nut, it may be clamped in any desired position.

The table is levelled (by moving the legs), by means of the long compass used with it, or by two small spirit-levels fixed at right angles to each other in a block of wood.

Rays are drawn to the various objects by means of a Sight-Rule or Alidade, which consists of a plain ruler about 1′ 6″ long, with a fiducial edge, and provided

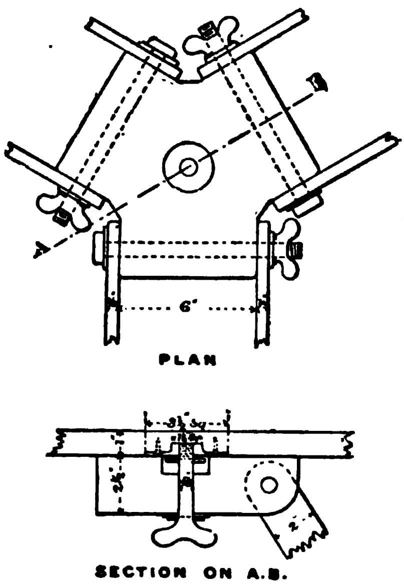


FIG. 133.

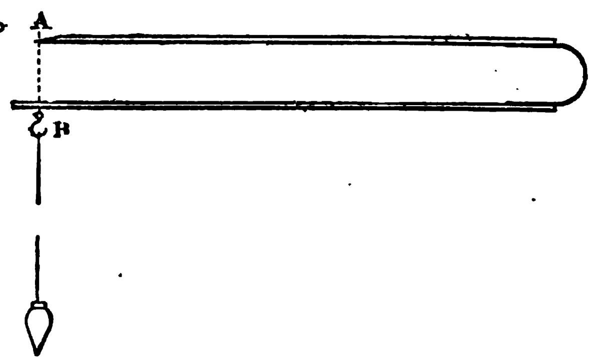


FIG. 134.

at each end with an ordinary sight-vane, like that of a compass or circumferenter. It is a great convenience, however, to attach the fiducial edge, to the body of the rule, after the manner of a sliding parallel ruler.

When working to a large scale, and when, consequently, the rays are short, a plumbing-fork is necessary (fig. 134). This consists of a metal fork. The opening of the fork is sufficiently wide, to take in the table and the paper on it. The lower leg of the fork is provided with a hook B, immediately below the point of the upper leg A. The plumb-bob is suspended from the hook B. When the point A coincides with any point on the paper, the plumb-bob hangs immediately below it, so that it may be brought vertically above the point which it represents on the ground. Or a point may be found on the papers which is vertically above the station point at which the table is set up.

A compass is a very necessary adjunct to the plane-table. The best form is a long (5 or 6-inch) bar needle enclosed in a rectangular box (fig. 135), which allows it to play about 5° or 10° on either side of the meridional line. Having once placed the table in its proper orientation, the compass-box is placed upon it, and moved about until the ends of the needle come to rest at zero, a line is then drawn round the box. By replacing the box within this rectangle, and turning the table, till the ends of the needle come to rest at zero, the table will be approximately oriented.

The paper must not be fixed to the table by means of paste or glue. The expansion and contraction of wood and paper are not the same, nor is that of wood the same 'with' and 'across' the grain. The paper used, should be of the very best quality, and well stretched in the way described in connection with traverse-surveying. It must never be rolled or folded, but must be carried flat in a portfolio.

Mode of attaching the paper.

To attach the paper to the table, by means of ordinary drawing-pins is inconvenient. The heads of the pins get in the way of the sight-ruler. Unless the pins are put in a new position every time the paper is put on the table, thus making

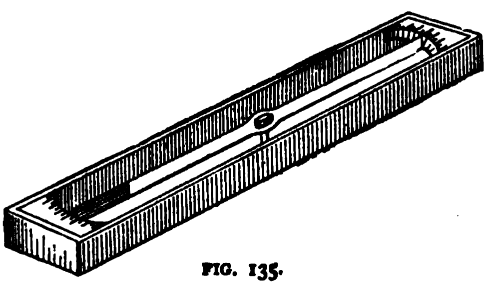


FIG. 135.

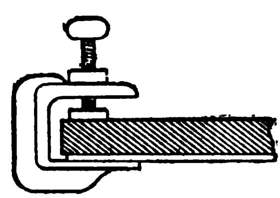


FIG. 136.

a number of holes, the perforations become enlarged, and the paper is apt to shift.

A better plan is to use screw clamps (fig. 136), if one of these gets in the way of the sight-rule it can be shifted. Four of these are necessary, but a set of six is convenient, so that one of the spare clamps may be put on, before taking off the one that is in the way. Another merit of screw clamps is, that a final adjustment in azimuth may be made by slacking three of the clamps judiciously, and moving the paper on the table.

A good plan for small scale surveys is, to mount the board with linen pasted round the margins, and then to mount the paper on to the linen, which spans over cracks in the wood, and a nice workable surface is obtained. This is the method adopted largely in India. Good millboard is in every way preferable to paper. It expands and contracts regularly, and wind does not cause it to flutter, as paper is apt to do. The surface should be of the best possible quality, so as to stand the rubbing and scrubbing, which it will have to receive.

White paper is painful to the eyes, pale drab, green (such as Willesden paper) or pale slate-colour, is more comfortable to work upon, in bright sunlight. A pale sepia, or burnt-umber colour is, according to the writer's experience, generally found to be the most pleasant.

Surveying between known Points projected on the Paper.

With the enumerated appliances, it is possible to produce a complete survey, the position of any two points being previously known. Usually, these will be previously determined trigonometrical points, and projected on the paper by co-ordinates in the manner already described, or they may be the extremities of a base line measured with a chain, the azimuth being determined astronomically. If trigonometrical points are available, it will be well to project three at least, and preferably more, so as to serve as checks to the work.

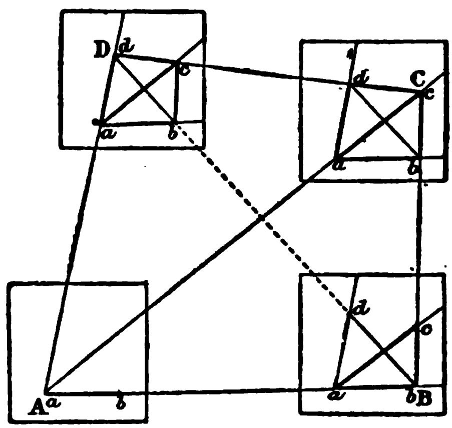


FIG. 137.

Let A and B (fig. 137) be the two known points on the ground, *a* and *b* their projections upon the plane-table sheet. Proceeding to A, the table is set up and levelled. The fiducial edge of the sight-rule is applied to the line *a b* on the sheet. The table is then turned in azimuth till the line of sight of the rule intersects the signal at B. Strictly speaking when the intersection is made, the point *a* on the sheet should be vertically above the point A on the ground. The effect of neglecting this condition will be discussed later on. Suffice it for the present to say that when working to a small scale the error due to defective centering will be negligible.

When the fiducial edge of the sight-rule coincides with the line *a b* on the sheet, and its line of sight intersects the point B, the table is correctly 'oriented.' The relative positions of the points on the plan coincide, in azimuth, with the points on the earth which they represent.

At this stage of the proceedings, the compass-box should be placed on the table, moved until the needle comes to rest at zero, and its position marked as described. When the table is set up at another station, it may be oriented, with the degree of accuracy which the compass is capable of (*vide* Traverse-surveying), by replacing the box within the rectangle described, and turning the table till the needle comes to zero.

Now, to fix some other new point such as C. If an ordinary sight-rule be used, the leg of a pair of dividers, or pin, or the point of a hard pencil is planted at (*a*) on the sheet to serve as a pivot (the nail of the forefinger often suffices), and the sight-rule is turned until the line of sight intersects the point C. The ray *a c* is then drawn, a note or reference, or sometimes a rough sketch of the object, being made in the margin, to ensure identification. Rays are also drawn to other objects, such as D, which are to be fixed.

The table is now transported to B, set up and levelled. It is then oriented, by placing the fiducial edge along *a b*, and turning the table in azimuth, till the line of sight intersects station A, and is clamped fast. Then the sight-rule is turned to C and the ray thereto drawn. The intersection of the rays *a c* and *b c* is the position of the point C on the plan. In like manner the positions of other points are fixed by intersecting the rays drawn at A, from (*a*) on plan, with rays through B, from (*b*) on plan.

Finally, a ray is drawn to some new point D destined to serve as a future station for the instrument.

The instrument is next set up at C, oriented on A, and the orientation checked by directing the sight-rule to B, and the ray *c d* is drawn to D cutting the ray *b d* and thus fixing D.

Thus, a system of triangulation may be carried on graphically. The three angles of the major triangles will be protracted directly on the table, analogous to the principal triangles of a trigonometrical survey, each point being observed to and from. Intermediate points will be determined by observations from these points, (but not observed *from*), corresponding to the *intersected* points of the trigonometrical survey.

If the object be to produce a topographical map, on a small scale, say $\frac{1}{60,000}$ or one inch to the mile, or less, the intermediate points will be so numerous that the eye may be trusted for sketching in intermediate detail, such as the courses of rivers or roads, the position and outlines of villages, the approximate configuration of the ground by eye-contours, or even more precisely by the use of a clinometer, such as Abney's level, in the manner practised in military reconnaissance.

If the map is ultimately to be published, on a scale of $\frac{1}{60,000}$ or less, it will be well to draw the original sheets to a larger scale, say 6 inches to the mile, and reduce by photography. It is not easy to produce in the field the fine drawing, that the small scale map requires. In this manner the topographical map of India was produced. The parallel ruler arrangement with the sight-vane greatly facilitates the intersection of points, and the drawing of rays. It is unnecessary to use a needle or pencil-point as a pivot for the sight-rule, making holes or marks on the paper. It is only necessary to place the sight-rule, so that when the intersection is made, its fiducial edge is near to the station point on the paper. When the line of sight intersects the objective point, the station point on the table can be intersected easily by extending the parallel bar, and the ray from it drawn. One hand is not occupied in holding the needle, or marking the point with the nail of the forefinger, but both are available for manipulating the sight-rule.

Errors due to inaccuracy in 'Centering' examined.

It has been stated that the point on the paper should be vertically above the point on the ground which it represents. It is not always easy to satisfy this condition perfectly, and at the same time level and orient the table. To do so might necessitate several shifts, and occasion much loss of time. It is therefore well to examine the nature and extent of the errors due to defective centering. Suppose that rays are to be drawn from some point (*a*) near the corner of the sheet, a second point (*b*) being already determined.

R

In what follows capital letters refer to points on the ground, the corresponding small letters to the points on the plan.

The table is set up with its centre over the station A (fig. 138). The angle C A B is that subtended at A by the rays to the station B and C. If rays were drawn with the sight-rule through (*a*) on the plan, they would include the angle C *a* B, materially different from C A B. The error caused by defective centering, depends upon the length of the ray, and on the scale of the plan. With a table 2 feet square it will be scarcely possible to be so far out of centre that the perpendicular A *o* exceeds 1 foot. If the point observed to, were distant 5000 feet the error in the ray A *a b* would be 41 secs. A similar error might occur in the ray A C so that the angle C *a b* might be in error by 1′ 22″. Both rays being 5000 feet long and the scale of the plan $\frac{1}{10,000}$ (nearly 6 inches to 1 mile), the rays as drawn,

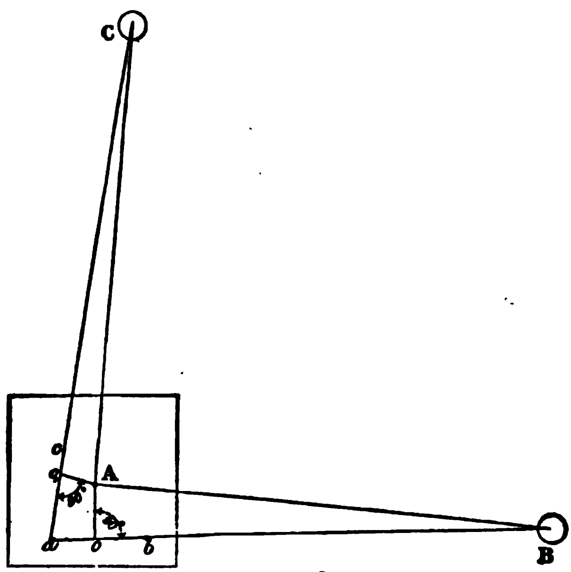


FIG. 138.

would be 6 inches long on paper. Now an error in the angle *c a b* of 1′ 22″ would displace the point *c* by ·00002 foot, a quantity quite inappreciable. Even if the rays were only 500 feet long, other conditions being the same, though the actual error in angle would be nearly ten times greater, (nearly 14 mins. and therefore appreciable), still the displacement of the point *c* would be inappreciable owing to the shortness of the rays, which would then be but ·05 foot in length.

Great care in Centering necessary, for Scales larger than $\frac{1}{2500}$

So we see that in surveys to a small scale, even as great as $\frac{1}{2500}$, extreme accuracy in centering the point on the paper over the station is not essential, *except for very short rays*. It is, however, desirable that centering should be performed as accurately as is possible (without great waste of time), for fear of accumulation of error.

If, on the other hand, the scale be large such as $\frac{1}{500}$, and the distance short, say

·50 feet, it is evident that an error of centering may produce a most perceptible error in the position of points upon the plan. The plane-table being useful for plans even on so large a scale as $\frac{1}{100}$, in such cases, accurate centering is very necessary.

The parallel sight-rule affords a ready means of eliminating the error of centering. Suppose that the plane-table when levelled, and approximately oriented, be so placed that the point *a* on plan (fig. 139) is not over the station on the ground which it represents. Find a point *o* on the paper which *is* vertically over A. Lay the sight-rule with its fiducial edge intersecting the point *o* exactly parallel to the ray *a b*. With the sight-rule in this position, turn the table in azimuth until the line of sight intersects the station. The table is now correctly oriented, though not correctly centered. Now with the fiducial edge intersecting *o* direct the sight-rule to C. Extend the parallel bar, and without moving the sight-rule, draw the ray *a c* through *a*. The angle *c a b* is equal to C A B and is therefore correct. In correcting the orientation, the position of *o* with regard to A may be slightly altered. Usually the movement will be so slight as to have no effect on the orientation. If it does affect it, find a new position for *o* and proceed

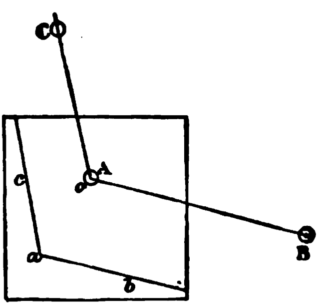


FIG. 139.

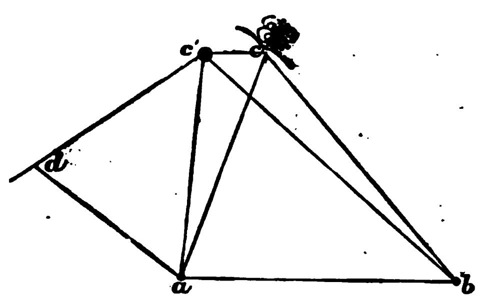


FIG. 140.

as before. The question as to whether any error of centering is negligible or not, may be solved at once, by considering whether the actual horizontal distance, as shown by the plumbing-fork between the point on the plan and the station point, is an amount which would be appreciable when drawn to the scale of the plan. Suppose the actual error in centering were 1 foot, and the scale of the plan $\frac{1}{10,000}$, then the actual error on the plan would be $\frac{1}{10,000}$ of a foot, and therefore wholly inappreciable. If the scale were $\frac{1}{500}$, the error would be just appreciable. If the scale were $\frac{1}{100}$, it would be on paper ·01 foot and therefore too serious to neglect.

Fixing intersected Points as for Minor △n.

In the course of plane-table surveying, it will often be convenient to fix points such as chimneys, steeples, trees, or corners of buildings, over which the plane-table cannot be set up. Nevertheless such points may be useful for the further development of the plan.

Let *c* (fig. 140) be a conspicuous tree, whose position has been determined by

R 2

the intersection of rays drawn from *a* and *b*. To make use of this station for further work, the plane-table is set up at a point c_1 near to the tree. It is then oriented carefully by means of the compass. Then the ray $c\,c_1$ is drawn, and the distance $c\,c_1$ is measured, and set off along $c\,c_1$, thus fixing with a close approximation to accuracy, the position of the table. It is evident that as the distance of $c_1\,c$ is small, an error in orientation by compass, of even a degree or two, will not affect the position of *c* on the plan. The point c_1 may now be taken as fixed with nearly as much accuracy as *c*. There is, however, uncertainty as to the correct orientation of the table. Lay the sight-rule with its fiducial edge intersecting c_1 and *b*. Turn it till the line of sight cuts B. Then check, by placing the sight-rule along $c_1\,a$, when the line of sight should cut A, and c_1 be correctly determined. Rays may now be drawn from c_1 intersecting different points such as *d*. If from *a* a ray *a d* has been drawn, *d* is determined. If, on the other hand, no ray has been drawn at *a* to *d*, still the position of *d* may be fixed without reference to other points, and without leaving a signal at c_1. Put up the table at *d*, orient with the compass. Draw rays to *a* and *c*. These rays should intersect on the ray drawn already at c_1. If they do not, the orientation is incorrect. The table must be moved in azimuth until they do.

The 'Three-Point Problem' solved graphically.

The three-point problem (*vide* p. 201) may be solved graphically with the plane-table, as follows:—

Let A, B, and C (fig. 141) be the three known points. Let O be the point at which the table is set up. Let A be on the left, B on the right, and C in the middle or behind the observer's back, as he regards A and B.

Lay the sight-rule through *b* and *a* on table, and direct on A, (*a* being towards A). Through *b* draw a ray *x b x* towards C.

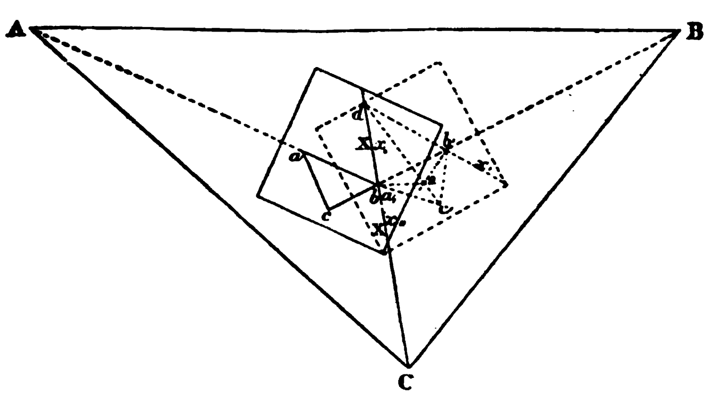


FIG. 141.

Lay the sight-rule again through $a_{,}$ and $b_{,,}$ and direct on B, *b* being towards B. Through $a_{,}$ draw a ray $x_{,,}\,a_{,}\,x_{,,}$ towards C.

Join the intersection *d* of the two rays drawn through *a* and *b* towards C (in the first and second positions of the table) with *c*.

Lay the sight-rule along this line. Turn the table till the line of sight cuts C, *c* being towards C. The table is now oriented. Draw rays through *a* and *b*. Their intersection fixes *o*, the point of observation on plan. These rays should intersect on the ray *d c* already drawn.

In the second position of the table the lines are dotted, and the letters accented.

The accuracy of this solution depends upon the length of the line from *c* to the intersection of the two auxiliary rays drawn from *a* and *b* to C. If this line is very short or if the angle of intersection be bad, the conditions are unfavourable to accuracy, and some other points must be used.

Strictly, the table should be moved at each operation so as to bring the point at which the angle is drawn over the station point. This precision is rarely necessary, and any error may be eliminated by means of the parallel sight-rule as already described.

Plane-Tabling with the aid of Chain or Tape.

Up to this point it has been assumed that plane-table surveying is conducted entirely by triangulation, without using any appliance for linear measurement, except perhaps a 50-foot tape for measuring short distances.

For topographical surveying on a small scale this method suffices. But when the survey is made to a large scale it would be laborious.

Suppose that a tortuous road had to be surveyed. Having fixed two points *a* and *b* (fig. 142), rays would be drawn from *a* to a number of leading points in the boundaries of the road. These points would be fixed by the intersection of rays, drawn from *b*.

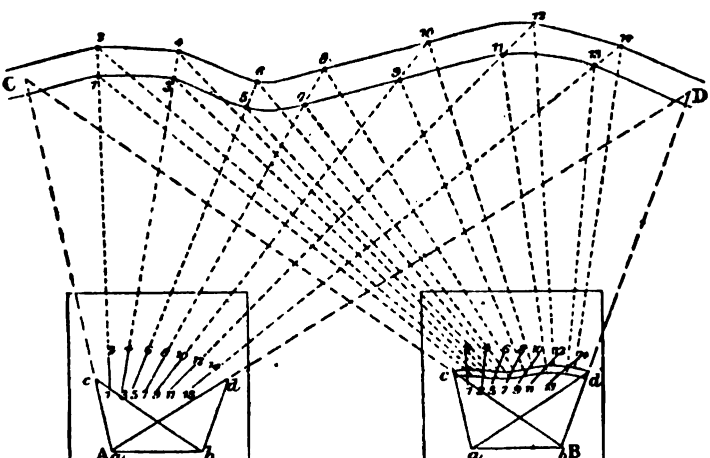


FIG. 142.

Now every point on the ground must be carefully marked and numbered in the first instance so that they may be identified when drawing the intersecting rays from *b*. The staff-holder must be careful not to miss a point, when the second set of rays is being drawn, otherwise hopeless confusion, and loss of time and temper, would result. The sheet will be covered with a maze of rays

If a chain or steel tape were added to the equipment of the plane-tabler, his procedure would be obviously simplified. With their aid traversing can be

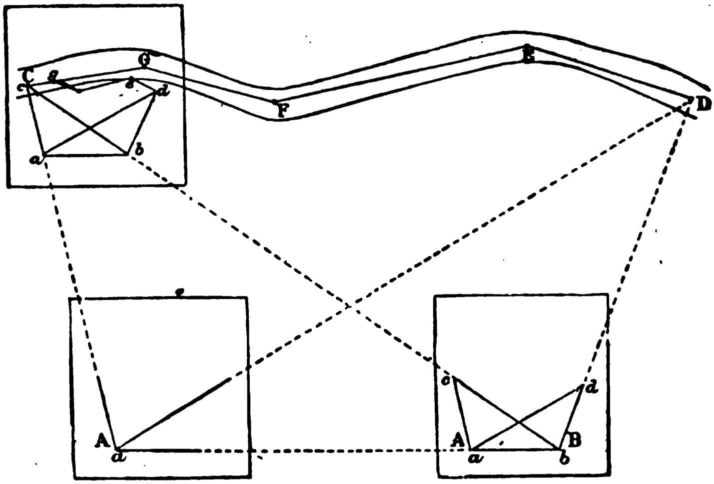


FIG. 143.

carried out with the plane-table quite as accurately, or more so, than would be the case if made with a theodolite and chain, and plotted with a protractor.

FIG. 144

Two points C and D (fig. 143) in the road would be determined by the intersection of rays from *a* and *b*. Putting up the table at D, and orienting on A and verifying on B, the position of *d* would be fixed. The ray *d e* would then be drawn and the distance D E measured and plotted. Intermediate distances and offsets would be taken as in chain-surveying. They might be plotted at once, or better, noted on a slip of paper, so that they could be plotted when the traverse is closed on C. Having laid off the distance *d e* the table is then set up at E, oriented on D, the orientation checked by rays to A and B, if these points be visible. The ray *e f* is drawn, and so on until closure is effected at C. This done, the offsets might be plotted, and the work, so far, would be complete. In this way the streets of a village might be easily surveyed. Unless the houses were flat-topped, so that the instrument could be put up in commanding positions, this would not be easy to perform by pure triangulation, and in any case, without a chain, the survey of a village would be very tedious.

Plane-Table and Telemeter.

The most useful appliance for linear measurement that can be used with the plane-table, is the telemeter or tacheometric telescope. Fig. 144 shows a plane-table as made by Messrs. Kern & Co., of Aarau, and fig. 145 is an enlarged view of the telemeter telescope.

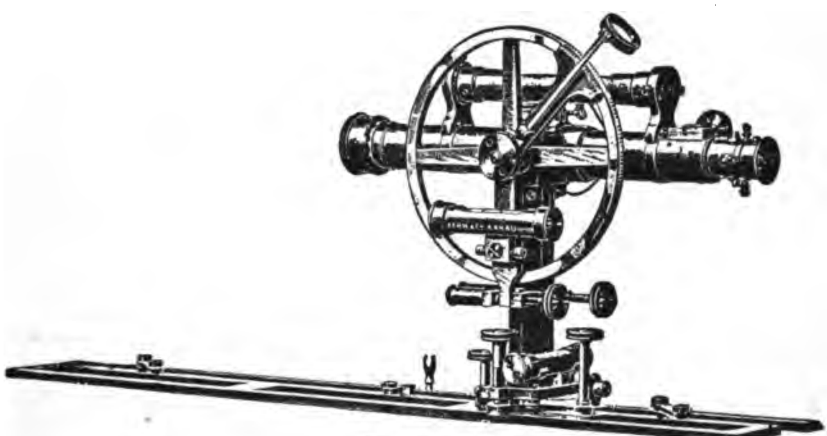


FIG. 145.

Thus equipped, the plane-table becomes an instrument of considerable precision, and useful for filling in detail, or even for making large scale surveys of considerable extent, working from a carefully measured base.

General Rules to be observed when using the P.T. with a Telemeter.

The following general rules should be adhered to, in surveying with a plane-table and telemeter.

(*a*) No new station of the table should be taken up by a ray and a distance only.

Every new station should be fixed either by one ray and a measured distance, and a ray drawn from an already determined station, verified by a back ray drawn at the new station to the second station, or, by a ray and distance from the station left, verified by back rays to two other fixed stations.

(*b*) Only as much ground should be surveyed from one position of the table, as will bring the longest distance well within the range corresponding to the permissible error. In short, there should be no hesitation to shift the table.

(*c*) In surveying a crooked boundary, or fence, take up a position for the table that will make the rays approximately tangents to the curves, rather than normals.

(*d*) The orientation of the table should be frequently checked by referring back to already fixed points. A slight pressure on the margin of the table produces considerable twist on the vertical axis. The clamp may slip slightly, unperceived, and the consequence will be, that an area will be surveyed correctly in itself, but displaced with regard to the rest of the plan.

In the preparation of an extensive survey, on the scale of $\frac{1}{2500}$, it is both desirable and economical to traverse the roads and leading boundaries, and to plot them on the sheet before commencing to use the plane-table. This gives a larger number of points to check upon, for if performed properly, traversing is only second in accuracy to minor triangulation. The chances of distortion referred to in the last paragraph are therefore materially reduced.

Plane-table surveying cannot be performed with economy, accuracy and despatch, during a high wind. The telescope vibrates too much, time is lost in waiting for a lull, and the staff is read hurriedly. In short, a high wind, even without rain, stops plane-table work. Traversing, on the other hand, can be carried out in windy, or even moderately rainy weather.

Plane-table work, even if the sheets are tinted as recommended, is trying to the eyes, and telescope-readings are fatiguing. It is desirable to rest occasionally when the weather is unpropitious for plane-table work, and the surveyor can then occupy himself with traversing, or perhaps, measuring main lines and angles, only leaving the re-measurement, and offset detail work to a subordinate (*vide* Traverse-surveying). Each surveyor should have two sheets in hand, one in the traverse stage, and the other being completed with the table. The computation and plotting of the traverses, will be performed at the head office.

Measuring differences of Level with the Telescopic

With the telescopic rule, differences of level may be measured, by means of the vertical angles. The staff-readings of the three horizontal wires are taken.

The apparent distance (a) is the difference of the reading of the top and bottom wire multiplied by 100.

The difference of level Δh is given by the expression

$$\Delta h = a \sin \theta \cos \theta.$$

An 'eye and object correction' must be applied as in triangulation.

Call rises +.
Falls −.
Height of instrument +.
Staff reading of middle wire −.

Sum algebraically, and the result is the difference of level.

A slide-rule is provided for making this calculation in the field. It usually forms part of the rule used for reducing the apparent, to the horizontal distance.

The rules, usually provided, are too small, and being of electrum, are trying to the eyes. A celluloid rule 2 feet, or even 3 feet long, would be better.

The vertical angles must be read to minutes, and the telescope should, after each observation, be brought to the horizontal position, and the index error read.

In a form of telescopic rule, devised by the writer, this is obviated by attaching a good level to the frame which carries the vernier. This frame is loose on the telescope axis, but is prevented from revolving by a tail-piece, the extremity of which is held between a spring and opposing-screw, attached to the rule. The line of collimation, and vernier level, can be adjusted as in the transit theodolite. Consequently the bubble can at any time be brought to the middle of its run, by turning the milled headed screw. The vertical angles are thus measured independently, and are not affected by dislevelment of the table. Index error can be eliminated by adjustment of the level. If any exists, it will be a constant quantity which need only be determined once and for all.

The graduations of the vertical arc are on a cylindrical surface. The vernier is placed at an angle of 45° from the vertical, so that it can be read by the observer without moving his position at the eye-end of the telescope. The divisions are boldly cut, so that minutes may be read without using a lens. This arrangement saves a second or two of time at each observation, a matter of importance when perhaps a thousand have to be made in the day. It also obviates the necessity for stepping round at each observation, and running the risk of kicking the legs. (*Vide* fig. 145.)

As originally made by Elliot & Co., this instrument was found to be too heavy. The weight might be reduced by hollowing some of the parts, or by the use of aluminium.

The telescope axis is borne in two Y's at the head of the pillar, and its weight is more or less balanced by the limb and level at the opposite ends of the axis. The rule is provided with top parallel bars, one at each side. Adjustment can be made exactly as in an Everest theodolite.

Complete Topographical Map prepared with P.T. and Tel. Rule.

With the telescopic rule and plane-table a complete topographical map may be prepared. The levels of a number of points may be fixed from each station, and entered on the plan. Contours may then be sketched in. The vertical angles between two successive positions of the table should be taken both forwards and backwards, and the mean used, in order to eliminate the effects of curvature and refraction. It would probably be advisable to book the main forward and backward readings, and work them out accurately at leisure. To facilitate this, the group of level points taken at each station of the table might be enclosed with a pencil line, and the levels of the points inscribed on the plan, correction would then be easy.

Desirability or otherwise of taking Vertical Angles with the P.T.

The desirability or otherwise of levelling with the plane-table has to be decided on the principles laid down in connection with triangulation. If the country is bold and rugged, and the main object is to produce a topographical plan showing the configuration of the ground, no better procedure than vertical angles, and deduced difference of levels, can be adopted.

If, on the other hand, the country is level and the main object is to produce a plan recording correctly roads, fences, buildings, &c., then the surveyors will have quite enough to do without determining heights. Levelling will be performed more cheaply, expeditiously, and accurately, as a separate operation, after the plan is finished.

Conditions under which the P.T. may be profitably employed.

The conditions under which the plane-table may be profitably employed may be discussed as follows.

The first question is that of climate. It would be absurd to use the plane-table in the Highlands of Scotland. It is a tropical instrument. Yet it is possible, that even in the British Isles, it would be useful for verifying and bringing to date Ordnance maps, in connection with Parliamentary plans.

If a cadastral map of a country much intersected with fences has to be produced, it is invaluable, as a secondary instrument, for filling in detail. It has the advantage when used with the telemeter that it involves no damage to crops. The staff-holder naturally walks round the margins of the fields. There is no chaining through standing crops, no smashing fences, and no annoying claims for compensation.

In an open country, especially if the survey is carried on by means of an organised party of staff surveyors, computers and draughtsmen, the plane-table is not so valuable as it would be with a less complete organisation.

To the engineer who has to study an unsurveyed country for the purpose of preparing a project for a road, railway, or other public work, the plane-table is invaluable. He can, unassisted, except by a few labourers, rapidly produce a contoured plan amply sufficient to locate a line of communication, in a far more satisfactory manner than if he proceeded at once with theodolite and level.

Examples of Great P.T. Surveys.

The preliminary survey of the St. Gothard railway was entirely made with the plane-table. A contoured plan of the valley was first made to a small scale. On this, the line, with its numerous tunnels, was studied and located.

The magnificent contoured map of Switzerland was produced by means of the plane-table, with triangulation as a basis.

Sketching from nature, as it were, the plane-table surveyor can give expression to features, better than if the drawing is done at the office. The engineer can note many points, such as the nature of the ground and the manner of overcoming difficulties.

The plane-table has been found useful for filling in the details of a town survey to a scale as large as $\frac{1}{100}$ (approximately $\frac{1}{8}$ of an inch to the foot), such as that of Kingston, Jamaica. Bolts were fixed in the ground at intersections of streets, and at intervals of 200 feet. The exact positions of these were determined accurately. Two such points were projected on each sheet: starting from one the section was surveyed, checking to the second. The streets of the town in question are straight and approximately at right angles to each other. The bolts in the north and south streets were ranged out with the theodolite, and the distances were measured with a long steel band, 10 chains in length. The plane-table was of the simple type described and figured at the commencement of this chapter. The sight-rule as figured was designed for use with it.

The various objects, corners, doors, veranda-posts, steps, &c., within 50 feet of the table were determined by rays and tape measurements, the distances, from place to place of the table, were measured with a good chain. As soon as finished the sheet was traced. In this manner a plan of each street was produced, suitable for the design of sewers and drains, and for their record after construction.

It was found that the detail could be drawn in to scale on the ground, in little longer time than would be taken to make sketches and measurements. There was less chance of omissions and error than with the usual plan.

Tacheometric Surveying with P.T.

The value of the plane table as a surveying instrument is very much enhanced when the alidade is provided with a subtense telescope, with which angles of inclination can be observed to points whose distances can also be ascertained, directly, by reading on a staff the divisions subtended by the cross hairs.

The heights are calculated on the spot with a slide rule. The above additions to the alidade as adopted in Germany, Switzerland, Egypt and the United States, consist of a telescope with a vertical arc mounted on the ruler, and fitted with an eye-piece with subtense hairs, and an anallatic lens (as described under the head of the Tacheometer, in the Chapter on OPTICS).

The Swiss survey, and those for locating the St. Gothard and Simplon railways, were made by this method.

To determine each point in position and altitude, the following five processes must be gone through:—

1. Set the instrument, and read the staff.
2. Read the vertical circle.
3. Calculate the horizontal distance, and difference of level, with the slide rule.
4. Plot distance ascertained, thus fixing the position of the point.
5. Note the height of each point above datum.

In this way about 40 points per hour can be fixed, and a square mile of country with from 1000 to 2000 points surveyed in from three to four days.

Tacheometric surveying is fully discussed in the second part of this treatise, and a description given of the various types of tacheometer.

CHAPTER VII.

LEVELLING AND CONTOURING.

Levelling.

LEVELLING is the art of determining the relative altitudes of points on the Earth's surface.

Were the Earth a true sphere, the altitude of a point might be defined as its distance from the Earth's centre; but, as it is spheroidal in shape, the definition is not exact in a general sense, though it is nearly so as regards a limited area. If a long pipe A B (fig. 146) were laid, from one place to another, with vertical pipes at intervals, and filled with water, the altitude of the water-surfaces in the upright tubes would be equal when the water has come to rest. Further, if a network of

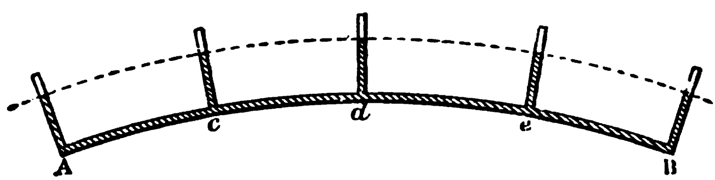


FIG. 146.

pipes were laid over a given area, connected together, and provided with upright pipes, the water-surface in each and every pipe would stand at the same altitude.

A surface passing through all water-surfaces in the several pipes, would be a surface of equal altitude, and would be an approximately spherical surface, whose radius is equal to the mean radius of the earth at the latitude of the place.

If levelling operations were conducted, from pipe to pipe, with perfect accuracy, the difference of the altitude of the water-surface, or water-level, in any two pipes, would be zero. The surface of a still lake is a surface of equal altitude. So, also, would be a surface, passing through 'mean sea level' at various places on the earth.

It is a property of a surface of equal altitude that a plane, tangential to it at any point, will be at right angles to a plumb-line, suspended at that point.

If the plumb-line be deflected from its normal position, for instance by the attraction of some large mountain, as sometimes happens, the plane of equal altitude will be deflected also.

The effect of the mountain mass, would be to produce local elevation in the surface of equal altitude (*vide* fig. 147, dotted lines), thus reducing the apparent height of the summit, above the general mean surface, in the neighbourhood thereof. But if a tunnel were driven through the mountain perfectly level, its invert would exactly satisfy the conditions of a line of equal altitudes as before described, and it would be *level* from a hydrostatic point of view, as well as from that of the engine-driver.

The Level. The levelling instrument, in its essential parts, consists of a telescope with a horizontal wire accurately centred in the optical axis (*vide* Chapter on INSTRUMENTS), thus affording a line of collimation, and two vertical wires to define a perpendicular portion of the field of view. A spirit-level is attached to the telescope, and it is so adjusted that, when the bubble is at the centre of its run, the line of collimation is horizontal, or in other words at right angles to a plumb-line suspended from the instrument.

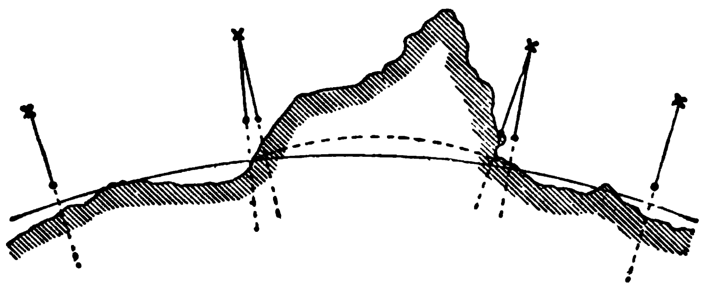


FIG. 147.

Levelling operations. To ascertain the difference of altitude (more commonly called the 'difference of level'), between two points, A and B (fig. 148), not far apart, it is only necessary to set up the instrument at one point A, adjust it so that the line of collimation is horizontal, direct it to a graduated staff at B, and read the height h at which the cross-wire cuts the staff.

Then, measuring the height of the centre of the telescope h above the point A, the difference between h and h_1 is the approximate 'difference level' between the points A and B.

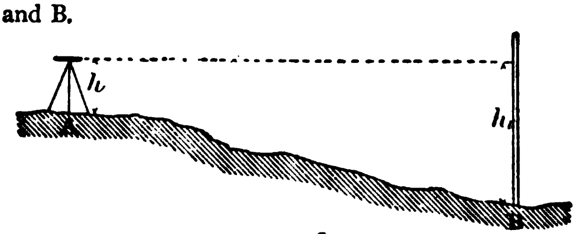


FIG. 148.

If h_1 is greater than h, then B is lower than A, if less, then A is lower than B.

This simple procedure would give for any two points, not more than one or two hundred feet apart, a practically accurate result, provided that the instrument were in perfect adjustment.

Effect of Curvature of the Earth on Levelling. For long distances, however, this procedure is inaccurate, on account of the curvature of the 'line of equal altitudes,' otherwise called the 'curvature of the earth.'

Let A C (fig. 149) be the curved line of equal altitudes, an arc of a circle, then B C is the true difference in level between the points A and B. Draw A E parallel to I D, the horizontal line of collimation of the instrument, when it is evident that the apparent difference of level between A and B as given by deducting h from h_1 is in excess of the true difference E C, due to the 'curvature of the earth.' This excess or error amounts to about 8″·o or o·67 feet in one mile, and increases as the square of the distance.

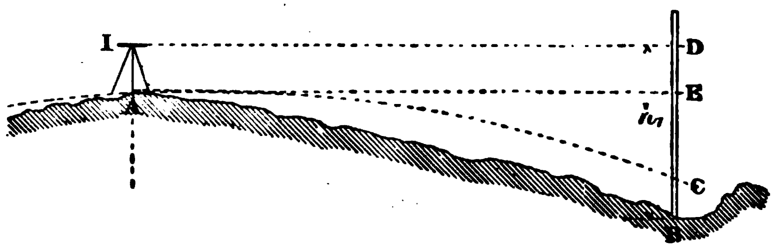


FIG. 149.

For ten miles it would be $0·67$ feet $\times 10^2 = 67$ feet.

For one-tenth of a mile, or 528 feet, it would be $0·67 \times \frac{1}{10^2} = 0·0067$ feet only. This error would, however, be reduced by about one-sixth to one seventh of its amount by 'refraction,' which raises the apparent position of a distant object, to an extent which is somewhat irregular. (See table K, p. 226.)

The error due to curvature, is not serious in a *single* sight taken at a distance compatible with the optical power of the telescope of an ordinary levelling instrument, but if one were to attempt to determine the difference of level of the first and last, of a long series of points, by proceeding successively in the manner above described for two points, the error would be cumulative, always tending in the same direction, and the final total error would be serious.

Elimination of Curvature and Refraction.

It is, however, possible to eliminate completely all errors due to curvature and refraction.

To this end it is only necessary to plant the level, not directly over one of the points, whose difference of level is to be determined, but midway between the two (fig. 150).

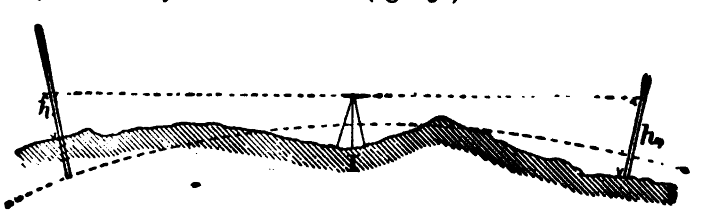


FIG. 150.

It is evident that by so doing, the error due to curvature and refraction will be equal in both directions, and that the difference between the two staff-readings will be the true difference between the levels of the two points A and B.

The effects of both curvature and refraction are thus *entirely* eliminated, and need no further consideration.

The line of equal altitudes so determined may be treated as a truly level line, even in the most extensive levelling operations.

The above is the usual practice in levelling operations, and involves no error as regards the physical results.

If a railway were set out in this manner, and accurately level, the top of the rails would be a curved line, concave to the centre of the earth. In a length of a few miles, this curvature would be very perceptible, but nevertheless a railway carriage on the line would not tend to move in one direction or the other, for the rails would at all points be at right angles to the direction of the plumb-line, that is to say, to the direction of the force of gravity.

Elimination of Collimation Error.

Another, and not less important merit of the midway position of the level is, that it eliminates completely, the effect of instrumental errors in collimation adjustment.

For example, suppose that the line of collimation is not parallel to the axis of the spirit-level, but makes some fixed angle θ therewith, thus (fig. 151).

It is evident that, when the instrument is exactly equidistant between the two staves S and S_1, the errors in reading $d \tan \theta$, are equal and opposite in direction, and therefore produce no error in the determination of the difference of level of

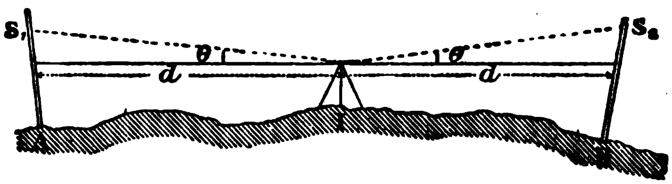


FIG. 151.

the points A and B. Provided always that the bubble remains, or is brought to the middle point, or to any one position in its run, when reading the staff at the two equidistant points, A and B.

Procedure to Obtain the Difference of Level between two points.

The following is the procedure to obtain the difference of level between two points, A and X, separated by a considerable distance. Let A be the starting point whose altitude or 'level' (above a selected datum) is known, or has at least some arbitrary or rational value assigned to it. Place the staff on the initial point A (fig. 152), and set up the instrument at a point I, so that whilst the graduations of the staff, held at A, can be distinctly read, as well as those of the staff held at some second point B (equidistant, or nearly so, from I), the position of the level can also be fixed and recorded, if necessary. With the level bubble in the centre of its run take readings on the staff at A and B respectively, when the difference of these readings will give the differences of level between the stations A and B. Next, whilst still holding the staff at B, remove the instrument to some point I_2, about midway between B and the next selected station C. Read the staff at B and C respectively as before, which gives the difference of level between B and C. Proceeding in a similar manner with the staff held at C, D, E, F, &c., and the instrument erected at I_3, I_4 .... &c., it is obvious that the difference of level between A and X, can be obtained by the algebraical summation of the successive differences of level between the intermediate points A, B, C, D, &c., to X.

In the operation of levelling, proceeding in the direction from A to X, the instrument being at I, the reading of the staff held at A is called a 'back-sight,' and that of the staff held at B a 'fore-sight.'

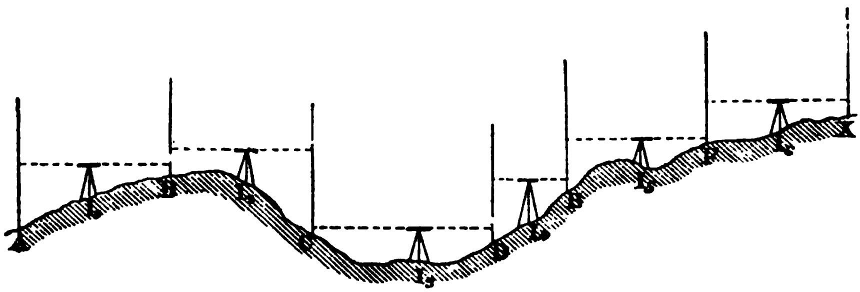


FIG. 152.

It is evident that the difference of level, between A and X, is the difference of the sums of the 'back-sights,' and 'fore-sights' respectively.

Method of simply finding the Difference of Level between two points.

When, therefore, it is merely necessary to ascertain the difference of level between two points, without reference to any intermediate points, the 'back-sights' are inscribed in one column in the field-book, the 'fore-sight' in another, thus

Station.	Back-sight.	Fore-sight.
I_1	6·11	3·86
I_2	2·93	6·32
I_3	1·76	0·83
I_4	7·32	0·96
I_5	6·95	5·18
I_6	5·38	4·19
	30·45	21·34
	21·34	
Difference . . . +	9·11 (a rise)	
Reduced level of A . .	136·14	
Reduced level of X . .	145·25	

The sum of the 'back-sights' being greater than the sum of the 'fore-sights,' then the difference of level is a rise. Now, since the 'required level' of A (that is to say, its altitude above mean sea-level, or some arbitrary point, adopted for the survey) is known, it is merely necessary to add or deduct the difference of level, as the case may be, to obtain the 'reduced level' or altitude of X. In the example, the sum of the 'back-sights' being greater than the sum of the 'fore-sights,' the difference of level is a rise, and must be added to the known level of A. A simpler way is to inscribe the reduced level of the starting point, at the head of the 'back-sight' column, thus

Station.	Back-sights.	Fore-sights.	
	136·14	..	.. Reduced level of A.
I_1	6·11	3·86	
I_2	2·93	6·32	
I_3	1·76	0·83	
I_4	7·32	0·96	
I_5	6·95	5·18	
I_6	5·38	4·19	
	166·59	21·34	
	21·34		
	145·25	..	.. Reduced level of X.

If, having determined the difference of level between two points, in the manner described, one were to level back by the same, or by some other route to the starting point, it is obvious that the sum of the 'back-sights' will be exactly equal to the sum of the 'fore-sights,' if the work be accurately performed.

This simple form of level-book suffices for 'trial-levels,' taken merely to determine the difference of level between two points. It also serves for the purpose of checking a series of levels taken in greater detail. Very frequently, however, levelling operations are conducted for the purpose of determining the levels of a number of points on a line (straight or curved) on the earth's surface, so that a section along that line may be prepared from which the volume of excavations or embankments, &c., may be calculated, for the construction of a railway, a road, a sewer, or a water-main. The form of field-book then becomes more complicated.

In such cases the procedure is somewhat as follows.

Procedure when Levelling for a Section. The line along which the section is to be taken, having been marked by ranging rods, a staff is set up at the starting point A (fig. 153), whose level is known or assumed.

The instrument is then set up as above described and adjusted.

The distance at which the instrument can be set, depends upon its optical power, and upon the nature of the ground.

It must not be so far that divisions of the staff cannot be read distinctly. On flat ground, this is the sole limit to the distance. On strongly undulating ground the distance must be shorter, in order that the line of collimation may intersect the staff. The instrument need not be set up on the line of section, but it may be placed at any convenient point to the right or left thereof. Some care and judgment is required, in selecting the position of the instrument. The ground should be firm, and fairly level, and the place should be so selected as to cover a sufficient space of ground, so as to give an approximately equal 'back-' and 'fore-sight,' whilst permitting the reading of the staff, in the several intermediate positions.

The telescope being directed to A, the staff is read, and the readings recorded. This reading is a 'back-sight,' as before defined.

Next, the instrument remaining at the same place, the staff is held at *b*, a point where there is a marked change in the slope of the ground. The staff is then read again and noted.

This reading is called an 'intermediate sight.'

The difference between the readings at A and *b* is clearly the difference between the levels of these two points.

The distance A *b* is then chained and recorded.

The staff is now moved to the point *c*, in the example, the crest of the bank of a brook; it is again read and noted, and the horizontal distance A *c* is measured, by continuous chaining from A.

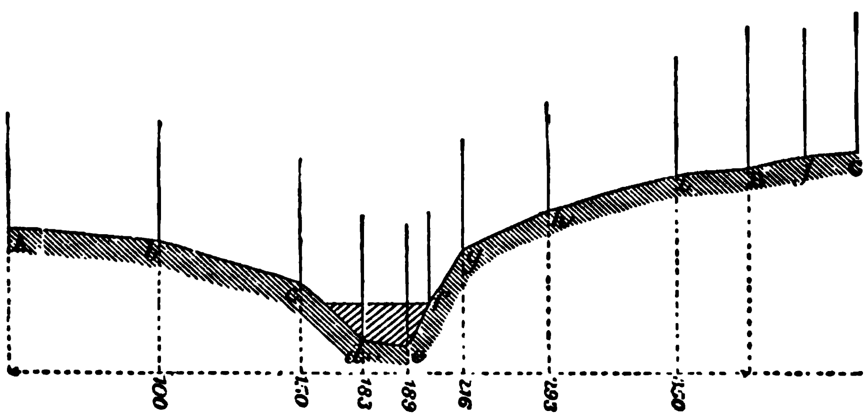


FIG 153.

The staff is next held successively at *d* on the bed, at *e*, also on the bed, at *f* at the water's level, at *g* the crest of the bank, at *h* and *i*, and finally at B. The horizontal distances to the several points are also measured. The point B, to which the final sight is taken, from the first position of the instrument, should be so situated that A and B are (approximately) at equal distances from the instrument, for the reasons already given.

The reading of the staff at B is called, as before, a 'fore-sight,' those at *b*, *c*, *d* .... *i* having been entered as 'intermediate sights.

The horizontal distances A *b*, A *c*, A *d*, &c., are determined by continued chaining from A.

Special form of F.B. for Section Levelling. Having thus levelled the section A B, the staff remaining at B, the instrument is moved forward, to a second position, and the process is repeated.

There are several forms in use for recording the staff readings and distances The following is one that in the opinion of the writer, is the most convenient and compact.

FIELD-BOOK FORM FOR SECTION LEVELLING.

Back-Sights.	Inter-mediate Sights.	Fore-Sights.	Height of Instru-ment.	Reduced Levels.	Distances.			Remarks.
					Left.	Centre.	Right.	
6·32	..	..	113·77	107·45	..	0·00	..	Reduced level of point A.
..	6·96	..	..	106·81	..	100	..	Point *b*. [stream.
..	7·32	..	..	106·45	..	150	..	,, *c*, crest of bank of
..	12·18	..	..	101·59	..	183	..	,, *d*, bed of stream.
..	13·11	..	..	100·66	..	189	..	,, *e*, ,, ,, [stream.
..	9·63	..	..	104·14	..	—	..	,, *f*, level of water in
..	4·73	..	..	109·04	..	216	..	,, *g*, crest of bank of
..	3·13	..	..	110·64	..	293	..	,, *h*. [stream.
..	2·19	..	..	111·58	..	350	..	,, *i*.
..	..	0·73	..	113·04	..	411	..	,, B.
9·36	..	..	122·40					
..	8·17	..	..	114·23	..	500	..	,, *j*.
..	..	7·42	..	114·98	..	600	..	,, C.
6·32	..	..	111·30	..	..	..	..	
..	..	3·11	..	118·19	..	..	..	Staff on B.M. top of milestone.
3·11	..	8·83	..	..	..	..	..	Check levels.
1·12	..	6·14	..	..	..	..	..	Close on point A.
26·23		26·23						

The staff reading at A, is entered in the column headed 'back-sights.' The readings taken at *b*, *c*, *d*, &c., *i*, are entered in the column headed 'intermediate sights,' whilst the final reading of the set, taken to the staff at B, is entered in the column headed 'fore-sights.' Then a bar is drawn, across the three columns back, intermediate, and fore, to indicate that the readings taken with the first position of the instrument are terminated :—

The computation of the levels is performed as follows.

The reading of the staff when held at A, is added to the reduced level of A. This gives the reduced level of the line of collimation of the instrument, and the sum (107·45 + 6·32 = 113·77) is entered in the column 'height of instrument.'

The readings of the staff at *b*, *c*, *d*, &c., deducted from this number, give the reduced levels of the several points.

For example, the reduced level of *b* is 113·77 − 6·96 = 106·81, which is entered in the column 'reduced level,' opposite to the reading, and to the appropriate distance 100.

Similarly, for the staff readings, at the points *c*, *d*, *e*, *f*, &c. Finally, the 'fore-sight' to staff at B is recorded in the proper column, is deducted from the height of instrument, and the remainder is entered as a 'reduced level.'

The computation of the first setting of the instrument is now complete.

The staff remaining at B, the instrument is carried forward, and set up and adjusted at some other point.

The 'back-sight' to B, is recorded in the 'back-sight' column, the 'intermediates' in their column, and finally the last sight to the staff at C (the last sight of the second setting), is entered in the column of 'fore-sights,' and so on.

The distances are entered on the same lines as the sights to which they refer.

The 'back-sight' taken to staff at B, added to the 'reduced level' of B, as obtained by the front setting, gives the height of instrument at its second setting.

The 'intermediate sights' and 'fore-sights' are deducted, as before, from the 'height of instrument,' and the remainders inserted in the column of 'reduced levels.'

Arrived at C, the surveyor is supposed to have completed his work, but he may wish to check its accuracy. To do so, he levels back from C to A, placing the staff and instrument at any convenient points. Between these points, perhaps following some different route, he also wishes to obtain, for future reference, the level of a permanent mark, such as the top of a milestone, near to C. He places the staff on it and reads, then, shifting his position, he continues his levelling back to A, by any convenient route, and holding the staff on any desirable points; but, since the levels of these intermediate points of this line of check-levels are not wanted, it suffices to enter the 'back-' and 'fore-sights,' one beneath the other, in their appropriate columns, as for check-levelling.

On reaching A, all the 'back-sights' and all the 'fore-sights' are summed up. If the work has been accurately performed, the sum of the 'back-sights' should be equal to the sum of the 'fore-sights.'

To check the accuracy of the computation of level of the main, or change points, up to the B M or milestone, sum the 'back-' and 'fore-sights' to the point.

To the sum of the 'back-sights,' add the 'reduced level' of the starting point, and from this sum subtract the sum of the 'fore-sights.' The remainder is the reduced level of the B.M.

	Back-sights.	Fore-sights.
Sums	22·00	11·26
Reduced level of A	107·45	
Sums	129·45	
Less 'fore-sights' . . . –	11·26	
Remainder or reduced level of B.M. .	118·19	

Usual form of F.B. for Section Levelling.

The following is a more usual form of level-book (for an example, see p. 262):—

The first three columns, 'back-sights,' 'intermediate,' and 'fore-sights,' are the same.

The next two columns are headed 'rise' and 'fall,' and the last 'reduced level.'

The staff readings are entered as in the first method.

Then, by deducting the successive staff readings taken at any one position of the instrument, from each other, always taking the smaller from the greater, the 'differences' of level, from point to point, are obtained and are entered in the columns, 'rise' and 'fall.'

If the reading at any point is less than that at the preceding point, then the 'difference of level' is a 'rise,' if greater, it is a 'fall.' The 'reduced levels' are obtained by *adding*, or *deducting* in succession from the preceding level, the 'rise' or 'fall,' as the case may be.

There is a complete check over the accuracy of work, both instrumental and arithmetical, as follows—

The difference between the sums of the 'back-' and 'fore-sights,' must be equal to the difference between the sums of the 'rises' and 'falls,' and also to the difference between the *first* and *last* 'reduced level.' If the levelling is carried back to the starting point, then, as before, the sum of the 'back-sights' must equal the sum of the 'fore-sights,' the sum of the 'rises' must equal the sum of the 'falls,' and the difference between the assumed, and final reduced level of the starting point, must be zero.

If the circuit does not close exactly, and if the arithmetical work has been checked as above, the difference must be due either to instrumental inaccuracy or to errors of observation.

This method of keeping a level-book, involves the use of more figures, and the performance of more additions and subtractions than that first described.

The 'rise' and 'fall,' between successive points, is a piece of information which is rarely if ever required.

The principal advantage of the second method is, the complete arithmetical check which it affords over each individual 'reduced level,' and not merely over the computation of the 'reduced levels,' corresponding to the 'back-' and 'fore-sights.'

Each 'reduced level' is derived from that immediately preceding it.

A slip in any one calculation is therefore carried forward, and comes to light in the final balance of the several columns.

In the first method, there is no final check on the computation of the 'intermediate' reduced levels, but an error made in deducting the staff reading from the 'height of instrument' affects, in the case of an 'intermediate,' the particular level only. It is not carried forward, so as to affect all the subsequent 'reduced levels,' and so appear in the final balance of the columns.

Whether the security against arithmetical error, which the second method affords, is worth the extra number of figures, and arithmetical operations, which it involves, is a matter of personal opinion.

The following is an example of the ordinary form of level-book, with rise and fall columns occupying two pages, so as to show how to continue a line of levels from one page to another.

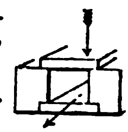
The reading 2·70 at the distance 28 chains is really an intermediate, but to check the arithmetical work on each page separately, this reading must be put in the *fore-sight* column on the first page and transferred to the *back-sight* column of the next page, the corresponding reduced level being also carried forward as shown.

When the 'height of instrument,' or 'collimation level' column is used instead of the rise and fall, simply carry forward from one page to the next *the last back-sight*, together with the corresponding collimation and reduced levels.

In either system, in fact, for the arithmetical checking of each page separately, every page, as well as every set of levels, must begin with a back-sight and end with a fore-sight.

LEVELS TAKEN FOR PROPOSED NEW ROAD GOING FROM...............

TO............ *Date*.........

Rise.	Back-sight.	Inter-mediate.	Fore-si ht.	Fall.	Reduced Level.	Distance (chains).	Remarks.
..	0·10	..	..	..	30·00	..	B.M. on covering flag of gullet, C on plan. 
..	..	0·40	..	·30	29·70	0·00	Point A on plan. Beginning of proposed road.
..	..	5·85	..	5·45	24·25	1·00	
..	..	9·60	..	3·75	20·50	2·00	
..	..	11·35	..	1·75	18·75	3·00	
..	..	12·10	..	·75	18·00	..	Level of highest known flood.
..	..	12·30	..	·20	17·80	4·00	
..	4·32	..	12·56	·26	17·54	5·00	
..	..	7·35	..	3·03	14·51	6·00	
..	..	7·80	..	·45	14·06	7·22	Top of left bank of stream.
..	..	12·14	..	4·34	9·72	..	Mean (summer) water level.
..	..	13·54	..	1·40	8·32	7·28	Bottom of stream.
·26	..	13·28	..	..	8·58	7·76	" "
4·68	..	8·60	..	..	13·26	7·85	Top of right bank of stream.
3·25	..	5·35	..	..	16·51	9·00	
4·08	11·78	..	1·27	..	20·59	10·00	
3·88	..	7·90	..	..	24·47	11·00	
2·94	12·37	..	4·96	..	27·41	12·00	
3·17	..	9·20	..	..	30·58	13·00	
3·15	..	6·05	..	..	33·73	14·00	
1·25	..	4·80	..	..	34·98	15·00	
1·45	..	3·35	..	..	36·43	16·00	
1·65	..	1·70	..	..	38·08	17·00	
1·13	12·85	..	0·57	..	39·21	18·00	
3·75	..	9·10	..	..	42·96	19·00	
1·50	..	7·60	..	..	44·46	20·00	
3·30	..	4·30	..	..	47·76	21·00	
2·22	13·14	..	2·08	..	49·98	22·00	
1·54	..	11·60	..	..	51·52	23·00	
1·50	..	10·10	..	..	53·02	24·00	
2·55	..	7·55	..	..	55·57	25·00	
1·95	..	5·60	..	..	57·52	26·00	
1·55	..	4·05	..	..	59·07	27·00	
1·35	..	..	2·70	..	60·42	28·00	Carried forward.
	54·56 24·14 30·42		24·14		30·42 30·00		

LEVELS TAKEN FOR PROPOSED NEW ROAD GOING FROM..............

TO........... *Date*........... *Continued.*

Rise.	Back-sight.	Inter-mediate.	Fore-sight.	Fall.	Reduced Level.	Distance (chains).	Remarks.
..	2·70	..	..	..	60·42	..	Brought forward.
·75	..	1·95	..	..	61·17	29·00	
·60	..	1·35	..	..	61·77	30·00	
·45	..	0·90	..	..	62·22	31·00	
·30	..	0·60	..	..	62·52	32 00	Highest point.
..	..	1·10	..	·50	62·02	33·00	
..	..	1·75	..	·65	61·37	34·00	
..	..	2·90	..	1·15	60·22	35·00	
..	..	4·00	..	1·10	59·12	36·00	
..	..	5·50	..	1·50	57·62	37·00	
..	..	7·50	..	2·00	55·62	38·00	
..	..	9·20	..	1·70	53·92	39·00	
..	..	10·00	..	·80	53·12	40·00	
..	..	11·45	..	1·45	51·67	41·00	
..	1·33	..	12·92	1·47	50·20	42·00	
..	..	3·00	..	1·67	48·53	43·00	
..	..	4·45	..	1·45	47·08	44·00	
..	..	6·20	..	1·75	45·33	45·00	
..	..	7·85	..	1·65	43·68	45·83	D on plan. Centre of existing road.
..	..	8·30	..	·45	43·23	1·00	On existing road, to left.
..	..	8·80	..	·50	42·73	2·00	
..	..	9·20	..	·40	42·33	3·00	
2·00	..	7·20	..	..	44·33	1·00	On existing road, to right.
·50	..	6·70	..	..	44·83	2·00	
·60	..	6·10	..	..	45·43	47·00	
1·65	..	4·45	..	..	47·c8	48·00	
·92	..	3·53	..	..	48·00	49·23	Point B on plan. End of proposed road.
..	1·18	..	12·44	8·91	39·09	..	Check levels.
..	..	..	10·26	9·08	30·01	..	Back at A.
	5·21		35·62		30·41		
			5·21		60·42		
			30·41				

The Use of the Levelling Staff.

The levelling-staff is a graduated wooden rod. For description and means of ensuring verticality, *vide* Chapter on INSTRUMENTS.

Formerly, it was provided with a sliding vane, which was moved up and down, by the staff-holder, according to signals given by the observer, until the centre line of the vane was intersected by the horizontal wire of the level-telescope.

The height to the centre of the vane, was then read off from graduations on the staff.

The staff was made telescopic, to facilitate the movement of the vane, when above the reach of the staff-holder. Such a staff is required, when an instrument unprovided with a telescope, such as the water-level, is used.

In the early part of the present century, the optical powers of level-telescopes was greatly increased, so that the graduations of the staff could be read by the observer direct.

This plan is now universally adopted in England, and on the Continent.

In America, the writer believes that the vane-staff is still employed.

The levelling instrument is usually entrusted to a subordinate, whose duty it is to adjust it, and to direct the staff-holder to move the vane to its proper position.

The chief surveyor reads the staff, and records the reading, as well as the chain measurements made from station to station.

He is therefore able to keep with the chain-men, supervise them, and select the alignment of the line of section, and the positions of the staff, without the necessity of going backwards and forwards between the instrument and the chain-men.

The observer keeps a subsidiary level-book, which serves to check the surveyor's book.

The surveyor will always do well to cause the staff to be turned round in his presence, as he passes it, and to make sure that it is replaced exactly to the original position on the ground.

If the ground is soft the staff at 'fore' and 'back-sights' should be placed on a stone or flat piece of wood, firmly embedded by the heel of the staff-holder.

When the highest accuracy is required the staff should, at the 'fore' and 'back-sights' respectively, be held on a firmly driven-peg with a rounded nail in its head.

Bench-Marks. In conducting levelling operations of an extensive character it is often desirable to ascertain the level, not merely of points on the ground along a line of section, or in the neighbourhood thereof, but to determine the level of some permanent objects or points, for future reference or for the purpose of checking. Such points of reference are called 'bench-marks.'

The Ordnance surveyors of Great Britain incise on walls, gate-posts, &c., the well known mark or broad arrow (fig. 154).

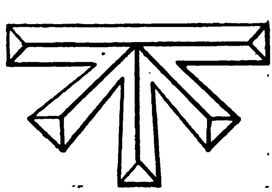


FIG. 154.

These are shown on the maps with their level inscribed.

The centre line of the horizontal V-cut incision is that to which the inscribed level refers, thus ⟵, so that when using an ordnance B.M. the heel of the staff should be held level with this line.

The Ordnance mark is not altogether convenient, since the staff is not supported by anything solid, and may therefore slip.

A better plan prevails in some Continental cities, where the official bench-marks consist of small cast-iron brackets built into the wall, on which the staff can be held.

A tablet with a reference number is provided, for identification, in the bench-mark book, and in some cases the level is inscribed on the tablet.

Private surveyors will rarely find it convenient to cut bench-marks, since they cannot generally afford the time to do so.

It is better to use existing objects on which the staff can be placed, for example, the tops of milestones, thresholds of doors (close to the jamb where there is no wear), window-sills, the pavement of the footpath at a street corner (close to the angle of the house), tops of guard-posts at street corners, cascabels of guns (often planted at street corners), the pavement or curb at the base of a lamp-post, hinges of gates, and the like.

A neat sketch of the object forming the bench-mark should be entered in the level-book, with an arrow indicating the position of the staff. Thus (fig. 155).

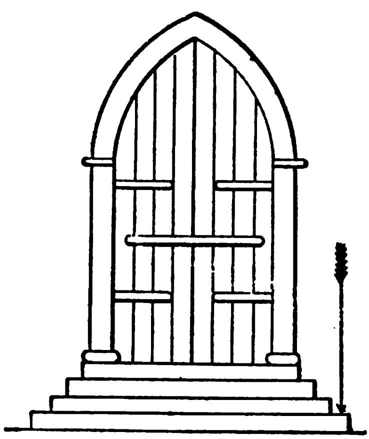


FIG. 155.

In extensive levelling operations, for Public Works, these sketches should be copied into a reference book, with full descriptions of position, and the reduced levels of each. This record will be of great service in future operations.

If, in a place where the British Ordnance bench-mark is in use, the surveyor desires to incise a bench-mark, he should adopt a different form of mark; for example, an arrow without the horizontal bar, thus /|\, in order to prevent confusion.

In levelling a long line of section, it is well to establish numerous bench-marks in convenient positions, say one at every half-mile, and this will greatly facilitate checking.

The surveyor, having completed a line of section, should check his work by levelling back to the B.M. from which he started, using the method of booking described as 'check-levelling.'

If, on closing, a serious error is found, then by comparing the first and second determinations of the levels of the several B.M.'s. en route, he will ascertain the section in which the error has occurred; and he need only verify or re-level that section, and not the whole work.

Datum. Levels are referred to some determined line or plane, of equal altitude, called a 'datum-line' or 'datum-plane.'

The use of the word 'plane' is not strictly correct, for a surface of equal altitude is a spherical surface, as already shown, though the difference between the plane and spherical form may be neglected.

The only natural datum-plane or line, is that of 'Mean Sea Level,' obtained at any place by taking the level of the sea, at intervals of an hour, for a period of a year or more.

The datum-plane of the English Ordnance Survey is supposed to be M.S.L. at Liverpool.

There being some doubt as to the accuracy of the determination of this level, Ordnance datum is practically an arbitrary datum, or plane, so many feet below certain marks.

Parliamentary regulations prohibit the adoption of Ordnance datum for sections of roads, railways, and other Public Works, submitted for sanction.

All levels must be referred to that of some permanent and easily recognised mark.

The datum-line must be definite, as being so many feet above or below some object, such as the step of a church, &c.

It is permissible to use an Ordnance B.M. as the primitive point, and to define the datum-line as being so many feet below the same, quoting, if the surveyor sees fit, the level of the B.M. as given by the Ordnance Survey, but he is not permitted to define the datum-line of a Parliamentary plan as 'Ordnance datum' or so many feet above or below it. Mean sea level is by no means convenient in all cases, as the datum for engineering purposes.

A town may be situated several thousand feet above sea level, and in such a case it would be obviously inconvenient to refer the levels of such a town to M.S.L., owing to the number of figures required, even though the altitude of some point in the town above M.S.L. were accurately known.

In such a case, some arbitrary datum should be adopted.

Again, in the case of sea-port towns, there will be constructions, such as sewers, docks and sea-walls, some parts of which will be above, others below mean sea level.

If mean sea level were adopted as datum, much inconvenience would arise owing to the necessity for inscribing + or − before the levels; and to ascertain differences of level, subtraction or addition must be used, as the case may be, causing confusion, and increased liability to error.

In such cases, a datum-plane, well below any probable structure, should be assumed. In the case of the City of Bombay, General Delisle adopted as a datum, a plane 100 feet below a certain plate of brass, in one of the steps of the Town Hall, a point a little above the average street level.

It was afterwards ascertained, by tidal observation, that mean sea level was 80·30 feet above that datum-plane.

Extreme high water of spring tides was about 89, and extreme low water about 71.

All the levels of the dock-works are therefore positive, coping level being 92·00 feet, say, and foundations 52 and so on.

Even the levels of the harbour-bottom in the fairway, are positive.

This proved a most satisfactory arrangement.

Cross-section Levelling.

In making a section for the construction of a line of road or railway, it often happens that the ground slopes transversely to the line of section; hence, to calculate the volume of excavation and cutting, cross-sections taken at right angles to the central section-line are necessary.

Provision is made for entering these in the sample field-book.

Under the heading 'distance,' there are three columns headed 'left,' 'centre,' and 'right.' In the 'centre' column, the chained distances along the central section line are inscribed, to point out in it what levels upon it were taken, and those at which cross-sections were run.

In the columns 'left' and 'right,' the distances measured at right angles to the line, to level points right and left of it respectively, are inscribed.

FORM OF FIELD-BOOK FOR CROSS-SECTION LEVELLING.

Back.	Inter-mediate.	Fore.	Height of Instrument.	Reduced Levels.	Distances.		
					Left.	Centre.	Right.
..	..	..	..	75·36	..	700	..
6·42	..	..	81·78	..	..	..	..
..	3·11	..	..	78·67	43	700	..
..	2·31	..	..	79·47	61	700	..
..	7·42	..	..	74·36	..	700	30
..	9·30	..	..	72·48	..	800	..
..	4·60	..	..	77·18	50	800	..
..	10·78	..	..	71·08	..	800	75

This reads as follows :—

The level of the ground on the section line at 700 is 78·67; at a point 43 feet from 700 to the left the level is 78·67.

Again, 61 feet left of 700, the level is 79·47; whilst 50 feet right of 700 the level is 74·36.

At 200 on centre line, the level is 72·48; 50 feet left of 800 it is 77·18; whilst 75 right of 800 the level is 71·08.

In very transverse sloping ground the measure of cross-sections with the level is laborious, inasmuch as one or more movements of the level may be required for each section.

In such cases it will often suffice to take the angle of slope with a clinometer.

A staff is provided with the vane fixed at the height of the observer's eye. The observer stands at a point on the line, and the vane-staff is held in a direction at right angles to the line at some distance right or left, so selected as to give a fair average measurement of the general slope.

The angle of 'elevation' or 'depression' of the vane is then observed, and may be inscribed opposite to the central distance in the columns headed 'left' and 'right,' as the case may be, marking elevations and depressions.

These angles may be plotted on the cross-sections by means of a common protractor.

The same form of level-book serves for taking numerous spot levels, over the site of some intended work.

The surveyor sets out some central axis line, driving pegs at intervals of 10 or 20 feet along it.

At each of these points he sets out a cross-section line, at right angles to the axis line, and pegs out distances of 10 or 20 feet along it also.

Having done this, he proceeds to level, taking the levels of as many points as come within the range of the instrument at one position; then he shifts the instrument, and proceeds to take the level of more points, and so on.

It is evident that the form given affords the means of recording the position of the level points, with the minimum amount of writing, and yet with perfect precision.

He may take the levels in any convenient order without any sacrifice of clearness.

Checking Levels.

When a long line of levels has to be run in haste, it is desirable to have some means of checking, which does not involve a second levelling operation.

One method of eliminating mis-readings, is to read the staff twice at points of importance, such as points of change; firstly, with the staff in the ordinary position, zero downwards; secondly, with the staff inverted, zero upwards; or by figuring the staff on both sides, the zero of one being different from that of the other. This double figuring is used on the 10-foot Ordnance Survey pattern. This staff can be read four times by reversing it and then turning it end for end. This cannot be done with the telescopic staff.

In the great levelling operations of India the following plan was adopted.

The staves were graduated on both sides. On one side the divisions were, black on a white ground, on the other red on a black ground. The divisions on one side, say that with white ground, had the zero at the bottom of the staff reading to 10·00 at top, as usual. On the other side, that with the red on black divisions, the graduations started from 5·55, at the bottom of the staff, reading to 15·55 at top.

The readings of the two sides of the staff will differ by the constant number 5·55, if correct.

This plan was well adapted to the case in which it was used, which was to determine, with the highest degree of accuracy, the difference of level between points some 1500 miles apart.

The precautions, taken at every position of the instrument, were numerous. The duplicate readings of the two sides of the staff were, however, not considered to be sufficiently complete, as a check on gross errors of reading, to dispense with the running of a second check-level over the same ground. As observed in Smith and Thuillier's Indian Manual, 'Errors of reading are, however, of such uncommon occurrence, and the results from black and white faces so constantly coincide" (i. e. differ by the correct amounts), "that the observer, in writing down the second result, is liable to be biassed by it, and to imagine that they coincide when in reality there is a difference."

The mistake might not be found out till it was too late to correct it.

For ordinary purposes check-levelling is to be preferred to multiplying the readings, as being the most efficient check.

Bench-marks should be numerous, so that if any error is discovered a short length only will require to be re-levelled.

Moreover, it is found that levelling in opposite directions tends to eliminate a slight cumulative error, which is positive when working in one direction, negative in the other.

When the utmost expedition is required, it will be well to employ a second check-leveller, to level from bench-mark to bench-mark, closely following the chief surveyor.

Any intelligent artisan can be taught to take check-levels in a few days (of course one who can do an addition sum). He will probably do so with as great accuracy and less liability to error than the chief surveyor, for he has nothing to do but to attend to the adjustment of the instrument, and to the accuracy of his readings. His mind is not occupied with such matters as selecting the line, taking cross-sections, controlling the chain-men, and the like. When, therefore, the greatest rapidity of work is desired, there should be no hesitation in incurring the moderate expense of a check-leveller, and one or two staff-men.

Degree of Accuracy in Levelling.

The closing error in levellings, with ordinary care, and reasonably favourable conditions as to weather and ground, should not exceed 0·10 feet per mile, increasing as the square root of the distance in miles. That is to say,

			feet
Permissible error in	1 mile	. . . .	0·10
"	" 4 miles	. . . .	0·20
"	" 9 "	. . . .	0·30
"	" 100 "	. . . .	1·00

If the surveyor levels a certain distance, and then back again by the same, or even by a different route, bench-mark to bench-mark, the permissible closing error is that due to the distance between the extreme points. If, for example, a distance of one mile were levelled, in both directions, the permissible error on closing would be 0·10, although the total distance levelled is two miles.

The reason of the above is, that certain errors tend to accumulate with opposite signs according to the direction in which the operation is performed; for instance, when levelling up-hill the back-staff readings are greater than the fore-staff readings, and any deviation from the vertical position of the staff, tends to augment the back-readings far more than the fore-readings. There is therefore a tendency to a cumulative error inasmuch as the back-readings are, if anything, in excess of their proper value. When levelling down-hill, however, the tendency will be to augment the fore-readings. The result will be that the two errors being in opposite directions, will tend to compensate each other. The final closing error may be quite permissible, yet the determined height of the hill may be materially too great.

In the closed circuit there is not necessarily the same regular tendency to compensation, and consequently a greater error per mile is to be expected.

Points to be attended to, to secure Accuracy in Levelling.

The following are the principal points that have to be attended to in order to secure accuracy in levelling:—

(*a*) Equality and moderate length of back- and fore-sights.

(*b*) Accurate, permanent and temporary, adjustment of the instrument.

(*c*) Verticality of the staff.

(*d*) Solidity of points of support of staff, together with care in preventing the staff from slipping or settling when turned in changing from fore- to back-sight.

(*e*) Solidity of ground on which the instrument stands.

(*f*) Care in reading to avoid clerical errors.

(*g*) Accurate focussing of the telescopes.

Before considering the above points in order, it will be well to indicate the degree of accuracy to be aimed at, in each instance. This is a question which can only be decided by individual judgment, and experience.

The beginner will do well to aim at a standard of accuracy in excess of the requirements of the case. The more accurate his levelling operations, the greater will be the appreciation of his work, for, a series of permanent bench-marks, established, say in taking a section for a road or railway, will be of great use during construction if the levelling be accurate. Work may be set out from them direct without going to the trouble of re-levelling. Again, a series of reliable bench-marks, distributed over a tract of country, will always be valuable, since the levels for projects of improvement, such as roads, sewers, and the like, may be picked up from them and referred to some common datum, without the labour and expense of distant levelling, possibly to some primitive bench-mark. For these reasons, the levelling operations of a Department charged with the construction of public works, should be uniformly conducted with all possible accuracy, and if this rule be adhered to, the district in question will gradually be dotted over with numerous bench-marks, all of which will be useful in the future.

On the other hand, it would be out of place to spend the same amount of time and money, when levelling for a mountain road, as would be essential in the case of a survey for a great irrigation canal, where the fall does not exceed a few inches in the mile. The golden rule is to be as accurate as the available appliances permit. With ordinary care, work good enough for all practical purposes may be done, with instruments of ordinary size and construction, without any unreasonable expenditure of time and money, by careful attention to certain points of detail.

Equality and moderate length of 'fore-' and 'back-sights.'

It has been already shown that by taking equal back- and fore-sights numerous errors are eliminated, namely, parallax, refraction, errors of permanent adjustment, that is to say want of parallelism between the bubble and line of collimation. Provided always that the temporary adjustment of the instrument is correctly made as hereafter mentioned. If the highest accuracy is required, such as when determining the surface slope of a large canal or river, or the difference of level between two sluice sills (cases in which an inch may be of scientific, or legal importance), then accurately chained equal distances should be adopted. It would also be well to adopt one and the same standard distance, throughout the line of levels. If for any reason the two staves have to be placed nearer together than the standard distance, then the instrument should be placed at the apex of an isosceles triangle, the space between the fore- and back-staff forming the base with the standard distance being the sides. By adopting a standard distance the focal distance will be constant, thus avoiding any error due to looseness, or inaccuracy in the draw-tube. The collimation error will also be constant. Lastly, optical errors will be eliminated by the equality of the sights.

In ordinary engineering levelling, it is scarcely practicable, and not essential to use strictly equal fore- and back-sights. Approximated equality should be attempted, as far as physical conditions permit.

One must guard against certain tendencies. In levelling up-hill there is a strong tendency to use longer back-sights than fore-sights, especially if a staff longer than ten feet be used, whilst in levelling down-hill the reverse is the case. This tends to produce an accumulative error, which is not eliminated or reduced by check-levelling. The result is a tendency to make hills too high. In levelling a section in flat or gently undulating country, there is a temptation to make the fore-sights longer than the back-sights, for the surveyor is naturally anxious to survey-in the greatest possible extent from one position of the instrument.

From every point of view, it is therefore desirable to equalise the fore- and back-sights. This may be done by pacing from the instrument to the fore-staff, and then from the same to the new position of the instrument, noting the distance in the margin of the level-book. The beginner is tempted to use too long sights. The longer the sight the greater will be the error introduced, or left uncompensated for inequality.

Length of Sights.

The maximum desirable length of the sight which should be taken, depends upon the optical power, and perfection of the telescope, and to some extent on the eyesight of the observer. The distance of the staff should be no greater than that at which the most minute divisions are distinctly visible. The writer has found that, with an ordinary 12-inch level, about 200 to 250 feet, or three to four chains, is the proper limit of distance between instrument and staff. Of course, relatively unimportant points, such as intermediate readings on a cross-section, may be taken at greater distances, for any error in them is not carried forward.

Long sights really effect little saving in time. After a little practice, the surveyor learns to set up and adjust the level very quickly, and with sights of moderate length he keeps in touch with his chain-men, saving time and fatigue in going forwards and back to read chainments, to align the chain, to take offsets, to select positions for the staff, &c. He also avoids the chance of errors which are very likely to occur when measurements are shouted to him from a distance, not to mention the loss of temper which such proceedings tend to produce, especially as sometimes happens when the surveyor is but imperfectly acquainted with the language of his assistants.

Accurate, permanent and temporary Adjustment of the Instrument.

It is naturally desirable that the permanent adjustments of the instrument should be as perfect as possible. Of these, the essential one is the parallelism of the level and line of collimation. By attending to the precautions above indicated, as to the equality of fore- and back-sights, any error due to imperfect permanent adjustment is eliminated. The writer, therefore, does not recommend the surveyor to torture his level with the view of obtaining perfect permanent adjustment. In so doing there is a chance of breaking or *stripping* a screw, thus perhaps crippling the instrument when far from any means of repair. If the level-bubble will remain, within one or two divisions of the central position, during a complete revolution of the telescope, and if the line of collimation be correct to the nearest hundredth of a foot at a distance of 100 or 150 feet, the writer would let well alone, and trust to the elimination of error, by attention to the precautions above described.

The temporary adjustments of the level consist in bringing the axis of the instrument into a perfectly vertical position, then if the permanent adjustments are perfect the level bubble should remain in the middle of its run throughout one revolution of the instrument, and the line of collimation should be horizontal in every position of the instrument. But all the permanent adjustments are rarely perfectly made, or indeed seldom remain stable. That which places the level and line of collimation at right angles to the vertical axis, rarely remains correct for any length of time. Some flexure takes place, either in carrying or on account of unequal heating by the sun. The writer has rarely found that a level would 'traverse,' that is, that the bubble will remain stationary in all positions of the telescope correctly after a few days' use. After all, correct traversing is a convenience rather than an essential point. The essential is that the line of collimation should be parallel to the bubble. Then it is only necessary to bring the bubble to the middle of its run when reading the staff to secure accuracy. Assuming that the permanent adjustments are approximately correct, this can easily be done by slightly touching one of the 'plate screws,' if a three-screw instrument be used, or two if it be provided with four screws.

It is a good practice to set up the instrument so that if it be a three-screw one arm of the tribrach (or if a four-screw a pair of screws) is in the general line of the work. In any case, *the bubble should be brought exactly to the middle of its run, when reading any important sight, such as a fore- or back-sight.*

Some instruments are provided with a mirror, mounted at an angle of a little less than 45° to the telescope above the level, by means of which the observer can see the level-bubble without moving from his position at the eye-piece. The writer cannot however say that he has found this to be a very satisfactory arrangement. The mirror is necessarily somewhat long, and is a good deal in the way and apt to get knocked about. It is better to step to the side and look directly at the bubble, the hands will then be in convenient position for touching the screws.

The same effect is produced far more effectively in the excellent level invented by Wagner (made by Tepsdorp in Stuttgard, Germany, and sold by Adie in London, *vide* description of instruments). In this an image of the level-tube and bubble is visible in the field of the telescope, to the right or left of the image of the staff. The observer therefore sees simultaneously the staff and the bubble, and by means of a fine micrometer screw he can bring the bubble to the middle of its run, at the instant of reading the staff. The writer has used these instruments for many years, and recommends them strongly, especially for work in hilly countries.

Verticality of Staff. This subject has been already dealt with when describing the use of the levelling staff. The longer the staff, the greater is the error produced by any given deviation from verticality, when reading near the top. Therefore let a staff of moderate length be used, say, ten or twelve feet long at most.

A line of levels may intersect a brook, a section of which is required. When held in the bed of the brook, the top of the staff may be below the level of the line of collimation. Now the level of the brook does not demand great accuracy. It would be waste of time to shift the instrument, merely for the levels of the brook. To meet such cases a square rod exactly five feet long, with small brackets or cleats nailed to the sides with their tops at one, two, three and four feet from the bottom is used.

By holding the staff on one of the cleats, it may be brought within range of the instrument, and the proper number of feet can be added to the staff-reading. This arrangement should not be used in the case of fore- or back-sights.

Solidity of the Ground on which the Instrument Stands.

It is obviously desirable that the instrument should be set up on solid ground. If it settles during the interval between the reading of the fore- and back-sights, an error will be introduced. In soft or boggy ground, the vibrations caused by the movements of the observer will prove sometimes annoying. Firm ground should therefore be selected for the erection of the instrument. If it is absolutely necessary to erect the instrument on soft ground, then the best plan is to support the legs on strong, firmly driven stakes. If the means of so doing are not available, then the only plan is to read the fore- and back-sights in rapid succession, each time bringing the bubble to the middle of its run, the observer shifting his position as little as possible.

Solidity of Support for Staff.

Displacement of the staff, when turning from fore- to back-sight, and taking up a new position of the instrument, is a fertile source of error. If the staff were held on the top of a clod in a ploughed field, it may, when turned round, sink or slip down for a tenth of a foot or more, thus introducing an error that will be carried forward through the rest of the work. In such cases, the staff should be held for the change on a peg, or on a stone firmly stamped down into the ground.

The appliance shown in fig. 156 is useful in such cases. The plate is firmly stamped down, and the staff is held on the spherical boss in the centre.

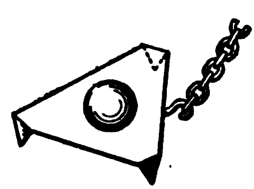


FIG. 156.

In hilly countries, if the staff be held on a sloping face of rock, it is apt to slip down when it is turned. It may rest on one angle only when the act of turning is pretty sure to let it down if the turning is carelessly performed.

The writer makes it a practice to turn the staff with his own hands, as he passes it, in going forward to a new position of the level. The surveyor should never hesitate to take an extra sight as a fore-sight, in order to secure a firm position for the change. Thus in levelling a section for a sewer, along even a good macadamised road, it is prudent to plant the staff for the change on the curb, and not on one of the level points on the road.

Settlement of the staff always tends to make an error in one direction, that is to say to increase the back-sight unduly. The error is therefore cumulative.

Care in Reading, to avoid clerical errors. After a little practice, errors in reading should be rare occurrences. When they do occur, they will generally be of considerable magnitude, such as a whole foot, a mistake of six for nine, or the like. Probably the best plan is to make sure of the whole feet and book them, then look to the bubble and read and enter the hundredths. The only effective way of detecting clerical errors is to re-level from bench-mark to bench-mark.

Accurate Focussing. It is all-important that the cross-wires should be exactly in the common focus of the eye-piece and object-glasses of the telescope. It is quite possible to adjust the object-glass so as to give a distinct image of the staff, even if the eye-piece be so far out of focus with regard to the cross-hairs that they are quite invisible.

With instruments of ordinary quality, it is also possible to obtain a distinct image of the staff, and a fairly distinct image of the cross-hairs, although the latter are considerably out of the common focus. When this is the case the image of the wires will shift with regard to that of the staff, when the eye is moved up or down or to the right or left. If, under these conditions, the staff be read with the centre of the eye, above or below the centre of the eye-piece, an error of several hundredths of a foot may be made. The eye-piece should be adjusted so that a distinct view of the cross-hairs is obtained and so that their image appears to be fixed on the staff, notwithstanding a movement of the head, as described in the chapter on instruments and their adjustments.

Plotting Sections. Many methods are in vogue for plotting sections. The following form of section is one that the writer finds convenient, and is plotted from the field-book on p. 262. The base line at the bottom is the datum-line or a line at some round number of feet above it. Beneath it are other lines forming columns in which are inscribed the following numbers (fig. 157):—

Distances, reduced levels: copied from field-book.

Formation level in the case of a road or railway, or invert level in the case of a sewer, calculated from the known points A, C, etc.

Height of bank, or depth of cutting.

The formation level is put on after the section is plotted, and the heights of bank and cutting are then computed and inscribed, and are then available for the computation of quantities.

Exaggeration of Vertical Scales. It is usual to plot the vertical heights, to a larger scale than the horizontal distance. The object of this is to give a clear idea of the variations of level and slope of the ground. If a section of a piece of ordinarily undulating ground were plotted to a natural scale, that is, if the same scale were used to plot both the vertical heights and the horizontal distances, the resulting section would be merely a slightly undulating line, varying little from a straight line; and giving no idea of the height of embankments or of the depths of cuttings. Another reason for increasing the vertical scale is that the measurement of the verticals is more important than that of the horizontal dimensions. For example, in the case of an embankment with sloping sides, an error of one foot in scaling a length of

one hundred feet, only affects the contents of the bank by 1 per cent. But the area of the cross section of a bank varies, roughly as the square of its height, consequently if the bank were ten feet high, an error of one foot in scaling this dimension would introduce an error of about 20 per cent. in its area. Generally, accuracy in vertical dimensions is more necessary than in horizontal dimensions, because the latter can be, for the most part, obtained direct from the inscribed figures. The ratio of vertical exaggeration must depend upon the formation of the ground, and on the purpose for which the section is taken. For sewer-sections an exaggeration of 10 to 1 is convenient, and the scales may be $\frac{1}{24}$ and $\frac{1}{240}$ respectively, corresponding to 2 feet and 20 feet to one inch on the duodecimal system. Horizontal measurements may be read to 0·1 foot, and vertical to 0·01 foot approximately.

Taking out Areas from Exaggerated Sections. Areas may be calculated from an exaggerated section, by equalising the section line in the manner described in the case of offsets in horizontal measurements, either graphically by the computing scale or by the planimeter.

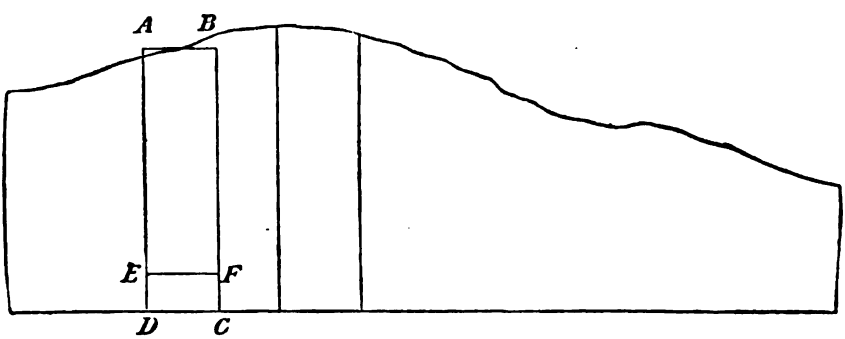


FIG. 158.

The area of the strip A B C D, the ratio of vertical exaggeration being *n* to 1, as obtained by multiplying the height and breadth, both measured off with the horizontal scale, would be exactly *n* times too great. The same would be the case for all other strips, and for the figure which they jointly constitute. To obtain therefore the correct area of a section, it is only necessary to obtain its area by any method, using the horizontal scale for all dimensions, and then dividing the result by the ratio of exaggeration.

CONTOURING.

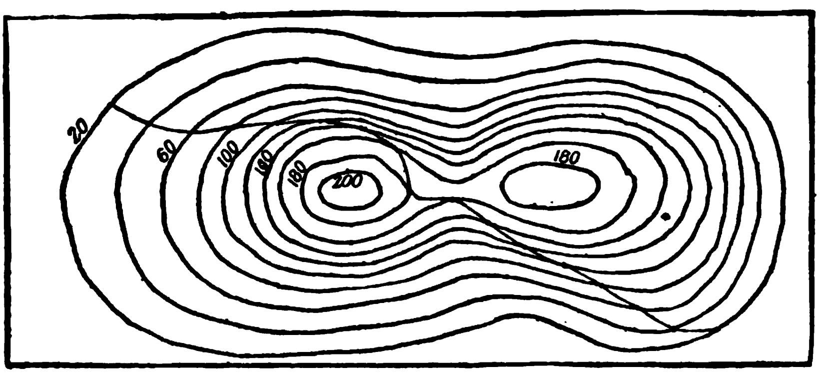
Definition. Contours are curved lines on a plan, which represent the interception of planes of equal altitudes at fixed vertical distances apart, with the undulating features of the country, district, or site surveyed.

Use of Contours. When a plan is contoured, it conveys an accurate idea of the features of the area surveyed, showing the exact position of the hills, valleys, watersheds and watercourses, and on such a plan it is possible to lay out the best general direction for a road or railroad between any two points. Such plans are also most valuable from a military point of view, as indicating the defensive or offensive capabilities of the district. Contoured sites of a more limited area are often required for the location of buildings, reservoirs, fortifications, &c.

Illustration by Model. Suppose now that a model of a piece of ground is prepared constructed to scale, and that it is placed in a tank or cistern with a level bottom (fig. 159).

Suppose further that water is poured into the tank to a depth of say one inch, representing according to the vertical scale of the model an altitude of 20 feet. Then let the water margin be traced on the model (fig. 159). Then let more

Section showing successive water levels.



Plan showing successive "contours."

FIG. 159.

water be poured in to a depth of two inches and another curve traced; and so on till the model is completely submerged. The successive curves or 'contours,' when projected on a plan, give an accurate idea of the conformation of such a model or the country it represents, provided always that the interval between them is sufficiently small. The interval between two contours divided by the distance between them, is the tangent of the general slope of the ground, or conversely by dividing the distance by the interval the gradient is obtained, expressed in the common way of one in thirty, &c., or inversely by dividing the distance by the interval the gradient is obtained, expressed in the common way of one in thirty. Supposing that it were desired to lay out a road on the ground delineated on the sketch (fig. 159), having a gradient of say 1 in 50. Then it is only necessary to take the distance 50 × 20 = 1000 in the dividers and step it from contour to contour, to obtain the general direction of the road.

Distance between Contours.

When contouring large areas, the distance between the planes of equal altitude may vary between 20 feet and 100 feet, according to the scale to which the finished plan is to be drawn. Such intervals should be inversely to the scale of the plan, so that if twenty feet intervals be adopted for a 6-inch scale, forty should be used for 3-inch scale, sixty feet for a 2-inch scale, and so on for smaller scales.

For small areas, such as building sites, an interval of from one foot to three or five feet, according to circumstances is usually adopted.

Method of determining and Surveying Contours for Large Areas.

Various methods are adopted for determining contour lines, depending on the accuracy which is desired; and the fact must never be lost sight of, that the accuracy of a contour line depends on the number of points in it which have been carefully fixed in altitude and position.

When contouring large areas surveyed to small scales, it suffices to fix the positions of numerous points in altitude by either vertical angles or levelling, and then to interpolate the contour lines by eye. The hilly districts of Scotland and Wales were contoured in this way when the Ordnance Surveys were made.

When greater accuracy was required, such as the more undulating slopes of the English counties, each contour was run with a level and contouring staff, the points so fixed being located from the details already surveyed, or from auxiliary chain lines when necessary.

Method of Contouring Small Areas.

The following three methods of contouring small areas or sites are recommended, the formation of the ground leading to a decision as to which should be adopted in any given case :—

1. By running the contours with a level, and surveying in the points fixed by pickets either from fixed details or with the chain or tape, with the aid of a compass or theodolite, as found most convenient. The heights of the terminal pickets at one or both ends of such contours should be determined by levels run from a known bench-mark.

2. By covering the area with a series of imaginary squares, pickets being driven at the corners and their heights determined with a level. At least one of the sides of the rectangular area so divided up into squares, should be fixed in position by reference to known points, previously surveyed, so that the position in space of the area may be known. From the spot levels so obtained, the positions of contour lines at fixed intervals can be interpolated. Having decided on the initial line from which to start work, and having noted its direction with reference to known points, pickets can be driven at intervals of say five, ten or twenty feet apart (*vide* fig. 160). Perpendiculars are then set off at the first and last picket of the series, and pickets are driven along them at the intervals adopted. The intermediate pickets are then lined in, and finally the level of each picket is determined.

3. By laying out radial lines from a central point of the site, and driving

pickets at the required vertical intervals, afterwards measuring the distances from picket to picket on each radial.

Set up a theodolite at the point A (fig. 161), whose position has been fixed by measurements to known points, then drive pickets on the bearings selected, at

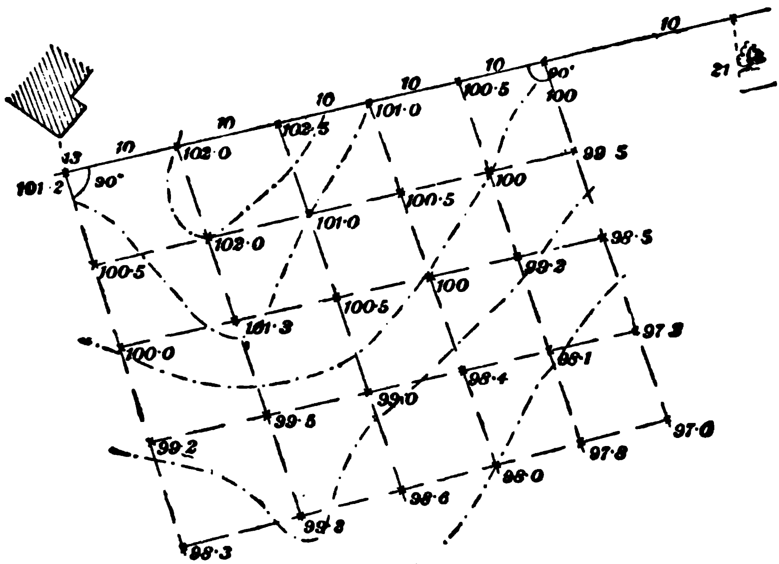


FIG 160

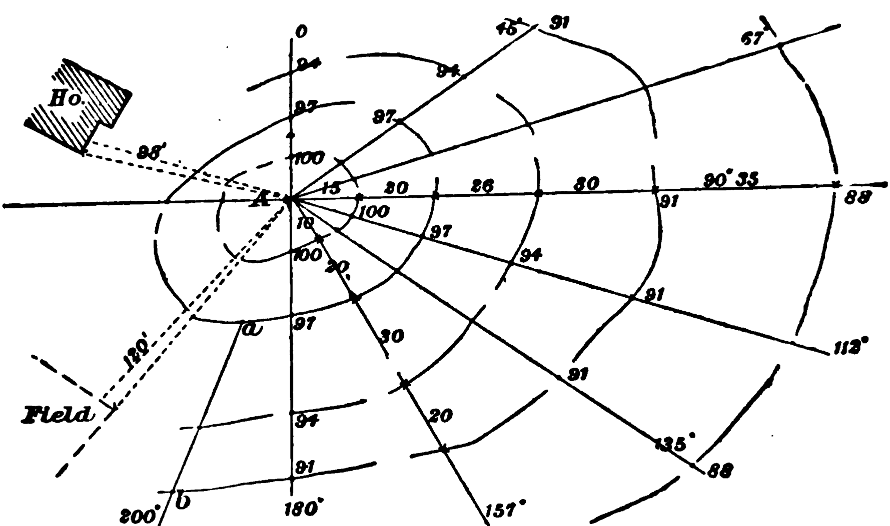


FIG. 161

the desired vertical intervals, as determined by a level and contouring staff (the theodolite level will often suffice), and note the distances from picket to picket on each radial or bearing.

The above method is most applicable for the contouring of a small area on the top of a hill. As the distance between the radials increases, new bearings can be interpolated as at *a b* (fig. 161).

Contouring Staff.

A contouring staff consists of a long square staff, figured at intervals of one foot and fitted with two sliding sight-vanes, which can be fixed at the desired points by a thumb-screw.

The Plane-Table in Contouring.

The plane-table, with the telemeter sight-rule (*vide* plane-tabling) lends itself admirably to contouring, especially when used in conjunction with the level. The telescope of the sight-rule may be used to fix the contour points. This is not, however, a convenient plan, for the sight-rule does not traverse easily on the paper. In the opinion of the writer, it is better to use a level to determine the contour points, leaving to the plane-table the duty of registering their position on the plan. The work is much facilitated by the employment of two observers, one for the level, and the other for the plane-table. The leveller fixes the points on the contour by a procedure similar to that described, with the exception that there is no necessity for planting pickets. The leveller determines the contour points, and the plane-tabler fixes them at once on the plan by drawing a ray and measuring the subtense on the same staff that is used by the leveller. The work proceeds with great rapidity, and this system has the great merit that the contours are sketched in, as it were, from nature. Minor accidents of the ground can be sketched in. It is also possible for the plane-tabler to sketch in by eye, with no small accuracy, contours intermediate to those determined instrumentally, thus preparing a very fine topographical plan, with great rapidity and *precision*.

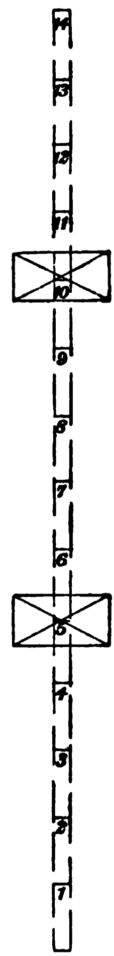


FIG. 162

As to the Practical Utility of Contouring.

There is no doubt as to the general utility of a contoured plan or map, provided always that the contours are traced at frequent intervals. Unfortunately the cost of tracing contours is so great that it is rarely possible to adopt a sufficiently close interval, to be of real service, in any but very precipitous ground. In tropical forests the tracing of contours would involve a prohibitory expenditure of time and money. It is rarely possible, as regards cost, to trace contours at such frequent intervals as to be of much use in engineering construction. The usual interval is from 50 to 100 feet. Such contours serve no doubt as an *excellent* guide for the selection of the general alignment of a road or a railway. Between them many features of ground may, however, occur which will necessitate material deviation from the preliminary lines selected from the map, in the office. It is certainly out of place to contour a whole country at so close intervals as to make the map available for the design of public works, in any part thereof, when as a fact works will be required in but few and limited portions thereof. All that can be done is to trace a few contours at considerable intervals such as 50 feet between them, many features may occur which therefore escape delineation, though they may be important from an engineering point of view.

For constructional purposes the desirability of tracing contour lines is questionable. As a general rule the writer believes that it is preferable to obtain the levels of a great number of points (spot-levels) distributed, at regular intervals, over the plot of ground. Some time is lost in finding the position of a contour-point, as the staff has to be shifted about until the exact leading is obtained. Whereas with a spot-level the staff is put up at the point, read, the reading recorded the operation is finished, as far as that point is concerned. One can therefore determine more spot-levels in a given time than one can fix contour points. Again, the determination of spot-levels involves no arithmetical calculation in the field, a proceeding always to be avoided.

To take the levels over a plot of land, for the construction of some work thereon, the best plan appears therefore to be to range out a base or axis line through the middle thereof, and to take numerous equidistant cross-sections holding the staff at equidistant points along the same. The levels may be recorded by the system described in connection with cross-sections. A clear record is obtained, and the levels of the several points may be worked out at leisure, in the office. If it be desired to survey a valley for the purpose of constructing a reservoir, the best plan is to range an axis line following, roughly, the valley line, and to take cross-sections at right angles to this line. The cross-sections, when plotted, moreover are directly available for computing the contents of the reservoir.

APPENDICES.

APPENDIX A.

ANOTHER great improvement in the construction of modern transit theodolites is the use of microscopes, wherewith to read the horizontal and vertical arcs, in place of the verniers and magnifying glasses fitted to the old pattern instruments.

It is much less fatiguing to the eye to read with microscopes: and with these the arcs of 5-inch and 6-inch instruments can be read to 6 seconds, as against 10 seconds with verniers.

The degrees on the arcs are divided into six parts, and the micrometer drum is divided into 100 parts.

APPENDIX B.

ADJUSTMENT OF HORIZONTAL AXIS OF ROTATION,

I.E. TO MAKE THE HORIZONTAL AXIS OF ROTATION TRULY HORIZONTAL.

Method 1.—Without a Striding Level.

THE vertical axis of rotation having been made truly vertical, turn the telescope on to some point at a considerable altitude and bisect it with the cross wires. Now turn the telescope to a nearly horizontal position, and note or mark some object which the cross wires now bisect. Next, change the face of the instrument by rotating it through 180°, and repeat above operation. If the object bisected by the cross wires when the telescope is brought to the horizontal position the second time does not coincide *very nearly* with that first bisected the horizontal axis is not *truly* horizontal. Select, *or mark*, a point midway between the objects bisected in the two positions of the telescope when nearly horizontal, and, by means of the clamp-screws, adjust the split, or movable Y, until the cross hairs will bisect both the object in altitude and the last selected point when nearly in a horizontal position. When this can be done it is obvious that the line of culmination travels in the plane of a truly vertical circle, and that the axis of rotation, or horizontal axis of the instrument, must be truly horizontal.

Method 2.—With a Striding Level.

Firstly, *to adjust the level.*—Place it in position on the trunnions of the horizontal axis, and turn the latter over a foot-screw. Bring the bubble of the striding level to the centre of its run by means of the foot-screw only; then reverse the striding level on the trunnions, and, should the bubble in its new position run out of centre, correct one half of its displacement by the foot-screw and one half by means of the screws acting on the split Y of the striding bubble, thereby lengthening or shortening that leg and raising or lowering that end of the bubble as required. Repeat till the bubble remains in the centre of its run in both positions.

Secondly, *to adjust the length of the* A *support*, which is fitted with the means of doing so.—Rotate the instrument through 180° on its vertical axis, and, should the bubble be displaced, correct *half* the displacement by means of the foot-screw and *half* by means of the adjustable A, either by tightening or loosening the split Y or by raising or lowering a movable Y, as the case may be. Repeat till perfect.

N.B.—The above adjustment of the horizontal axis is only necessary when observing to an object, such as an elevated peak or a star, at a very considerable altitude, hence the means of making this adjustment are only provided with the transit pattern of theodolite, and not with the Everest or Y patterns.

APPENDIX C.

Magnetic Elements.

(By permission of the Proprietors of 'Whitaker's Almanack.')

The following table of mean magnetic elements is derived from the observations made at Greenwich in the respective years, and apply to Greenwich only.

The diurnal variation of the magnetic declination at Greenwich is about 12′ in summer and 7′ in winter. The needle occupies its mean position about 10*h*. a.m., and again about 6*h*. p.m., throughout the year. It reaches its most westerly position about 2*h*. p.m., and its most easterly position during the night or early morning, according to the season of the year. The inclination or dip also varies, from hour to hour, in a similar manner to the declination. The declination and dip are also subject to secular variations, the duration of which is not accurately known. Accidental perturbations, due to magnetic storms, affect the needles. These variations in the position of the magnets occur with great suddenness, deflecting the needle right and left with great rapidity, almost like ordinary telegraphic signalling, and are generally coincident with the passage of great outbursts of sunspots across the sun's central meridian.

Year.	Mean Magnetic Declination at Greenwich West.	Horizontal Magnetic Force in C. G. S. Units at Greenwich.	Mean Inclination or Dip of Needle at Greenwich.	Year.	Mean Magnetic Declination at Greenwich West.	Horizontal Magnetic Force in C. G. S. Units at Greenwich.	Mean Inclination or Dip of Needle at Greenwich.
	° ′		° ′		° ′		° ′
1889	17 34·9	·1821	67 24·9	1896	16 52·0	·1833	67 15·0
1890	17 28·6	·1823	67 22·9	1897	16 46·0	·1836	67 13·0
1891	17 23·4	·1825	67 21·4	1898	16 39·0	·1837	67 12·0
1892	17 17·4	·1826	67 19·8	1899	16 34·2	·1842	67 10·2
1893	17 11·4	·1829	67 17·8	1900	..	..	..
1894	17 4·6	·1829	67 17·3	1901	..	..	..
1895	16 57·0	·1832	67 14·7				

INDEX.

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